

~~Part B~~

Question 2

Solution:

Sum of 2 has 1 way 1, 1

Sum of 3 has 2 ways 1, 2 and 2, 1

Sum of 4 has 3 ways 1, 3; 2, 2; 3, 1

5 has 4 ways

6 has 5 ways.

8 has 5 ways

9 has 4 ways

10 has 3 ways

11 has 2 ways

12 has 1 way

Those are 15/36 for each side
with a sum of 30/36

That leaves a $6/36 = 1/6$

probability for sum of 7.

Q5

The probability function for binomial random variable is

$$b(x, n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

This is the probability having x successes in a series of n independent

$$E(X) = \sum_{x=0}^n x \binom{n}{x} p^x (1-p)^{n-x}$$

$$\sum_{x=0}^n x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$\sum_{x=1}^n \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

Since $x=0$

let $y = x - 1$

$m = n - 1$

Subbing $x = y + 1$ and $n = m + 1$

into the last sum

$$\begin{aligned} E(x) &= \sum_{y=0}^m \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y} \\ &= (m+1)p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\ &= np \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \end{aligned}$$

The binomial Theorem says that:

$$(a+b)^m = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Setting $a=p$ $b=1-p$

$$\sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} = \sum_{y=0}^m \frac{m!}{y!(m-y)!}$$

$$a^y b^{m-y} = (a+b)^m = (p+1-p)^m = 1$$

So that

$$E(x) = np$$

Similarly, but the time using $y = x - 2$

$$m = n - 2$$

$$E(x(x-1)) = \sum_{x=0}^n x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^n \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^n \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= n(n-1)p^2 (p + (1-p))^m$$

$$= n(n-1)p^2$$

$$= n(n-1)p^2 (p + (1-p))^m$$

$$= n(n-1)p^2$$

So $E(x^2) - E(x)^2 = E(x(x-1)) + E(x) - E(x)^2 = n(n-1)p^2$

$$+ np - (np)^2 = \underline{\underline{np(1-p)}}$$

Q4

Since the c_i 's form a partition of the sample space we can apply the law of total probability for ANB

$$P(ANB) = \sum_{i=1}^M P(ANB/c_i) P(c_i)$$

$$P(ANB) = \sum_{i=1}^M P(A/c_i) P(B/c_i) P(c_i)$$

(A and B are conditionally independent)

$$P(ANB) = \sum_{i=1}^M P(A/c_i) P(B) P(c_i)$$

= B is independent of all c_i 's)

$$P(ANB) = P(B) \sum_{i=1}^M P(A/c_i) P(c_i)$$

$$P(ANB), P(B)P(A)$$

(law of total probability)

Hence A and B are independent

Question 1

Solution:-

Let A be the event that the sum is 7

Let B the sum is odd

So,

$$A: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$$

$$B: (1,2), (1,4), (1,6), (2,1), (2,3), (2,5)$$

... (6,1), (6,3), (6,5) and

$$A \cap B: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$$

Now, there are 36 outcomes in all in tossing 2 fair dice, Hence

$$P(A) = 6/36 = 1/6 \quad P(B) = 18/36 = 1/2$$

$$\text{and } P(A \cap B) = 6/36 = 1/6 \text{ so, } P(A|B)$$

$$= P(A \cap B) / P(B) = 1/3$$

Bi-nomial Distribution

A binomial distribution can be thought of as simply the probability of a success or failure outcome in an experiment or survey that is repeated multiple times.

$$P(X=x) f(x) = \binom{n}{x} p^x q^{n-x}$$

Bi-nomial Frequency Distribution

If the binomial probability distribution is multiplied by N , the number of experiment or sets, the resulting distribution is known as

$$N \binom{n}{x} p^x q^{n-x}$$

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PROBABILITY :-

Q7

Coefficient of variation

For Data Set A :-

$$CV = \frac{6}{4} \times 100$$

$$CV = \frac{3}{4} \times 100$$

$$CV = 6.7$$

For Data Set B :-

$$CV = \frac{11}{60} \times 100$$

$$CV = 18.3$$

For Data Set C

$$CV = \frac{5}{50} \times 100$$

$$CV = 100$$

For Data Set D $CV = \frac{15}{25} \times 100$

$$CV = 60$$

Question 3

$$\sum_{x=3}^6 \binom{8}{x} \binom{2}{3}^x \binom{1}{3}^{8-x}$$

$$\binom{8}{3} \binom{2}{3}^3 \binom{1}{3}^5 + \binom{8}{4} \binom{2}{3}^4 \binom{1}{3}^4$$

$$+ \binom{8}{5} \binom{2}{3}^5 \binom{1}{3}^3 + \binom{8}{6} \binom{2}{3}^6 \binom{1}{3}^2$$

$$= \frac{8}{3^8} [56 + 140 + 224 + 224]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$