Name	Shoh AB malook
I.D	7878
Subject	Differential equation
Section	"A"
Exam	mid trem.

(1)

Q1:- $\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2)$ $\quad y(0)=0$

$y(0)=0$ So $x=0$ $y=0$

$dy = e^y \cdot e^{-1} \sec(y)(1+t^2)\,dt$

$\frac{1}{e^y \cdot \sec(y)} dy = (1+t^2)e^{-t}\,dt$

As $\cos(y) = \frac{1}{\sec(y)}$

$\int e^{-y} \cos y\,dy = \int (1+t^2)e^{-1}\,dt$

using integration by parts.

$e^{-y}\int \cos y\,dt - \int\left(\int \cos y \frac{d}{dy} e^{-y}\right) =$

$\cancel{t}$ $(1+t^2)\int e^{-t} - \int\left(\int e^{-t} \frac{d}{dt}(1+t^2)\right)$

eq (1)

L.H.S

$e^{-y}\int \cos y\,dx - \int\left(\int \cos y \cdot \frac{d}{dy} e^{-y}\right)$

$e^{-y} \sin y - \int (\sin y\, e^{-y}(-1))$

$e^{-1} \sin y + \int (\sin y \cdot e^{-y})$

$e^{-1} \sin y + \int e^{-y} \sin y)$

again using integration by parts.

Q1 a Page 2

$e^{-y}\sin y + e^{-y}(-\cos y) - \int(\sin y \frac{d}{dy} e^{-y})$

$e^{-y}\sin y + e^{-y}\cos y - \int(\cos y\, e^{-y})$

Since $\int(\cos y\, e^{-y} = L \cdot H \cdot S$

Since is again Same to the first one So L.H.S will become

$L \cdot H \cdot S = e^{-y}(\sin y - \cos y) - L \cdot H \cdot S$

$2L \cdot H \cdot S = e^{-y}(\sin y - \cos y)$

$L \cdot H \cdot S = \frac{e^{-y}(\sin y - \cos y)}{2}$

Now taking R.H.S

$\int(1+t^2)e^{-t}\,dt$

$(1+t^2)\int e^{-t} - \int(\int e^{-t} \frac{d}{dt}(1+t^2))$

$(1+t^2)e^{-t} - \int(-e^{-t}(2t)$

$= (1+t^2)e^{-t} + \int(2t)e^{t}$

again using integration by Parts.

Q1 page 3

$$-(1+t^2)e^{-1}+\left(2t\int e^{-t}-\int\left(\int e^{-t}\frac{d}{dt}2t\right)\right)$$

$$=-(1+t^2)e^{-t}+\left(-2te^{-t}-\int(-e^{-1}2)\right)$$

$$=-(1+t^2)e^{-t}+\left(-2te^{-t}+\int 2e^{-t}\right)$$

$$=-(1+t^2)e^{-t}+\left(-2te^{-t}-2e^{-t}\right)+c$$

$$-(1+t^2)e^{-t}-2te^{-t}-2e^{-t}+c$$

$$-e^{-t}-e^{-t}t^2-2te^{-t}-2e^{-t}+c$$

$$-(t^2+2t+3)e^{-t}+c=R\cdot H\cdot S$$

Now take $L\cdot H\cdot S=R\cdot H\cdot S$

$$\frac{e^{-y}(\sin y-\cos y)}{2}=-(t^2+2t+3)e^{-t}+c$$

We know that

$t=0 \quad y\,0$

put in above

$\Rightarrow \ (0-1)=-3+c$

$c=5/2$

put value of c

$$\boxed{\frac{e^{-y}(\sin y-\cos y)}{2}=-(x^2+2t+3)e^{-t}+5/2}$$

Scanned with CamScanner

(1)

Q1 $\dfrac{dy}{dt} = e^{y-t}\sec(y)(1+t^2) \quad y(0)=0$

Solution.

$$\frac{dy}{dt} = \frac{e^{y}e^{-t}}{\cos(y)}(1+t^2)$$

$$e^{-y}\cos(y)\,dy = e^{-t}(1+t^2)\,dt$$

$$\int e^{-y}\cos(y)\,dy = \int e^{-t}(1+t^2)\,dt$$

$$\frac{e^{-y}}{2}\left(\sin(y)-\cos(y)\right) = -e^{-t}(t^2+2t+3)$$

Applying the initial condition gives:

$$\tfrac{1}{2}(-1) = -(3)+C \qquad C = \tfrac{5}{2}$$

$$\frac{e^{-y}}{2}\left(\sin(y)-\cos(y)\right) = -e^{-t}(t^2+2t+3)$$

it is not possible to find an explicit Solution for this problem and so we will have to leave the solution it its implicit from finding intervals of validity from implicit Solution can often by very difficult So we will also not bother with that for this problem.

Q2 page 1

$\sqrt{x+y} + \sqrt{x-y}\, dx - \sqrt{x+y} - \sqrt{x-y}\,)\, dy = 0$

$$\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \longrightarrow (1)$$

this is homogeneous Differential eq. in x and y to Solve this put

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

they eq (1) becomes

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1 + \cancel{v} + 1 - \cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}\left(1 + \sqrt{1-v^2}\right)}{\cancel{2}v}$$

Scanned with CamScanner

Q2 page 2

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

taking integrals on b/s.

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

- put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v}} = -dt.$$

Q2 page 3

$$\int -\frac{dt}{t} = \int \frac{dx}{\cancel{x}}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1-v^2} = \ln cx$$

$$\ln(1+\sqrt{1-v^2} = -\ln cx$$

$$\ln(1+\sqrt{1-v^2}) = \ln(cx)^{-1}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$$x+\sqrt{x^2-y^2} = C_1 \quad \because \frac{1}{c} = C_1$$

which is a Required Soultion

Scanned with CamScanner

Q_3 page 1

$$(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$$

Sol:- $(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$

$\Rightarrow f(D)y = f(x)$

As it is non homogenous linear equation

So solution will be.

$$y = y_c + y_p \quad \text{—(1)}$$

Complementary Solution y_c.

$D^4+D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either $D^2 = 0 \Rightarrow \boxed{D=0}$

$D^2+1 = 0 \Rightarrow D^2 = -1$

$D = \sqrt{-1} \Rightarrow \boxed{D=i}$ or $D = \boxed{0+i}$

Roots are real and complex.

$y_c = c_1e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$

$y_c = c_1 + c_2\cos x + c_3 \sin x$

$$y_p = \frac{1}{f(D)} F(x)$$

Q_3 page 2

$$y_P = \frac{1}{D^4+D^2}(3x^2+4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$f(D) = D^4 + D^2$

at $D = 0 \Rightarrow f(0) = 0$

So $f'(D)$ $4D^3 + 2D$

Now also for $D = 0 \Rightarrow f(D) = 0$

again differentiating.

$f''(D) : 12D + 2$

So for $D = 0$

$f''(0) = 12(0) + 2 = 2$

So replacing $1/f(D)$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_P \; \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} 4\sin x - \frac{x^2}{12D+2} 2\cos x$$

Scanned with CamScanner

Q_3 page 3

putting $D=0$ in all

$$y_P = \frac{x^2\, 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2 \cos x}{12(0)+2}$$

$$y_P = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3x^4}{2} + 2x^2 \sin x - x^2 \cos x$$

So putting in equation (i)

$$y = C_1 + C_2 \cos x + C_3 \cos x + \tfrac{3}{2} x^4 + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = C_1 + (C_2 - x^2) \cos x + (C_3 + 2x^2) \sin x + \tfrac{3}{2} x^4}$$