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Section # B

Subject # Applied Calculus

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Deptt # BE Civil Engg

# Q#01

**Ans**:- $P(4,1,3) = 4\hat{i} + \hat{j} + 3\hat{k}$

$Q(1,2,4) = \hat{i} + 2\hat{j} + 4\hat{k}$

Now distance b/w PQ

so $|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

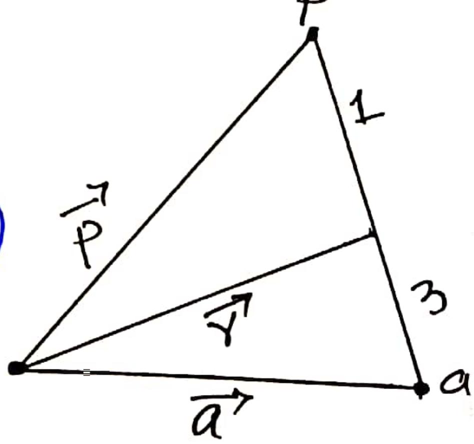
$= \sqrt{(4-1)^2+(1-2)^2+(3-4)^2}$

$= \sqrt{9+1+1} = \sqrt{11}$

Now finding The poistion Vector of The point dividing PQ in The ratio of 1:3

$a : b = 1 : 3$

$\vec{r} = \dfrac{b\vec{p} + a\vec{a}}{b+a}$




$= \dfrac{3(4\hat{i}+\hat{j}+3\hat{k})+1(\hat{i}+2\hat{j}+4\hat{k})}{3+1}$

$= \dfrac{12\hat{i}+3\hat{j}+9\hat{k}+\hat{i}+2\hat{j}+4\hat{k}}{4}$

$$= \frac{13\hat{i} + 5\hat{j} + 13\hat{k}}{4}$$

$$\boxed{\vec{r} = \frac{13}{4}\hat{i} + \frac{5}{4}\hat{j} + \frac{13}{4}\hat{k}}$$ Ans

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**Q#02** $$\int \frac{4x^3 + 10x + 4}{2x^2 + x}\, dx$$

**Ans:-**

$$\begin{array}{r|l}
 & \quad 2x - 1 \\
\hline
2x^2 + x & 4x^3 + 10x + 4 \\
 & \pm 4x^3 \pm 2x^2 \\
\hline
 & -2x^2 + 10x + 4 \\
 & \overset{-}{\underset{+}{}} 2x^2 \mp x \\
\hline
 & 11x + 4
\end{array}$$

$$\text{So} \quad 2x - 1 + \frac{11x + 4}{2x^2 + x} = \frac{4x^3 + 10x + 4}{2x^2 + x}$$

$$\Rightarrow \int \frac{4x^3 + 10x + 4}{2x^2 + x} = \int 2x - 1 + \int \frac{11x + 4}{2x^2 + x} \rightarrow ①$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x + 4}{2x^2 + x}\, dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x + 4}{x(2x + 1)}\, dx \rightarrow ②$$

Now find

$$\int \frac{11x+4}{x(2x+1)}\,dx.$$

$$= \frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow (A)$$

$$= \frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$= 11x+4 = A(2x+1)+Bx \longrightarrow (3)$$

put $x=0$ in eq (3)

$$4 = A$$

Now put $x = -\frac{1}{2}$ in eq (3)

$$= 11\left(-\frac{1}{2}\right)+4 = B\left(-\frac{1}{2}\right)$$

$$= -\frac{11}{2}+4 = \frac{-B}{2}$$

$$= \frac{-11+8}{\cancel{2}} = \frac{-B}{\cancel{2}}$$

$$= -3 = -B \Rightarrow \boxed{B=3}$$

putting the value of A & B in (A)

$$= \frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

⇒ Taking Integral on both Sides

$$= \int \frac{11x+4}{x(2x+1)}dx = \int \frac{4}{x}dx + \int \frac{3}{2x+1}dx$$

$$= 4\int \frac{1}{x}dx + 3\int \frac{1}{2x+1}dx$$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

putting These value in ②

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln(2x+1)$$

Now put These value in eq ①

$$\int \frac{4x^3+10x+4}{2x^2+x}dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

Required Answer

Q#03 $\int_0^2 x^2 e^x dx$

Ans:- Now first find Integration

$$= \int x^2 e^x dx$$

$$= x^2 \int e^x dx - \int \left( \int e^x dx \, \frac{d}{dx} x^2 \right) dx$$

$$= x^2 e^x - \int e^x (2x) dx$$

$$= x^2 e^x - 2 \int x e^x dx$$

$$= x^2 e^x - 2 \left[ x \int e^x dx - \int \left( \int e^x dx \, \frac{d}{dx} x \right) dx \right]$$

$$= x^2 e^x - 2 \left[ x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2x e^x + 2 e^x$$

Now put Limits

$$= \left| x^2 e^x - 2x e^x + 2 e^x \right|_0^2$$

$$= (2^2 e^2 - 2(2) e^2 + 2 e^2 - (0 - 0 + 2 e^0)$$

$$= 4\cancel{e^2} - 4\cancel{e^2} + 2e^2 - 2$$

$$\boxed{= 2e^2 - 2}$$

Ans

(b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Ans ⇒ first find Integration

$= \int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = ? \rightarrow (1)$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\boxed{2\,dy = \frac{1}{\sqrt{x}}\,dx}$$

put in 1

$= \int \sin(y)(2dy) = 2\int \sin(y)\,dy$

$= 2(-\cos y)$

$=$ put $y = \sqrt{x}$

$= -2\cos\sqrt{x}$

put limits

$= -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{2} - \cos 1)$

$$\boxed{= -2\cos\sqrt{2} + 2\cos(1) \quad \text{Ans}}$$

Q#4

Ans:- The Laplace equation in 3-d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow (A)$$

So $u(x, y, z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x, y, z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x(-3/2)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \rightarrow (1)$$

Now

$$\frac{\partial u}{\partial y} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \rightarrow (2)}$$

$$\frac{\partial u}{\partial z} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\boxed{\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2} \rightarrow (3)}$$

putting ① ② & ③ in eq Ⓐ

$$= 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2} - \cancel{x^2} - \cancel{y^2} - \cancel{z^2} + \cancel{3y^2} - \cancel{x^2} - \cancel{y^2} - \cancel{z^2} + \cancel{3z^2} - \cancel{x^2} - \cancel{y^2} - \cancel{z^2}\right]$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

so the given $u(x,y,z)$ is solution of laplace equation.