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Paper Name :- Differential Equation.

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Question number (ONE) (01)

Solve the following objects.

(i) The order of Matrix A is $m \times p$ and the order of B is $p \times n$. Then order of Matrix AB is?

Ans The order of ~~A~~ AB is $m \times p$.

(ii) The number of Non-Zero rows in an Echelon form?

Ans: It determines the Rank of Matrix.

iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a Singular Matrix then $a = ?$

Ans: $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix} \Rightarrow |B| = 0$

$\Rightarrow |B| = a - 8$

$\Rightarrow a - 8 = 0$

$\Rightarrow a = 8.$

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$.

Ans: $\Rightarrow |A| = (2i)(-i) - (i)(i)$

$\Rightarrow |A| = -2i^2 - i^2.$

$\Rightarrow |A| = -2(-1) - (-1)$

$\Rightarrow |A| = 2 + 1 = 3$

$\Rightarrow |A| = 3.$

The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

Ans The Matrix Scaler Matrix.

(vi) Solation of $\frac{dy}{dx} + 2xy = y$?

Ans $\frac{dy}{dx} + 2xy = y$

$$\frac{dy}{dx} = y - 2xy$$

$$dy = y(1-2x)\,dx$$

$$\frac{dy}{y} = (1-2x)\,dx$$

Integrale.

$$\int \frac{dy}{y} = \int (1-2x)\,dx.$$

$$\ln y = \int 1\,dx - 2\int x\,dx.$$

$$\ln y = x - \frac{2x^2}{2} + c$$

$$\ln y = x - x^2 + c$$

$= e^{\ln y} = e^{(x-x^2)}\, e^{c_1}$

$y = ce^{x-x^2}.$

(vii) The order and degree of differential equation.

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is?}$$

Ans: Order = 1.

degree = 3.

(viii) The order and degree of differential equation.

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is?}$$

Ans Order = 2.

degree = 1.

x) The differential equation

$$2\frac{dy}{dx} + x^2 y = 2x+3, \; y(0) = 5 \text{ is?}$$

Ans:

Not possible.

x) $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ is?

Ans: Expanding with first row.

$\Rightarrow 1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} - a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$.

$\Rightarrow 1(bc^2 - cb^2) - a(c^2 - b^2) + a^2(c-b)$

$\Rightarrow bc(c-b) - a(c-b)(c+b) + a^2(c-b)$

$\Rightarrow (c-b)[bc - a(c+b) a^2]$

$\Rightarrow (c-b)[bc - ac - ab + a^2]$

$\Rightarrow (c-b)[c(b-a) - a(b-a)]$

$\Rightarrow (c-b)(b-a)(c-a)$

Question number (Two) (02) (part 1)

(1) Express the determinant

$$= \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

= as the product of factors which are linear a.b.c-

1) $$= \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

= Taking a common from C1

= b from C2 and c from C3.

= expanding by R1

$$= abc\left[1\begin{vmatrix} b & c \\ b^2 & c^2 \end{vmatrix} - 1\begin{vmatrix} a & c \\ a^2 & c^2 \end{vmatrix} + 1\begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix}\right]$$

$$= abc\left[bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b\right]$$

$$= abc\left[bc^2 - ac^2 - b^2c + a^2c + ab^2 - a2b\right]$$

$$= abc\left[c2(b-a) - C(b^2 - a2) + ab(b-a)\right]$$

$$= abc\left[c^2(b-a) - c(b+a)(b-a) + ab(b-a)\right]$$

$$= abc\,(6-a)\left[c^2 - c(ba) + ab\right]$$

$$= abC\,(b-a)\left[c^2 - cb - Ca + ab\right]$$

$= abc\,(b-a)\left[c\,(c-b) - a\,(c-b)\right]$

$= abc\,(b-a)\,(c-b)\,(c-a)$

Answer.

Question (Two) (02) part (B) (2)

(ii) Find the Eigen value.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

For eigen values consider

$|A - \lambda I| = 0$

$$\Rightarrow A-\lambda I = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$\Rightarrow \left| \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} \right| = 0$$

$\Rightarrow$ Using $R_3 - R_2$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ 0 & -4+\lambda & 4-\lambda & 0 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$-1-3+\lambda$
$3-\lambda+1$

$-1-3+\lambda$ $\quad -1-3+\lambda$
$3-\lambda+1$ $\quad -4+\lambda$
$4\ \lambda$ $\quad 3-\lambda^2$

$\Rightarrow$ Expand by column first.

$$2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -4+\lambda & 4-\lambda & 0 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & 0 \\ -4+\lambda & 4-\lambda & 0 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by C_3 $\qquad$ Expand by C_3

$$= (2-\lambda)\left[-1\begin{vmatrix}-4+\lambda & 4-\lambda \\ -1 & \lambda-1\end{vmatrix} + (2-\lambda)\begin{vmatrix}3-\lambda & -1 \\ -4+\lambda & 4-\lambda\end{vmatrix}\right]$$

$$+\ 1\left[(2-\lambda)\begin{vmatrix}-1 & -1 \\ -4+\lambda & 4-\lambda\end{vmatrix}\right] = 0$$

$$\Rightarrow -(2-\lambda)(4-\lambda+4-\lambda)+(2\lambda)^2\big((3-\lambda)(4-\lambda)-4+\lambda\big)$$

$$+1\Big((2-\lambda)(\lambda-4-4+\lambda)\Big) = 0$$

$$\Rightarrow (\lambda-2)(8-2\lambda)+(\lambda-2)^2(12-7\lambda+\lambda^3-4+\lambda)$$

$$+\ \Big((2-\lambda)(2\lambda-8)\Big) = 0.$$

$$\Rightarrow (\lambda-2)(8-2\lambda)+(\lambda-2)^2(\lambda^2-6\lambda+8)$$

$$+\ (\lambda-2)(8-2\lambda) = 0$$

$$\Rightarrow (\lambda-2)\{8-2\lambda+(\lambda-2)(\lambda^3-6\lambda+8)$$

$$+\ 8-2\lambda\} = 0$$

$$\Rightarrow (\lambda-2)\{16-4\lambda+\lambda^3-6\lambda^2+8\lambda-2\lambda^2+12\lambda$$

$$-16\} = 0$$

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$= \lambda = 2 = 0 \quad . \quad \lambda^3 - 8\lambda^2 + 16\lambda = 0$

$\Rightarrow \boxed{\lambda = 2} \quad \lambda(\lambda^2 - 8\lambda + 16) = 0$

$= \boxed{\lambda = 0} , \quad \lambda^2 - 8\lambda + 16 = 0$

$\Rightarrow \lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\Rightarrow \lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$\Rightarrow (\lambda - 4)(\lambda - 4) = 0$

$\Rightarrow \boxed{\Rightarrow \lambda = 4, 4}$

Answer- $\Rightarrow 0, 2, 4, 4$

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Question number (Three) (03)

The rate of change in the form of differential equation is given by $(x^2+3y^2)\,dx - 2xy\,dy = 0$. Find the general solution at $x=2$ and $y=6$.

Solution:

$$(x^2+3y^2)\,dx = 2xy\,dy \quad , \quad y(2) = 6$$

$$2xy\,dy = (x^2+3y^2)\,dx$$

$$2xy\frac{dy}{dx} = x^2+3y^2$$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy} \rightarrow (1) \text{ homogenes}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$= v + x\frac{dv}{dx} = \frac{x^2 + 3v^2x^2}{2x \cdot vx}$$

$$= v + x\frac{dv}{dx} = \frac{x_2(1 + 3v^2)}{2x^2 v}$$

$$= v + x\frac{dv}{dx} = \frac{1 + 3v^2}{2v}$$

$$= \text{eq } (A) \Rightarrow \frac{x^2 + y^2}{x^3} = 5$$

$$= x^2 + y^2 = 5x^3$$

$$= y^2 = 5x^3 - x^2$$

$$= y = \sqrt{5x^3 - x^2} \text{ Answer.}$$