Course Title:- Calculas and analytical Geometry.

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Q No 1 (a) Identify $\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$ ①

Solution:- $\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$

$$\Rightarrow \frac{\sqrt{2+h}-\sqrt{2}}{h} \frac{(\sqrt{2+h}+\sqrt{2})}{(\sqrt{2+h}+\sqrt{2})}$$

$$\Rightarrow \frac{\cancel{2}+h-\cancel{\sqrt{2}\sqrt{2+h}}+\cancel{\sqrt{2}\sqrt{2+h}}-\cancel{2}}{h(\sqrt{2+h}+\sqrt{2})}$$

$$\Rightarrow \frac{\cancel{h}}{\cancel{h}(\sqrt{2+h}+\sqrt{2})}$$

$$\Rightarrow \frac{1}{\sqrt{2+h}+\sqrt{2}}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{2+0}+\sqrt{2}}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$\boxed{\lim_{h\to 0} \frac{1}{2\sqrt{2}}}$$ Ans

QNo1 (b) Find the first Order derivatives of the function $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

Solution:- $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

$$\frac{d}{dx}[f(x)] = f'(x)$$

$$\Rightarrow \frac{d}{dx}\left[\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)\right]$$

Apply the product Rule;

$$\Rightarrow \left(x + \frac{1}{x}\right) \cdot \frac{d}{dx}\left[x - \frac{1}{x} + 1\right] + \left(x - \frac{1}{x} + 1\right) \cdot \frac{d}{dx}\left[x + \frac{1}{x}\right]$$

$$\Rightarrow \left(x + \frac{1}{x}\right)\left(\frac{d}{dx}[x] - \frac{d}{dx}\left[\frac{1}{x}\right] + \frac{d}{dx}[1]\right) + \left(x - \frac{1}{x} + 1\right)\left(\frac{d}{dx}[x] + \frac{d}{dx}\left[\frac{1}{x}\right]\right)$$

$$\Rightarrow \left(x + \frac{1}{x}\right)\left(1 + \frac{\frac{d}{dx}[x]}{x^2} + 0\right)\left(x - \frac{1}{x} + 1\right)\left(1 - \frac{\frac{d}{dx}[x]}{x^2}\right)$$

$$\Rightarrow \left(x + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right) + \left(x - \frac{1}{x} + 1\right)\left(1 - \frac{1}{x^2}\right)$$

$$\Rightarrow \boxed{\frac{2x^4 + x^3 - x + 2}{x^3}}$$ Ans.

QNo 2 A dynamite blows up a heavy rock with launch velocity of 160 m/sec reaches a hight of $S = 160t - 16t^2$ after t Sec.

(i). How hig does the rock go.

(ii). Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down

(iii). Find the acceleration of the rock at tim 5 sec.

Solution:- (i) How high does the rock go;

The instant that the rock reaches its highest point its velocity is zero. To find the max hieght determin the first derivative, equate to zero and find the times when the velocity is zero.

Using this info to find S(t). The rock hieght;

Velocity $\Rightarrow S(t) = 160 - 32t$

time $\Rightarrow 160 - 32t = 0 \Rightarrow t = 5$ Sec

rock hieght $\Rightarrow s(5) = 160(5) - 16(5^2)$

$\Rightarrow s(5) = 800 - 400$

rock hieght. $\boxed{\Rightarrow s(5) = 400 \text{ feet}}$

(4)

(ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down.

To find the rock velocity at 256 feet on the way up and down, determine the two value of "t" such that:

$s(t) = 160t - 16t^2 = 256$

time $\Rightarrow -16t^2 + 160t - 256 = 0$

$\Rightarrow -16(t-2)(t-8) = 0$

$\Rightarrow t = 2$ sec and $t = 8$ sec

Using the info from part (a)

$v(2) = s(2) = 160 - 32(2) = 160 - 64 = \boxed{96 \text{ ft/sec}}$

$v(8) = s(8) = 160 - 32(8) = 160 - 256 = \boxed{-96 \text{ ft/sec}}$

(iii) Find the acceleration of the rock at time 5 sec.

At any time during its flight following explosion, the rock acceleration is a constant acceleration.

$\Rightarrow \boxed{s''(t) = -32 \text{ ft/sec}}$

Q No 3 :- Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent if so where?. (5)

Solution:- The horizontal tangent, if any occur where the slope $dy/dx$ is zero.

To find these points, we

(i) Calculate $dy/dx$ :-

$$\Rightarrow dy/dx = \frac{d}{dx}(x^4 - 2x^2 + 2)$$

$$= 4x^3 - 4x$$

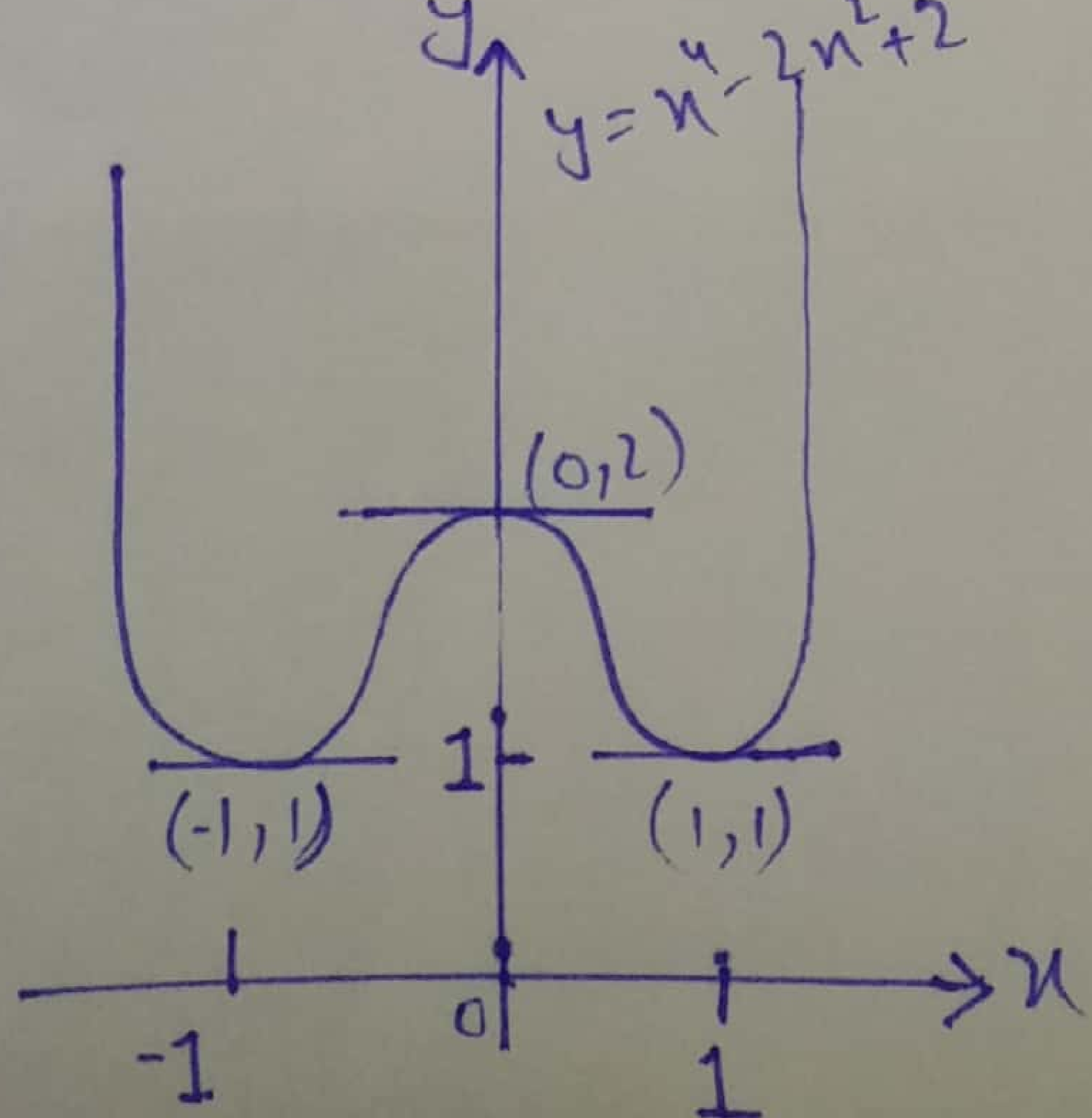
(ii) Solove the equition $dy/dx = 0$ for $x$;

$$\Rightarrow 4x^3 - 4x = 0$$

$$\Rightarrow 4x(x^2 - 1) = 0$$

$$\Rightarrow x = 0, 1, -1$$

The curve $y = x^4 - 2x^2 + 2$ has horizontal tangent at $x = 0, 1,$ and $-1$. The corresponding points on the curve are $(0, 2)$ $(1, 1)$ and $(-1, 1)$.




The curve $y = x^4 - 2x^2 + 2$ and its horizontal tangents.