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SECTION: A

CIVIL ENGINEERING DEPARTMENT

APPLIED CALCULUS

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TOPICS: Application of Derivatives.

Integration in Engineering.

Application of Derivatives:

There are various applications of derivatives not only in maths and real life but also in other fields like science, engineering, physics, etc. In previous classes, you must have learned to find the derivative of different functions, like, trigonometric functions, implicit functions, logarithm functions, etc. In this section, you will learn the use of derivatives with respect to mathematical concepts and in real-life scenarios. This is one of the important topics covered in Class 12 Maths as well.

Derivatives have various important applications in Mathematics such as:

- Rate of Change of a Quantity
- Increasing and Decreasing Functions
- Tangent and Normal to a Curve
- Minimum and Maximum Values
- Newton’s Method
- Linear Approximations

Applications of Derivatives in Maths:

The derivative is defined as the rate of change of one quantity with respect to another. In terms of functions, the rate of change of function is defined as dy/dx = f(x) = y'.

The concept of derivatives has been used in small scale and large scale. The concept of derivatives used in many ways such as change of temperature or rate of change of shapes and sizes of an object depending on the conditions etc.,

Rate of Change of a Quantity:

This is the general and most important application of derivative. For example, to check the rate of change of the volume of a cube with respect to its decreasing sides, we can use the derivative form as dy/dx. Where dy represents the rate of change of volume of cube and dx represents the change of sides of the cube.

Increasing and Decreasing Functions:

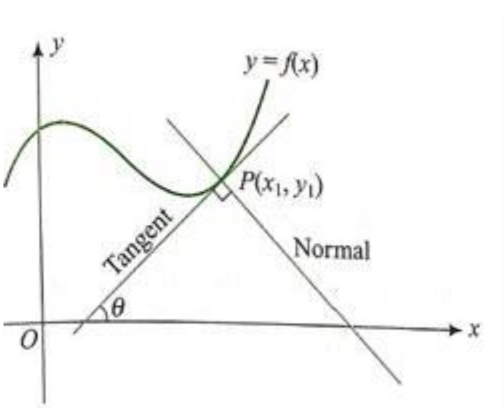
To find that a given function is increasing or decreasing or constant, say in a graph, we use derivatives. If f is a function which is continuous in [p, q] and differentiable in the open interval (p, q), then,

- f is increasing at [p, q] if $f'(x) > 0$ for each $x \in (p, q)$
- f is decreasing at [p, q] if $f'(x) < 0$ for each $x \in (p, q)$
- f is constant function in [p, q], if $f'(x)=0$ for each $x \in (p, q)$

Tangent and Normal to a Curve:

Tangent is the line that touches the curve at a point and doesn't cross it, whereas normal is the perpendicular to that tangent.

Let the tangent meet the curve at $P(x_1, y_1)$.



Now the straight-line equation which passes through a point having slope m could be written as;

$y - y_1 = m(x - x_1)$

We can see from the above equation, the slope of the tangent to the curve y = f(x) and at the point $P(x_1, y_1)$, it is given as dy/dx at $P(x_1, y_1) = f'(x)$. Therefore,

Equation of the tangent to the curve at $P(x_1, y_1)$ can be written as:

$y - y_1 = f'(x_1)(x - x_1)$

Equation of normal to the curve is given by;

$y - y_1 = [-1/ f'(x_1)] (x - x_1)$

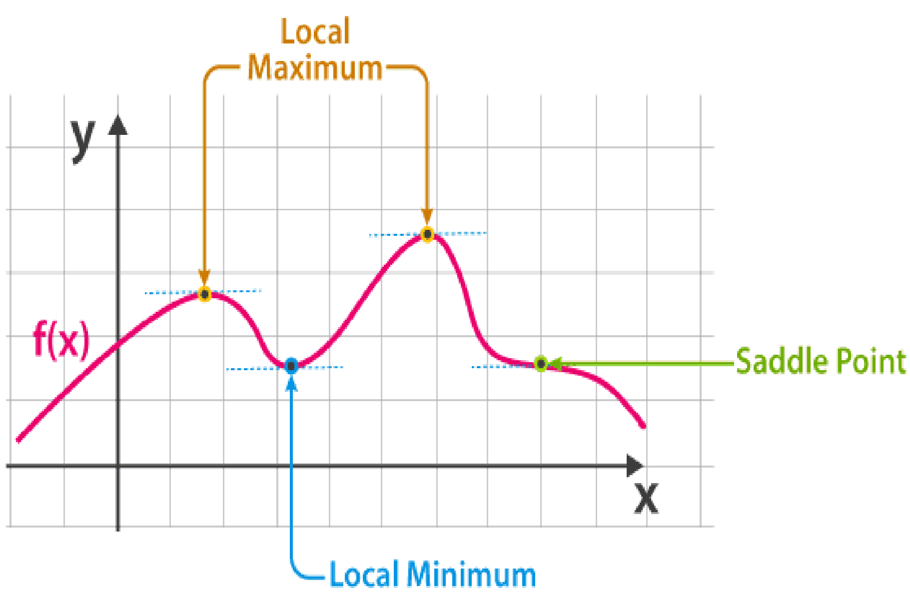
Or

$(y - y_1) f'(x_1) + (x-x_1) = 0$

Maxima and Minima:

To calculate the highest and lowest point of the curve in a graph or to know its turning point, the derivative function is used.

- When x = a, if f(x) ≤ f(a) for every x in the domain, then f(x) has an Absolute Maximum value and the point a is the point of the maximum value of f.
- When x = a, if f(x) ≤ f(a) for every x in some open interval (p, q) then f(x) has a Relative Maximum value.
- When x= a, if f(x) ≥ f(a) for every x in the domain then f(x) has an Absolute Minimum value and the point a is the point of the minimum value of f.
- When x = a, if f(x) ≥ f(a) for every x in some open interval (p, q) then f(x) has a Relative Minimum value.



Monotonicity:

Functions are said to be monotonic if they are either increasing or decreasing in their entire domain. $f(x) = e^x$, $f(x) = n^x$, $f(x) = 2x + 3$ are some examples.

Functions which are increasing and decreasing in their domain are said to be non-monotonic

For example: $f(x) = \sin x$, $f(x) = x^2$

Monotonicity Of A function At A Point

A function is said to be monotonically decreasing at x = a if f(x) satisfy;

$f(x + h) < f(a)$ for a small positive h

- f'(x) will be positive if the function is increasing
- f'(x) will be negative if the function is decreasing
- f'(x) will be zero when the function is at its maxima or minima

Approximation or Finding Approximate Value:

To find a very small change or variation of a quantity, we can use derivatives to give the approximate value of it. The approximate value is represented by delta △.

Suppose change in the value of x, dx = x then,

dy/dx = △x = x.

Since the change in x, dx ≈ x therefore, dy ≈ y.

Point of Inflection:

For continuous function f(x), if $f'(x_0) = 0$ or $f'''(x_0)$ does not exist at points where $f'(x_0)$ exists and if f''(x) changes sign when passing through $x = x_0$ then x_0 is called the point of inflection.

(a) If $f''(x) < 0$, $x \in (a, b)$ then the curve y = f(x) in concave downward

(b) if $f''(x) > 0$, $x \in (a, b)$ then the curve y = f(x) is concave upwards in (a, b)

For example: f(x) = sin x

Solution: f'(x) = cos x

$f''(x) = \sin x = 0$ $x = n\pi$, $n \in z$

Application of Derivatives in Real Life:

- To calculate the profit and loss in business using graphs.
- To check the temperature variation.

- To determine the speed or distance covered such as miles per hour, kilometre per hour etc.
- Derivatives are used to derive many equations in Physics.
- In the study of Seismology like to find the range of magnitudes of the earthquake.

Integration:

Integration is taking a number of parts and summing them together into a whole. In calculus, integration sums all the pieces of area under a curve into the total area under the curve.

Applications of Integration in Engineering:

Area between curves:

We have seen how integration can be used to find an area between a curve and the x-axis. With very little change we can find some areas between curves; indeed, the area between a curve and the x-axis may be interpreted as the area between the curve and a second “curve” with equation y = 0. In the simplest of cases, the idea is quite easy to understand

Distance, Velocity, Acceleration:

We next recall a general principle that will later be applied to distance-velocity-acceleration problems, among other things. If F(u) is an anti-derivative of f(u), then Z b a f(u) du = F(b) − F(a). Suppose that we want to let the upper limit of integration vary, i.e., we replace b by some variable x. We think of a as a fixed starting value x0. In this new notation the last equation (after adding F(a) to both sides) becomes: F(x) = F(x0) + Z x x0 f(u) du. (Here u is the variable of integration, called a “dummy variable,” since it is not the variable in the function F(x). In general, it is not a good idea to use the same letter as a variable of integration and as a limit of integration. That is, Z x x0 f(x)dx is bad notation, and can lead to errors and confusion.) An important application of this principle occurs when we are interested in the position of an object at time t (say, on the x-axis) and we know its position at time t0. Let s(t) denote the position of the object at time t (its distance from a reference point, such as the origin on the x-axis). Then the net change in position between t0 and t is s(t) − s(t0). Since s(t) is an anti-derivative of the velocity function v(t), we can write s(t) = s(t0) + Z t t0 v(u)du. Similarly, since the velocity is an anti-derivative of the acceleration function a(t), we have v(t) = v(t0) + Z t t0 a(u)du.

Volume:

We have seen how to compute certain areas by using integration; some volumes may also be computed by evaluating an integral. Generally, the volumes that we can compute this way have cross-sections that are easy to describe.

Average value of a function:

The average of some finite set of values is a familiar concept. If, for example, the class scores on a quiz are 10, 9, 10, 8, 7, 5, 7, 6, 3, 2, 7, 8, then the average score is the sum of these numbers divided by the size of the class:

Average score = 10 + 9 + 10 + 8 + 7 + 5 + 7 + 6 + 3 + 2 + 7 + 8 12 = 82 12 ≈ 6.83.

Suppose that between t = 0 and t = 1 the speed of an object is sin(πt). What is the average speed of the object over that time? The question sounds as if it must make sense, yet we can't merely add up some number of speeds and divide, since the speed is changing continuously over the time interval. To make sense of "average" in this context, we fall back on the idea of approximation. Consider the speed of the object at tenth of a second intervals: sin 0, sin(0.1π), sin(0.2π), sin(0.3π),. . ., sin(0.9π).

Work:

A fundamental concept in classical physics is work: If an object is moved in a straight line against a force F for a distance s the work done is W = F s

Center of Mass:

Suppose a beam is 10 meters long, and that there are three weights on the beam: a 10 kilogram weight 3 meters from the left end, a 5 kilogram weight 6 meters from the left end, and a 4 kilogram weight 8 meters from the left end. Where should a fulcrum be placed so that the beam balances? Let's assign a scale to the beam, from 0 at the left end to 10 at the right, so that we can denote locations on the beam simply as x coordinates; the weights are at x = 3, x = 6, and x = 8.

Suppose to begin with that the fulcrum is placed at x = 5. What will happen? Each weight applies a force to the beam that tends to rotate it around the fulcrum; this effect is measured by a quantity called torque, proportional to the mass times the distance from the fulcrum. Of course, weights on different sides of the fulcrum rotate the beam in opposite directions. We can distinguish this by using a signed distance in the formula for torque. So with the fulcrum at 5, the torques induced by the three weights will be proportional to (3 − 5)10 = −20, (6 − 5)5 = 5, and (8 − 5)4 = 12. For the beam to balance, the sum of the torques must be zero; since the sum is −20 + 5 + 12 = −3, the beam rotates counter-clockwise, and to get the beam to balance we need to move the fulcrum to the left. To calculate exactly where the fulcrum should be, we let ¯x denote the location of the fulcrum when the beam is in balance.

Kinetic energy:

Improper integrals Recall example 9.5.3 in which we computed the work required to lift an object from the surface of the earth to some large distance D away. Since F = k/x2 we computed

Z D r0 k x 2 dx = − k D + k r0 .

We noticed that as D increases, k/D decreases to zero so that the amount of work increases to k/r0. More precisely,

lim D→∞ Z D r0 k x 2 dx = lim D→∞ − k D + k r0 = k r0 .

We might reasonably describe this calculation as computing the amount of work required to lift the object "to infinity," and abbreviate the limit as

lim D→∞ Z D r0 k x 2 dx = Z ∞ r0 k x 2 dx.

Such an integral, with a limit of infinity, is called an improper integral. This is a bit unfortunate, since it's not really "improper" to do this, nor is it really "an integral"—it is an abbreviation for the limit of a particular sort of integral. Nevertheless, we're stuck with the term, and the operation itself is perfectly legitimate. It may at first seem odd that a finite amount of work is sufficient to lift an object to "infinity", but sometimes surprising things are nevertheless true, and this is such a case. If the value of an improper integral is a finite number, as in this example, we say that the integral converges, and if not we say that the integral diverges.

Probability:

You perhaps have at least a rudimentary understanding of discrete probability, which measures the likelihood of an "event" when there are a finite number of possibilities. For example, when an ordinary six-sided die is rolled, the probability of getting any particular number is 1/6. In general, the probability of an event is the number of ways the event can happen divided by the number of ways that "anything" can happen. For a slightly more complicated example, consider the case of two six-sided dice. The dice are physically distinct, which means that rolling a 2–5 is different than rolling a 5–2; each is an equally likely event out of a total of 36 ways the dice can land, so each has a probability of 1/36. Most interesting events are not so simple. More interesting is the probability of rolling a certain sum out of the possibilities 2 through 12. It is clearly not true that all sums are equally likely: the only way to roll a 2 is to roll 1–1, while there are many ways to roll a 7. Because the number of possibilities is quite small, and because a pattern quickly becomes evident, it is easy to see that the probabilities of the various sums are:

P(2) = P(12) = 1/36 P(3) = P(11) = 2/36 P(4) = P(10) = 3/36 P(5) = P(9) = 4/36 P(6) = P(8) = 5/36 P(7) = 6/36

Here we use P(n) to mean "the probability of rolling an n." Since we have correctly accounted for all possibilities, the sum of all these probabilities is 36/36 = 1; the probability that the sum is one of 2 through 12 is 1, because there are no other possibilities.

Arc Length:

Here is another geometric application of the integral: find the length of a portion of a curve. As usual, we need to think about how we might approximate the length, and turn the approximation into an integral. We already know how to compute one simple arc length, that of a line segment. If the endpoints are P0(x0, y0) and P1(x1, y1) then the length of the segment is the distance between the points, p (x1 − x0) 2 + (y1 − y0) 2, from the Pythagorean theorem.

Surface Area:

Another geometric question that arises naturally is: "What is the surface area of a volume?" For example, what is the surface area of a sphere? More advanced techniques are required to approach this question in general, but we can compute the areas of some volumes generated by revolution.

As usual, the question is: how might we approximate the surface area? For a surface obtained by rotating a curve around an axis, we can take a polygonal approximation to the curve, as in the last section, and rotate it around the same axis. This gives a surface composed of many "truncated cones;" a truncated cone is called a frustum of a cone. Figure 9.10.1 illustrates this approximation.

Uses of Integration in Engineering:

Many aspects of civil engineering require calculus. Firstly, derivation of the basic fluid mechanics equations requires calculus. For example, all hydraulic analysis programs, which aid in the design of storm drain and open channel systems, use calculus numerical methods to obtain the results. In hydrology, volume is calculated as the area under the curve of a plot of flow versus time and is accomplished using calculus.