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Submitted to

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(Q1) Answer the following short questions briefly.

Part.

i) Write down the Maxwell's equations which comprise the fundamental tenets of electromagnetic theory. Explain.

Ans) Maxwell's equations describe how electric charges and electric currents create electric and magnetic fields. Further, they describe how an electric field can generate a magnetic field and vice versa. The first equation allows you to calculate the electric charge.

* Maxwell's equation. Differential form

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

* Integral form

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{a} = \boxed{- \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}}$$

$$\oint \vec{B} \cdot d\vec{a} = 0$$

$$\oint \vec{B} \cdot d\vec{a} = \mu_0 I_{enc} + \boxed{\mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a}}$$

⇒ with the publication of A dynamical theory of the electromagnetic field in 1865 Maxwell demonstrated that electric and magnetic field travel through space as waves moving at the speed of light. He proposed that light is an undulation in the same medium that cause of electric and magnetic phenomena.

Part

Pg#4

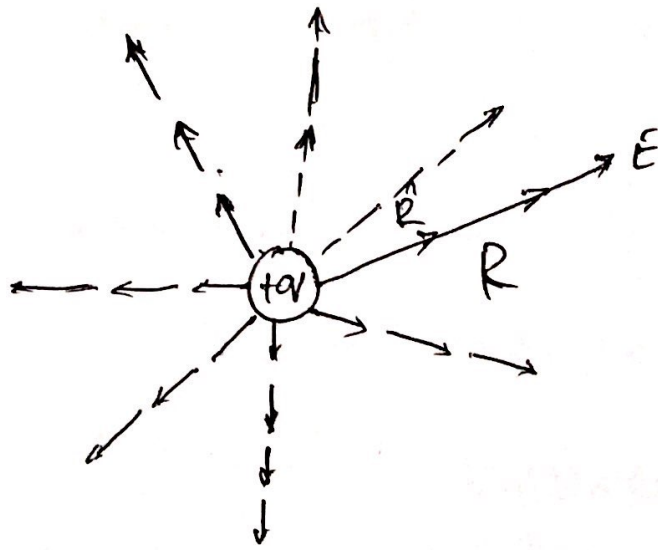
ii) Explain the Coulomb's Law?
Also state it with help of expressions?

(Ans) Coulomb's Law states that

An isolated charge q induces an electric field E at every point in space, and at any specific point P , E is given by.

$$E = \vec{R} \frac{q}{4\pi\epsilon R^2} \text{ V/m}$$

⇒ Where R is a unit vector pointing from q to P (in the dir) R is the distance between them and ϵ is the electrical permittivity of the medium containing the observation point. P



(2) In the presence of electric field E , at a given point in space which may due to a single or a distribution of many charges the force acting on a test charge q : when the charge is placed at that point is given

$$\vec{F} = q' \vec{E} \quad (N)$$

\Rightarrow When F measured in newtons (N) and q in Coulombs (C) the unit of E is (N/C) which is same as volt per meter (V/m)

\Rightarrow For a material with electrical permittivity the electric field quantities D and E are related by $D = \epsilon \vec{E}$

Part

(iii) What is the difference between convection and conduction currents?

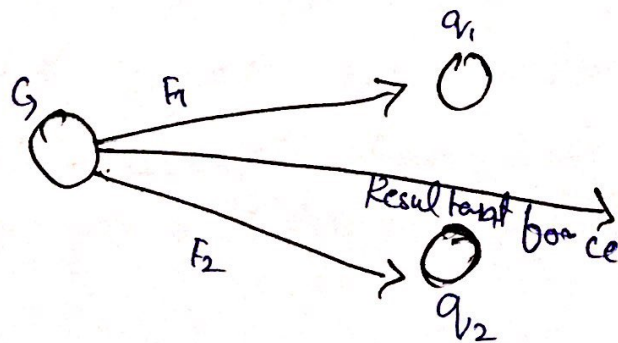
(Ans) Convection and Conduction Difference:

⇒ The difference between conduction and convection is that in convection heat is actually transferred by moving particles as when a fan is used to move heat from one place to another by blowing air.

⇒ In conduction heat is transferred through solids by means of the vibrating molecules in a substance.

Part
(iv) State the principle of (linear superposition) as it applies to the electric field due to a distribution of electric charges.

(Ans) The principle of superposition states that every charge in space creates an electric field at point that independent of the presence of other charges in that medium. The resultant electric field is a vector sum of electric field due to individual charges.



(Part v) what is Biot-savart Law's
Also state it with help of
expressions.

(Ans) Biot - savart Law:-

The Biot-savart law is an equation describing the magnetic field generated by a constant electric current. It relates the magnetic field to the magnitude, direction, length, and proximity of electric current.

Biot savart law is consistent with both Ampere's circuital law and Gauss's theorem

The Biot savart law is fundamental magnetostatics playing a role similar to that of Coulomb's law in electrostatics

* The Biot savart law statement and Derivation

The Biot - savart law can be stated as

$$\text{Hence, } dB \propto \frac{Idl \sin \theta}{r^2} \quad \text{or } dB = k \frac{Idl \sin \theta}{r^2}$$

where, k is a constant, depending upon the magnetic properties of the medium and system

$$k = \frac{\mu_0 \mu_r}{4\pi}$$

Q2) Part (a) A 2mm diameter copper wire with conductivity of $5.8 \times 10^9 \text{ S/m}$ and electrons $\dots \dots$ $\dots \dots$ $(\text{m}^2/\text{V}\cdot\text{s})$ $\dots \dots$ the free electrons.

(Ans) Given data

The density of diameter copper wire conductivity of $5.8 \times 10^9 \text{ S/m}$ and mobility of $0.0032 \text{ (m}^2/\text{V}\cdot\text{s)}$

We know carries concentration (n)

$$\frac{\text{Avogadro number} \times \text{Density}}{\text{Atomic weight}}$$

The conductivity of copper is

$$n = 84.6 \times 10^{25} \text{ m}^{-3}$$

The electrical conductivity

$$\sigma = \frac{1}{\rho} = \frac{1}{1.73 \times 10^8}$$

We know

$$\sigma = \frac{n e^2}{m}$$

Average time collision

$$\tau = \frac{m}{n e^2}$$

Q2
Part

(b) The x - y plane is a charge free boundary separating two dielectric media with permittivities ϵ_1 and ϵ_2 as shown in fig 4-19

If the electric field in medium 1 is $E_1 = x E_{1x}$.

$y E_{1y} + z E_{1z}$ find

(a) the electric field E_2 in medium 2 and

(b) the angles α_1 and α_2

(Ans) free interface the tangential components of E and the normal components of D are continuous

$$E_{2x} = E_{1x} \quad E_{2y} = E_{1y}$$

and

$$D_{1z} = D_{2z} \quad \text{or} \quad \epsilon_2 E_{2z} = \epsilon_1 E_{1z}$$

Hence

$$E_2 = x E_{2x} + y E_{2y} + z \frac{\epsilon_1}{\epsilon_2} E_{1z}$$

The tangential components of E_1 and E_2

$$E_{1t} = \sqrt{E_{1x}^2 + E_{1y}^2} \text{ and}$$

$$E_{2t} = \sqrt{E_{2x}^2 + E_{2y}^2} \text{ the angles}$$

θ_1 and θ_2 are then given by

$$\tan \theta_1 = \frac{E_{1t}}{E_{1n}} = \frac{\sqrt{E_{1x}^2 + E_{1y}^2}}{E_{1n}}$$

$$\tan \theta_2 = \frac{E_{2t}}{E_{2n}} = \frac{\sqrt{E_{2x}^2 + E_{2y}^2}}{E_{2n}}$$

$$= \frac{\sqrt{E_{1x}^2 + E_{1y}^2}}{(E_1 / E_2) E_{1n}}$$

and the law angles are related by

$$\frac{\tan \theta_2}{\tan \theta_1} = \frac{E_2}{E_1}$$

(Q3) The semi circular conductor
Part (a) shown in Fig.
 wire and b
 the force F_2 on the curved

* Solution :- (a) To evaluate F_1 consider that the straight section of the circuit is of length $2r$ and its current flows along the $+x$ direction. Application of Eq (5.12) with $l = \hat{x} 2r$ gives

$$F_1 = \hat{x} (2/r) \times \hat{y} B_0 = \hat{x} 2/r B_0 (N)$$

The \hat{z} direction in Fig 5.4 is out of the page.

(b) To evaluate F_2 consider a segment of differential length dl on the curved part of the circle. The direction of dl is chosen to coincide with the direction of the current. Since dl and B are the both in the xy plane.

Their cross product $d\mathbf{l} \times \mathbf{B}$ point in the negative \hat{z} direction and the magnitude of $d\mathbf{l} \times \mathbf{B}$ is proportional to $\sin \phi$ - where ϕ is the angle between $d\mathbf{l}$ and \mathbf{B} . Moreover, the magnitude of $d\mathbf{l}$ is $d\mathbf{l} = r d\phi$. Hence

$$\begin{aligned} \vec{F}_2 &= I \int_{\phi=0}^{\pi} d\mathbf{l} \times \mathbf{B} \\ &= -\vec{z} I \int_{\phi=0}^{\pi} r B_0 \sin \phi d\phi = -\vec{z} 2 I r B_0 \text{ (N)} \end{aligned}$$

\Rightarrow The \hat{z} -direction of the force acting on the curved part of the conductor is into the page. We note that $\vec{F}_2 = -\vec{F}_1$ implying that no net force act on the closed loop, although opposing forces act on it two sections.

Q3 part (B) A free standing linear conductor of length l carries current I in the $x-y$ plane. distance r in the

Solution:

The differential length vector $d\vec{l} = dz \hat{z}$
 Hence, $d\vec{l} \times \vec{R} = d\vec{l} \times (R \hat{r}) = dl R \sin\theta \hat{\phi}$ where ϕ is the azimuth direction and θ is the angle between $d\vec{l}$ and \vec{R}

$$H = \frac{1}{4\pi} \int \frac{d\vec{l} \times \vec{R}}{R^2} = \phi \frac{1}{4\pi} \int \frac{\sin\theta}{R^2}$$

Both R and θ are dependent on the integration variable z but the radial distance r is not

$$R = r \csc\theta \quad (5.26a)$$

$$z = -r \cot\theta \quad (5.26b)$$

$$dz = r \csc^2\theta d\theta \quad (5.26c)$$

Where θ_1 and θ_2 are the limited angle at $z = -l/2$ $z = l/2$ respectively from the right triangle in Fig it follows that

$$\cos\theta_2 = -\cos\theta_1 = \frac{r/2}{\sqrt{r^2 + (l/2)^2}}$$

Hence -

$$B = \mu_0 I \hat{\phi} \frac{4\pi r^2}{2\pi r \sqrt{r^2 + l^2}}$$

$$B = \mu_0 H = \phi \frac{\mu_0 I L}{2\pi r \sqrt{4r^2 + l^2}}$$

12g #15

For an infinitely long wire with $l \gg r$

$$B = \phi \frac{\mu_0 I}{2\pi r} \text{ Ans.}$$

(Q4)

Part (a)

Explain the Faraday's Law?

Also explain in brief its differential and integral forms.

(Ans) Faraday's Law states that the absolute value or magnitude of the circulation of the electric field E around a closed loop is equal to the rate change of the magnetic flux through area enclosed by the loop.

The equation below expresses Faraday's Law in mathematical form.

Starting with differential form of Faraday's

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

It is a local statement. we find integrate on both sides about on arbitrary surface.

$$\sum \int_{\Sigma} \vec{\nabla} \times \vec{E} \cdot d\vec{a} = - \int_{\Sigma} \frac{\partial B}{\partial t} \cdot d\vec{a}$$

on the left hand side of the above equation we use Stokes theorem.

$$\int_{\Sigma} \vec{\nabla} \times \vec{E} \cdot d\vec{a} = \oint_{\partial \Sigma} \vec{E} \cdot d\vec{l} \text{ where } \partial \Sigma \text{ is the boundary of the surface}$$

on the right hand side. we argue that the surface doesn't change with time, therefore the derivation sign can be moved outside of the integral sign in addition, the integral is ~~new~~ only a function of time therefore it is justified to use the total derivative symbol so we obtain

$$\oint_{\partial \Sigma} \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_{\Sigma} B \cdot d\vec{a}$$

which is integral form of the Faraday's Law.

(Q4) Determine voltages v_1 and v_2 across the 2Ω and 4Ω resistors shown in Fig. The loop is located in the xy plane. Its area is 4m^2 . The magnetic flux density is $B = -20.3t$ (T) and the internal resistance of the wire may be ignored.

(Ans) The flux flowing through the loop is

$$\Phi = \int B \cdot d\vec{s} = \int (-20.3t) \cdot \vec{z} \cdot d\vec{s}$$

$$= -0.3t \times 4 = -1.2t \text{ (wb)}$$

and the corresponding transformer emf is

$$V_{\text{emb}} = -\frac{d\Phi}{dt} = 1.2 \text{ (V)}$$

