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SECTION : B

SEMESTER : 4th

CLASS TIMING : FRIDAY 11-02pm

PAPER : LINEAR ALGEBRA

QUESTION # 1

Consider the given matrix as the argumented matrix of a linear system. Explain in your words the next elementry row operation that should be performed in order to solve this system - - - - - - - - - - - - - - - -

$$\begin{bmatrix} 1 & 1D3 & 3 & 0 & 5 \\ 0 & 1 & 1D.1D1T & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 1D3 \end{bmatrix}$$

SOLUTION:-

$$\begin{bmatrix} 1 & 5 & 3 & 0 & 5 \\ 0 & 1 & 6 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

$x_1 + 5x_2 + 3x_3 = 5 \longrightarrow ①$

$x_2 + 6x_3 = 7 \longrightarrow ②$

$x_3 = -6 \longrightarrow ③$

$x_4 = 5 \longrightarrow ④$

- equ(2) + equ(3)(-3)

$x_2 + 6x_3 = 7$

$-6x_3 = +36$

$x_2 = +29 \longrightarrow +29$

$x_1 + 5x_2 + 3x_3 = 5 \longrightarrow ①$

$x_2 = +29 \longrightarrow ②$

$x_3 = -6 \longrightarrow ③$

$x_4 = 5 \longrightarrow ④$

- equ (3)(3) − equ (1)

$$\begin{array}{r} x_1 + 5x_2 + 3x_3 = 5 \\ -3x_3 = +18 \\ \hline x_1 + 5x_2 = 5 \\ -18 \\ \hline x_1 + 5x_2 = 23 \longrightarrow ⑤ \end{array}$$

- equ (5) − équ (1)

$$\begin{array}{r} x_1 + 5x_2 = 23 \\ -5x_2 = -145 \\ \hline x_1 = -122 \end{array}$$

$x_1 = -122$

$x_2 = +29$

$x_3 = -6$

$x_4 = 5$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -122 \\ 0 & 1 & 0 & 0 & 29 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

$x_1 + 5x_2 + 3x_3 = 5$

$= -122 + 5(+29) + 3(-6) = 5$

$= -122 + 145 - 18 = 5$

$= -140 + 145 = 5$

$= \boxed{5 = 5}$

QUESTION : 2

a) Find the elementry row operation that transforms the first matrix into second

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

SOLUTION :-

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

In order to transform first matrix into the second we will multiply -2 with row 2 and then add the row 2 to row 3.

$-2R_2 + R_3$:-

$-2 \times \text{row } 2 = 0 \quad 1 \quad -4 \quad 2 \quad \times 2$

$= 0 \quad -2 \quad +8 \quad -4$

Now adding it to row 3 :-

$$\begin{array}{cccc} 0 & -2 & 8 & -4 \\ 0 & 2 & -5 & -1 \\ \hline 0 & 0 & 3 & -5 \end{array}$$

putting the newly format row 3 into the first matrix.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Hence we have transferred first matrix into second.

Now adding reverse row operation and turning the second matrix into the first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Multiplying 2 with row 2 and then adding it to row 3.

$2R_2 + R_3$:-

$$\begin{array}{cccc} 0 & 2 & -8 & 4 \\ 0 & 0 & 3 & -5 \\ \hline 0 & 2 & -5 & -1 \end{array}$$

Putting back the new formate obtained above in second matrix and transferring it into matrix no first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

QUESTION # 2

b) Given are some matrixes. Find whether these are in the forms written in front of them or not. Explain in your own words.

a) $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \text{II} & 0 & 0 \\ 0 & 0 & -\text{II} & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

b) $\begin{bmatrix} 1 & 0 & \text{II} \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

c) $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

a) $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$

The matrix is not in echelon form because the most below row have "e" element which is not zero and for echelon form there must be all zero element in the last line.

b) $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

This one is the echelon form because the pivot element having zero in their below row and the last row have also zero element.

c) $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

This is not reduced echelon form because having non-zero rows at the bottom.

d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

This is reduced echelon form if when we transpose $R_2 \leftrightarrow R_3$ then the bottom line will be have all zero element.

QUESTION # 3

a) The row echelon is used to solve the linear equations. What is the difference the row echelon and reduced row - - - - - . Give one example.

ROW ECHELON FORM :-

A matrix is said to be in row echelon form for when it satisfies the following conditions.

- The first non-zero element in each row called leading entry is 1.
- Each leading entry is in a row to the right of the leading entry in the previous row.
- Rows with all zero elements if any are below (after) the row having a non-zero element.

EXAMPLE:-

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

REDUCED ROW ECHELON FORM:-

A matrix is said to be in reduced row echelon form if it satisfies the following conditions:

- The matrix satisfies for the row echelon form.
- The leading entry in each row is the only non-zero entry of this row.

EXAMPLE:-

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore we can say that each reduced row echelon form is a row echelon form but vice versa is not always true.

QUESTION # 3

b) Find an echelon form for the below matrix using row operations. Where ID2 is second digit in your ID e.g ------ -------------

$$\begin{bmatrix} 1 & ID2 & 8 \\ 2 & 8 & -1 \\ -ID3 & 0 & 0 \\ 1 & -4 & ID\text{-First last} \end{bmatrix}$$

SOLUTION:-

$$\begin{bmatrix} 1 & 4 & 8 \\ 2 & 8 & -1 \\ -5 & 0 & 0 \\ 1 & -4 & 16 \end{bmatrix}$$

$R_2 - 2R_1$:

$$\begin{bmatrix} 1 & 4 & 8 \\ 0 & 0 & -17 \\ 0 & 20 & 40 \\ 1 & -4 & 10 \end{bmatrix}$$

$R_4 \longleftrightarrow R_2$:

$$\begin{bmatrix} 1 & 4 & 8 \\ 1 & -4 & 10 \\ 0 & 20 & 40 \\ 0 & 0 & -17 \end{bmatrix}$$

$R_2 - R_1$:-

$$\begin{bmatrix} 1 & 4 & 8 \\ 0 & -8 & 2 \\ 0 & 10 & 40 \\ 0 & 0 & -17 \end{bmatrix}$$