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① THUR ☐ FRI ☐ SAT ☐ SUN ☐

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4th Seme

Q 1 (b)

If $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Sol:-

$y[z] = H(z)\ Y(z)$

Find $y[n]$

$X(z) = 2 - 4z^{-2} + 2z^{-3}$

$H(z) = 3 + z^{-1} + 2z^{-2}$

Now

$$Y(z) = H(z) * X(z)$$

$$= (2 - 4z^{-2} + 2z^{-3})(3 + z^{-1} + 2z^{-2})$$

$$= 6 + 2z^{-1} + 4z^{-2} + 12z^{-2} + 4z^{-3} + 8z^{-4} + 6z^{-3} + 2z^{-4} + 4z^{-3}$$

$$= 6 + 2z^{-1} - 2z^{-2} - 2z^{-3} + 6z^{-4} + 4z^{-5}$$

To find $y[n]$ use the delay property.

$$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] - 2[n-3] + 6\delta[n-4] + 4\delta[n-5]$$

Q 2 -:

$$f(x) = \begin{cases} -\pi/2, & -\pi \le x < 0 \\ \pi/2, & 0 \le x \le \pi \end{cases}$$

Retrive the fourier series.

Sol:-

Fourier Transform of differenation Integration of continous-time.

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Q 1 (a)

Ans :-

Fourier Transform of Differentiation integration of continous - time.

Let $x(t)$ be a continous - time signal with a fourier transform of $x(jw)$.

i.e

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(jw)\, e^{jwt}\, dw$$

Differentiat b/s w.r.t (t)

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(jw) \frac{d}{dt} \{ e^{jwt} \} dw$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} x(jw) \{ e^{jwt} . jw \} dw$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \{ jw\, x(jw) \} e^{jwt} dw$$

=>

$$\mathcal{F} \left\{ \frac{d}{dt} x(t) \right\} = jw\, x(jw)$$

Result :-

we concluded if a function is differentiated in time domain, It is multiplied by jw in frequancy domain.

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Q 3 :- If $X(z) = \frac{2z^2 + 2z}{(z^2 + 2z - 3)}$

Retrieve $x[n]$

Ans :-

$$X_z = \frac{2z^2 + 2z}{(z^2 + 2z - 3)}$$

$$X_z = \frac{2z(z + 1)}{(z^2 + 2z - 3)}$$

$$\frac{X_z}{z} = \frac{2z + 2}{z^2 + 2z - 3}$$

$$\frac{2z + 2}{z^2 + 2z - 3} = \frac{A}{(z+1)} + \frac{B}{(z-3)} \quad \text{— (i)}$$

or

$$2z + 2 = A(z-3) + B(z+1)$$

put $z = 3$

$$2(3) + 2 = A(3-3) + B(3+1)$$

$$8 = 0 + B(4)$$

$$\boxed{B = 2}$$

put $z = -1$

$$2(-1) + 2 = A(-1-3) + B(-1+1)$$

$$0 = A(-4)$$

$$\boxed{A = 0}$$

Now put A, B in (i)

$$\frac{2z + 2}{z^2 + 2z - 3} = \frac{0}{z+1} + \frac{2}{z-3}$$

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$$X_z = \frac{9z}{z-3}$$

inverse z transform.

$$x[n] = 9(-3)^k$$

Q 9 — $\begin{cases} -\pi/9 & -\pi \le x \le 0 \\ \pi/9 & 0 \le x \le \pi \end{cases}$

As

$$a_0 = \frac{1}{9\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{9\pi} \left[\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right]$$

$$= \frac{1}{9\pi} \int_{-\pi}^{0} -\pi/9\, dx + \int_{0}^{\pi} \pi/9\, dx$$

$$= 1/9\pi \left[-\pi/9 \int_{-\pi}^{0} dx + \pi/9 \int_{0}^{\pi} dx \right.$$

$$= \frac{1}{9\pi} \left[-\pi/9\, x \Big|_{-\pi}^{0} + \pi/9\, x \Big|_{0}^{\pi} \right]$$

$$= 1/9\pi \left[\pi/9 \left[0 - (-\pi) + \pi/9 (\pi - 0) \right] \right]$$

$$= 1/9\pi \; -\pi/9 (\pi) + \pi/9 (\pi)$$

$$= 1/9\pi \left[-\pi/9 + \pi/9 \right]$$

$$= \frac{1}{9\pi} \left(\frac{0}{9} \right) \Rightarrow \boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= 1/\pi \int_{-\pi}^{0} f(x) \cos nx\, dx + \int_{0}^{\pi} fx \cos nx\, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} -\pi/2 \cos nx\, dx + \int_{0}^{\pi} \pi/2 \cos nx\, dx$$

$$= \frac{1}{\pi} \left[\pi/2 \frac{\sin nx}{n} \Big|_{-\pi}^{0} + \pi/2 \frac{\sin nx}{n} \Big|_{0}^{\pi} \right.$$

$$= \frac{1}{n\pi} \left[-\pi/2 \sin n(0) - \sin n(-\pi) \right] + \pi/2 \left[\sin n(\pi) - \sin n(0) \right]$$

$$= \frac{1}{n\pi} \left[-\pi/2 (0) + \pi/2 (0) \right]$$

$$= \frac{1}{n\pi} (0).$$

$$\boxed{a_n = 0}$$

Now $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin nx\, dx$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x) \sin nx\, dx + \int_{0}^{\pi} f(x) \sin nx\, dx \right.$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} -\pi/2 \sin nx\, dx + \int_{0}^{\pi} \sin nx\, dx \right.$$

$$= \frac{1}{\pi} \left[-\pi/2 \cdot \frac{\cos nx}{n} \Big|_{-\pi}^{0} + \pi/2 \frac{-\cos nx}{n} \Big|_{0}^{\pi} \right.$$

$$= \frac{1}{n\pi} \left[-\frac{\pi}{2} \left[-\cos n(0) + \cos n(-\pi) \right] + \pi/2 \left[-\cos n(\pi) + \cos n(0) \right] \right.$$

$$= \frac{1}{n\pi} \left[-\frac{\pi}{2} -1 + \cos n \cdot (-\pi) \right] + \pi/2 \left[-\cos n\pi + \cos n(0) \right]$$

$$\pi/2 = \frac{1}{n\pi} \left[-1 \left[-1 + \cos n(-\pi) \right] + 1 \left[-\cos nx + 1 \right] \right]$$

$$= \frac{1}{2n} \left[1 - \cos n\pi - \cos n\pi + 1 \right]$$

$$= 1/2n \left[2 - 2 \cos n\pi \right]$$

Now

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{4}{2n} & \text{if } n \text{ is odd} \end{cases}$$

$$\left\{ b_n = \frac{4}{2n} \right\}$$

$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots$
$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots$

$f(x) = 0 + (0)\cos x + 0 \cos(2x) + \cdots$
$= \frac{4}{2} \sin x + (0)\sin^2 x + \frac{4}{3(2)} \sin 3x + \cdots$

$$\left\{ \frac{4}{2} \sin x + \frac{4}{6} \sin 3x + \cdots \right\}$$

Q No. 4 $A = \begin{bmatrix} -2 & -1 \\ 2 & 0 \end{bmatrix}$, $B \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = (1, 2)$, $D = (0)$

Sol:- we know that

$$\frac{Y(s)}{X(s)} = H(s)$$

$$H(s) = C(SI - A)^{-1} B + D$$

put value

$$H(s) = [1 \quad 2] \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= [1 \quad 2] \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1}$$

$$\Rightarrow [1 \quad 2] \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\text{Adj} = (s+2)s + 1 = s^2 + s + 2s + 2$
$= s^2 + 3s + 2$

$$H(s) = [1 \quad 2] \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \times \frac{1}{s^2 + 3s + 2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H(s) = \frac{[1 \quad 2] \begin{bmatrix} s \\ 1 \end{bmatrix}}{s^2 + 3s + 2}$$

Q No. 5 $x(t) = e^{-a|t|} u(t)$

Sol:-

Fourier transform is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

Note-:

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{fo } t < 0 \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$\Rightarrow \int_{-\infty}^{0} e^{(a-j\omega t)}\, dt + \int_{0}^{\infty} e^{-(a+j\omega t)}\, dt$$

$$\Rightarrow \frac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)}}{-(a+j\omega}\Bigg|_{0}^{\infty}$$

$$\Rightarrow \frac{1}{a-j\omega}(e^{0} - e^{-\infty}] - \frac{1}{a+j\omega}[e^{-\infty} - e^{0}]$$

$$\Rightarrow \frac{1}{a-j\omega}[1-0] - \frac{1}{a+j\omega}[0-1]$$

$$\Rightarrow \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$\Rightarrow \frac{a+j\omega + a - j\omega}{a^2 - (j\omega)^2}$$

$$\boxed{\Rightarrow \frac{2a}{a^2+\omega^2}}$$