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NAME :: Qaiser Siddique

ID :- "7863"

SECTION :- "B"

Smster :: Summer (2020)

Subject :- Differential eq.

Submmited To ::

Dr Shumaila Mazhar

Q NO:1

⇒ Find the fourier Series representat-ion If: $f(t) = 1+t \quad , -\pi \le t \le \pi.$

⇒ **Solution:-**

$f(t) = 1+t \quad , -\pi \le t \le \pi$

Here we use the formula.

$f(x) = a_0 + \sum_{n=1} a_n \cos t + \sum_{n=1} b_n \sin t \rightarrow eq ①.$

$a_0 = \frac{1}{2\pi} \int_{\pi}^{\pi} f(t)\, dt.$

$a_0 = \frac{1}{2\pi} \int_{\pi}^{\pi} (1+t)\, dt$

$a_0 = \frac{1}{2\pi} \left| t + \frac{t}{2} \right|_{-\pi}^{\pi}$

$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi}{2} - \left(-\frac{\pi^2}{2} \right) \right)$

$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)$

$a = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)\, dt$$

$$a_n = \frac{1}{\pi} \left[(1+t) \frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{\sin nt}{n\, dt} (1+t) \right) \right.$$

$$a_n = \frac{1}{\pi} \left[(1+t) \frac{\sin nt}{n} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right.$$

$$a_n = \frac{-1}{n^2\pi} (\cos n\pi - \cos n(-\pi))$$

$$a_n = \frac{-1}{n^2\pi} (-1-(-1))$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) \sin nt\, dt$$

$$b_n = \frac{1}{\pi} \int (1+t) \int_{-\pi}^{\pi} \sin nt - \int \left(\sin nt \cdot \frac{d}{dt} (1+t) \right)$$

$$b_n = \frac{1}{\pi} \left(\frac{(1+t)(-\cos nt)}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{-\cos t}{n} (1) \right) \right)$$

$$b_n = \frac{1}{\pi} \left(\frac{-(1+t)(\cos nt)}{n} \Big|_{-\pi}^{\pi} + \left(\frac{\sin nx}{n^2} \Big|_{-\pi}^{\pi} \right) \right)$$

$$b_n = \frac{-1}{n\pi} \left((1+\pi)(\cos n\pi) - ((1+(-\pi))(\cos n\pi) \right.$$

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$$b_n = \frac{-1}{n\pi}\left(\cos n\pi + \pi\cos n\pi - \cos n\pi + \pi\cos n\pi\right)$$

$$b_n = \frac{-1}{n\pi}\left(2\pi\cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

So, eq becomes:

$$f(x) = \frac{1}{2\pi}(2\pi+\pi^2) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx.$$

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Q.No.2

⇒ calculate the characteristics of 7 eigen value of system. where A is given by

Solution :-

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values = ?

⇒ Step # 1 :

We have;

$(A - \lambda I)\, x = 0$ ∴ A is a given matrix.

→ Step # 2 :

We have; the characteristic equation is given by.

$$|A - \lambda I| = 0.$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 0 & 1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{bmatrix} = 0.$$

⇒ Step # 3 :-

$\lambda^3 - |\text{Sum of daignal elmt}|\, \lambda^2 + |\text{Sum of diagonal (minors)}|\, \lambda - |A| = 0$ — B

Sum of Diagonal elements: $1+1+2=4$

Sum of Diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= \cancel{-6+2}\ -3$

By putting values in eq B.

$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0$ — (C)

$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

⇒ By putting values in eq (C)

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$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0.$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0.$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0.$$

$\Rightarrow$ using quadratic equation.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \qquad \begin{matrix} a = 1 \\ b = -4 \\ c = -3. \end{matrix}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}.$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2}. \Rightarrow \frac{4 \pm \sqrt{28}}{2}.$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

We have eigen values.

$$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2}\right)$$

$\Rightarrow$ Required Solution.

Q No. 3

Solve the following system of linear equation.

$$
\begin{aligned}
5x + 0 + 4z + 2m &= 3 \\
x - y + 2z + m &= 1 \\
4x + y + 2z + 0 &= 1 \\
x + y + z + m &= 0
\end{aligned}
$$

Solution.

$$
\left[\begin{array}{cccc|c}
5 & 0 & 4 & 2 & 3 \\
1 & 1 & 2 & 1 & 1 \\
4 & 1 & 2 & 0 & 1 \\
1 & 1 & 1 & 1 & 0
\end{array}\right] \quad R_4 \leftarrow R_2
$$

$$
\left[\begin{array}{cccc|c}
5 & 0 & 4 & 2 & 3 \\
1 & -1 & 2 & 1 & 1 \\
4 & 1 & 2 & 0 & 1 \\
0 & 2 & -1 & 0 & -1
\end{array}\right]
$$

$$
\left[\begin{array}{cccc|c}
5 & 0 & 4 & 2 & 3 \\
1 & -1 & 2 & 1 & 1 \\
0 & -1 & +6/5 & +4/5 & 3/5 \\
0 & 2 & -1 & 0 & -1
\end{array}\right] \quad \leftarrow -1/5 \times R_3
$$

$$
\left[\begin{array}{cccc|c}
5 & 0 & 4 & 2 & 3 \\
1 & -1 & 2 & 1 & 1 \\
0 & -1 & 6/5 & 4/5 & 3/5 \\
0 & 0 & 7/5 & 8/5 & 1/5
\end{array}\right] \quad \leftarrow 5 \times R_3 \text{ and } 5 \times R_1
$$

$$
\left[\begin{array}{cccc|c}
5 & 0 & 4 & 2 & 3 \\
1 & -1 & 2 & 1 & 1 \\
0 & -1 & 6/5 & 4/5 & 3 \\
0 & 0 & 7/5 & 8/5 & 1
\end{array}\right] \quad 5R_3 \text{ and } 5R_4
$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1/5 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2 \times 5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad R_2 \times -5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 5 & 26/21 \\ 0 & 0 & 1 & 0 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$\frac{5}{4}$ [illegible]

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \frac{5}{4} \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 13/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, w) = \left(3/4,\ 31/21,\ -11/21,\ 1/3\right)$$

$$x = 3/4$$

$$y = 31/21$$

$$z = -11/21$$

$$w = 1/3$$

Q NO.4 verify that;

$u(x,t) = \sin(x+2t)$.

$\Rightarrow$ is a solution of one-dimensional equation.

Solution:-

GIVEN DATA:-

$u(x,t) = \sin(x+2t)$.

$\Rightarrow$ Differential w.r.t x partialy.

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t).$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)(1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \cdot \frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)\,(1+0)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t).$$

and

$$u(x,t) = \sin(x+2t)$$

Differentiat w.r.t "t"

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t) \quad (0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t).$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t) \quad (0+2).$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)}$$

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⇒ We know that one dimentional wave equation is;

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

$$-4 \sin(x+2t) = c^2(-\sin(x+2t))$$
$$-4 \sin(x+2t) = -c^2 \sin(x+2t) = 0$$
$$-4 \sin(x+2t) + c^2 \sin(x+2t) = 0$$

For the orbitray constant $c = \pm 2$

$$-4 \cdot \sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4 \sin(x+2t) + 4 \sin(x+2t) = 0$$

$$0 = 0.$$

⇒ Then it will be verified for the orbitarary constant.

$$c = 2.$$

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THE END