Department of Electrical Engineering
Assignment
Date:13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	**Pir Meher Ali Shah**	**Total Marks:**	30

Student Details

Name: **ADNAN SHAH** **Student ID:** **13692**

Q1.	(a)	Consider the following analog signal $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate F_s = 100Hz.What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal y_a(t) we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $x(n) = \begin{cases} 0.5n , & n \geq 0 \\ 0, & n < 0 \end{cases}$ This is signal is sampled at the rate F_s= 200Hz. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $x[n] = \{ 2, \underset{\uparrow}{1}, -2, 3, -4\}$, $h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $\boldsymbol{x(n)=}\begin{cases} \boldsymbol{\alpha^{n+1}}, & \boldsymbol{-3 \le n \le 5} \\ \boldsymbol{0}, & \boldsymbol{elsewhere} \end{cases}$ $\boldsymbol{h(n)=}\begin{cases} \boldsymbol{2^{n}}, & \boldsymbol{0 \le n \le 4} \\ \boldsymbol{0}, & \boldsymbol{elsewhere} \end{cases}$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i) $X(n)\begin{cases} (\frac{1^n}{4}) & , n \ge 0 \\ (\frac{1}{3})^{-n} & , n < 0 \end{cases}$ ii) $X(n)=\begin{cases} (\frac{1}{2})^n - 3^n & , \quad n \ge 0 \\ 0 & , \quad elsewhere \end{cases}$	Marks 10 CLO 2

Adnan Shah(13692)

Ans.1 (a)(i)

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

Solution:

minimum Sampling rate will be;

According to Nyquist criteria:

$f_s \geq 2 f_{max}$

Here; $f_1 = \frac{w_1}{2\pi}$

and $w_1 = 100\pi$

So $f_1 = \frac{\cancel{100}^{50}\pi}{\cancel{2}\pi}$

$\{ f_1 = 50 \text{ Hz} \}$

And $f_2 = \frac{\cancel{200}^{100}\pi}{\cancel{2}\pi}$

$\{ f_2 = 100 \text{ Hz} \}$

we know that $f_2 > f_1$

So $f_s \geq 2 \times f_2$

$f_s \geq 2 \times 100$ ⇒ minimum Sampling rate required to avoid aliasing.

Ans ① ⓐ ⓘⓘ Sampeling rate $F_s = 100\ Hz$

Solution:

$$F_s = 100\ Hz$$

So f_1' will be;

$$f_1' = \frac{f_1}{F_s} \qquad \because f_1 = 50\ Hz$$

$$f_1' = \frac{\cancel{50}^{1}}{\cancel{100}_{2}}$$

$$f_1' = 0.5\ Hz$$

Hence f_2 will be;

$$f_2' = \frac{f_2}{F_s}$$

$$f_2' = \frac{100}{100}$$

$$f_2' = 1\ Hz$$

So $\omega_1' = 2\pi f_1'$

$$\omega_1' = 2\pi(0.5)$$

$$\{\omega_1' = \pi\}$$

And $\omega_2' = 2\pi f_2'$

$$\omega_2' = 2\pi(1)$$

$$\{\omega_2' = 2\pi\}$$

$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$

So the discrete time Signal after Sampling will be;

$x[n] = 3\cos \pi n + 4\sin 2\pi n$

So the effect on newly generated discrete time signal is that there will be no Aliasing effect. So when we reconstruct the Signal there will be no distortion. and we can get the desired ~~a~~ signal.

1 a (iii) Yes, we can reconstruct the orignal Signal if we use ideal interpolation.

the folding frequency is

$= \frac{Fs}{2}$

$= \frac{100}{2}$

$= 50\,HZ$

The origina1 signal frequencies

~~is~~ are;

$f_1 = 50 HZ$, $f_2 = 100 HZ$

Hence these frequencies are equal or greater than folding frequency.

So the signal will be;

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

The signal is exactly the same to orignal Because we use ideal interpolation.

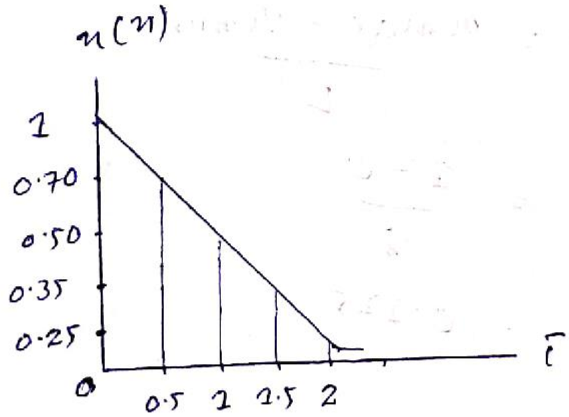
Ans 1 (b) (i) $x(n) = \begin{cases} 0.5^n & , n \geq 0 \\ 0 & , n < 0 \end{cases}$

$F_s = 2\ Hz$

Solution:

$x(n)$	0.5^n
0	$0.5^0 = 1$
0.5	$0.5^{0.5} = 0.707$
1	$0.5^1 = 0.5$
1.5	$0.5^{1.5} = 0.353$
2	$0.5^2 = 0.25$

The Signal will be;



(1) (15) (ii) Given data:

carry 3 bits per Sample

Required:

Quantization level &
Quantization resolution.

Solution:

Quantization level =

Bits = $n = 3$

$L = 2^n$

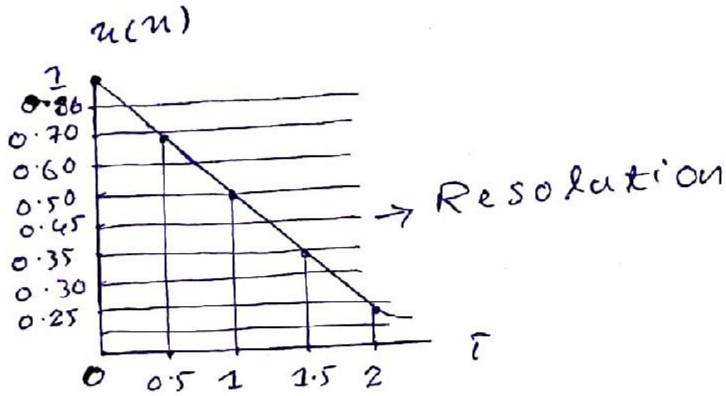
$L = 2^3$

$L = 8$ levels

$$Q.\ Resolution = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8}$$

$$= 0.125$$



Quantization level

①ⓑⓘⓘⓘ

	Discrete time Signal	Truncation	Rounding off	Error
0	1	1.0	1.0	0
1	0.853	0.8	0.9	-0.1
2	0.707	0.7	0.7	0
3	0.603	0.6	0.6	0
4	0.50	0.5	0.4	0.1
5	0.426	0.4	0.4	0
6	0.353	0.3	0.3	0
7	0.176	0.1	0.2	-0.1

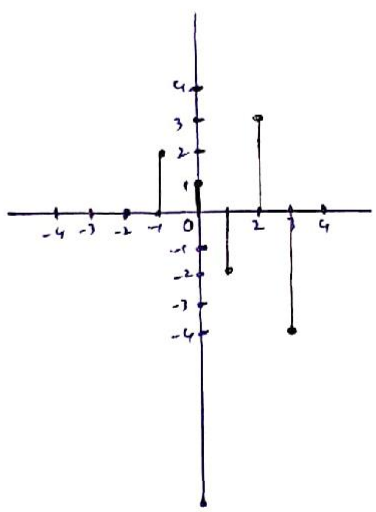
Ans ② ⓐ $x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$

$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$

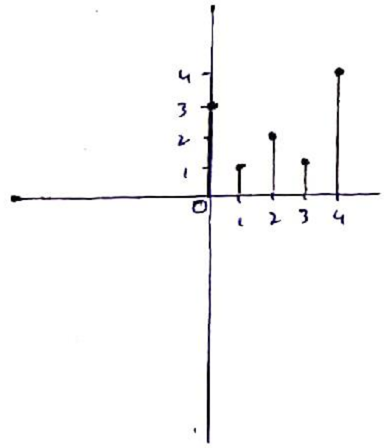
Solution:

To find $y[n]$, we convolve $x[n]$ and $h[n]$

$x[n]$



$h[n]$



So to find $y[n] = x[n] * h[n]$

$$y[n] = \sum_{n=-\infty}^{\infty} x[n] * h[n]$$

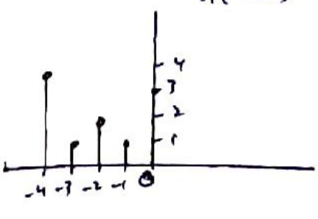
Now in convolution we replace n with K

$$y[K] = \sum_{K=-\infty}^{\infty} x[K]\, h[-K]$$

Now introduce shifting of n_0 in $h[-K]$

$$y[n_0] = \sum_{K=-\infty}^{\infty} x[K]\, h[n_0 - K]$$

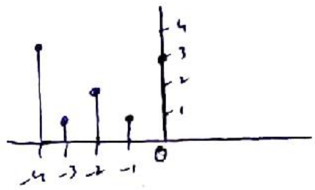
$h[-K]$



For $n_0 = 0$

$$y[0] = \sum_{k=1}^{0} x(-1)\,h(-1) + x(0)\,h(0)$$

$h[-k]$



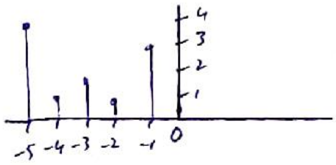
$= (2 \times 1) + (1 \times 3)$

$= 2 + 3$

$= 5$

For $n_0 = -1$

$h[-1-k]$



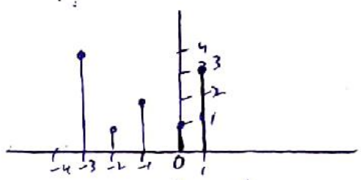
$h(-1-k)$

$y(-1) = x(-1)h(-1)$

2×3

$= 6$

$h[1-k]$



For $n_0 = 1$

$h(1-k)$

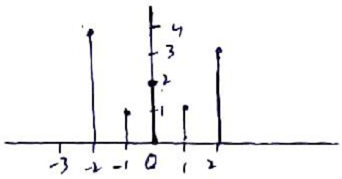
$y(1) = x(-1)\,h(-1) + x(0)\,h(0) + x(1)\,h(1)$

$= (2 \times 2) + (1 \times 1) + (-2 \times 3)$

$= 4 + 1 + (-6)$

$= -1$

$h[2-k]$



For $n_0 = 2$

$h(2-k)$

$y(2) = x(-1)\,h(-1) + x(0)\,h(0) + x(1)\,h(1) + x(2)\,h(2)$

$= (2 \times 1) + (1 \times 2) + (-2 \times 1) + (3 \times 3)$

$= 2 + 2 + (-2) + 9$

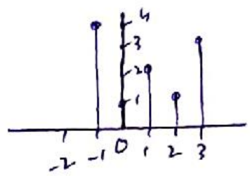
$= 2 + 9$

$= 11$

For $n_0 = 3$

$y(3) = x(-1)h(-1) + x(0)h(0) + x(-1)(h(-1)) +$
$x(2)h(2) + x(3)(h(3))$

h[3-k]



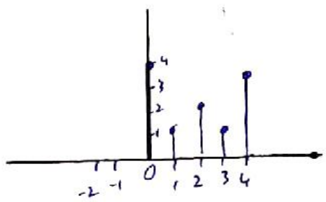
$= (2 \times 4) + (1 \times 1) + (-2 \times 2) +$
$(3 \times 1) + (-4 \times 3)$

$= 8 + 1 - 4 + 3 - 12$

$= 9 - 4 - 9$

$y(3) = -4$

h[4-k]



For $n_0 = 4$

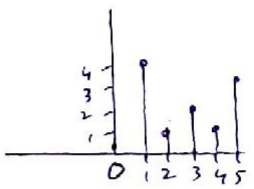
$y(4) = x(0)h(0) + x(1)h(1) +$
$x(2)h(2) + x(3)(h(3))$

$= (1 \times 4) + (-2 \times 1) + (3 \times 2) + (-4 \times 1)$

$= 4 - 2 + 6 - 4$

$y(4) = 4$

h[5-k]



For $n_0 = 5$

$y(5) = x(1)h(1) + x(2)h(2) +$
$x(3)h(3)$

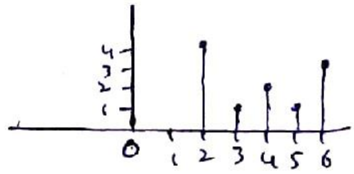
$= (-2 \times 4) + (3 \times 1) + (-4 \times 2)$

$= -8 + 3 - 8$

$= -13$

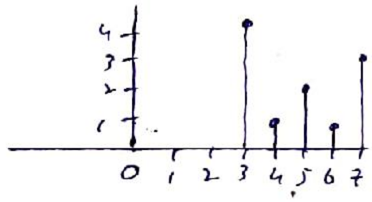
For $n_0 = 6$

$h[6-k]$



$$y(6) = x(2)\,h(2) + x(3)\,h(3)$$
$$= (3 \times 4) + (-4 \times 1)$$
$$= 12 - 4$$
$$= 8$$

$h[7-k]$

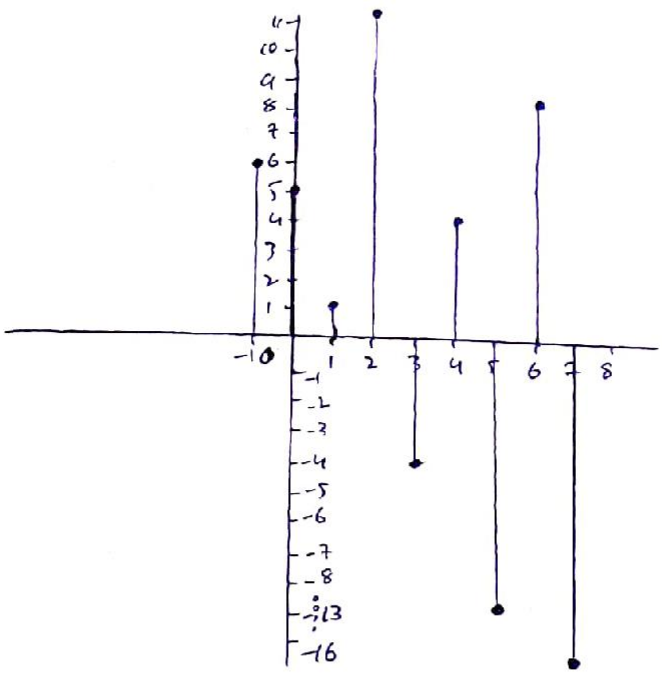


For $n_0 = 7$

$$y(7) = x(3)\,(h(3))$$
$$= -4 \times 4$$
$$= -16$$

So there is no more overlaping.

$$y[n] = \{6, \underset{\uparrow}{5}, -1, 11, -4, 4, -13, 8, -16\}$$



Ans (2) (b)

$$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

Solution:

$$x(n) = \alpha^{n+1}$$

$$= \alpha^{-3+1} = \alpha^{-2}$$

$$= \alpha^{-2+1} = \alpha^{-1}$$

$$= \alpha^{-1+1} = \alpha^{0} = 1$$

$$= \alpha^{0+1} = \alpha^{1}$$

$$= \alpha^{1+1} = \alpha^{2}$$

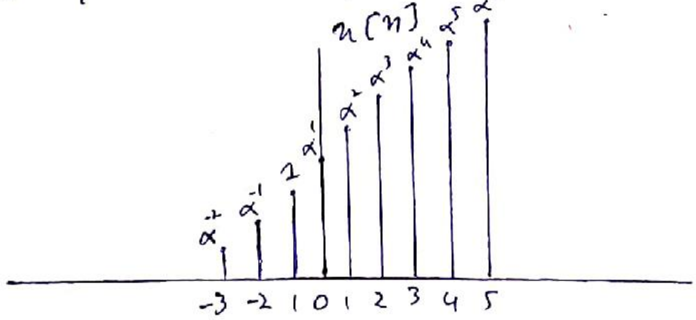
$$= \alpha^{2+1} = \alpha^{3}$$

$$= \alpha^{3+1} = \alpha^{4}$$

$$= \alpha^{4+1} = \alpha^{5}$$

$$= \alpha^{5+1} = \alpha^{6}$$

$$x(n) = \{\alpha^{-2}, \alpha^{-1}, 1, \underset{\uparrow}{\alpha^{1}}, \alpha^{2}, \alpha^{3}, \alpha^{+4}, \alpha^{5}, \alpha^{6}\}$$



$h(n) = 2^n$

$2^0 = 1$

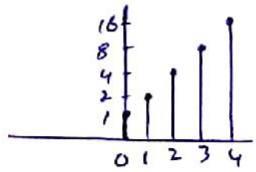
$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

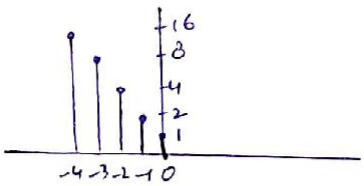
$h[K]$



$h(n) = \{ \underset{\uparrow}{1}, 2, 4, 8, 16 \}$ Replacing n with K

$$y(n_0) = \sum_{K=-\infty}^{\infty} x(K)\, h(n_0 - K)$$

$h[-K]$



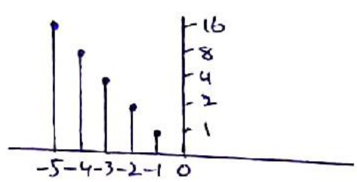
For $n_0 = 0$

$y(0) = x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0)$

$= (\alpha^{-2} \cdot 8) + (\alpha^{-1} \cdot 4) + (1 \times 2) + (\alpha \cdot 1)$

$= 8\alpha^{-2} + 4\alpha^{-1} + 2 + \alpha$

$h[-1-K]$



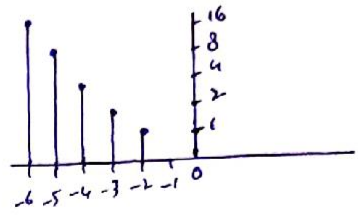
For $n_0 = -1$

$y(-1) = x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1)$

$= (\alpha^{-2} \cdot 4) + (\alpha^{-1} \cdot 2) + (1 \times 1)$

$= 4\alpha^{-2} + 2\alpha^{-1} + 1$

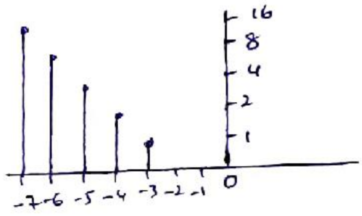
$h[-2-k]$



For $n_0 = -2$

$$y(-2) = x(-3)h(-3) + x(-2)h(-2)$$
$$= (\alpha^{-2}\cdot 2) + (\alpha^{-1}\cdot 1)$$
$$= 2\alpha^{-2} + \alpha^{-1}$$

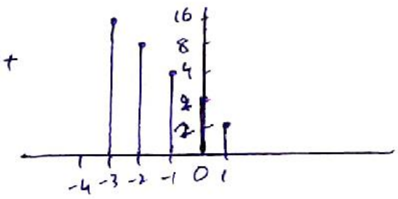
$h[-3-k]$



For $n_0 = -3$

$$y(-3) = x(-3)h(-3)$$
$$= (\alpha^{-2}\cdot 1)$$
$$= \alpha^{-2}$$

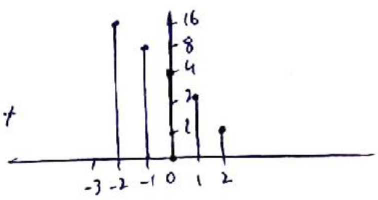
$h[1-k]$



For $n_0 = 1$

$$y(1) = x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1)$$
$$= (\alpha^{-2}\cdot 16) + (\alpha^{-1}\cdot 8) + (1\times 4) + (\alpha\cdot 2) + (\alpha^{2}\cdot 1)$$
$$= 16\alpha^{-2} + 8\alpha^{-1} + 4 + 2\alpha + \alpha^{2}$$

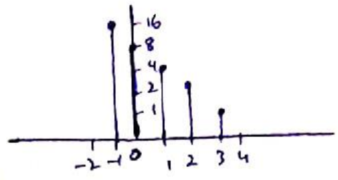
$h[2-k]$



For $n_0 = 2$

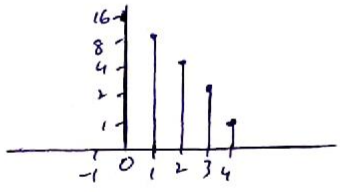
$$y(2) = x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$$
$$= (\alpha^{-1}\cdot 16) + (1\times 8) + (\alpha\cdot 4) + (\alpha^{2}\cdot 2) + (\alpha^{3}\cdot 1)$$
$$= 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^{2} + \alpha^{3}$$

$u[3-k]$



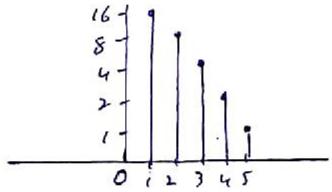
For $n_0 = 3$

$$y(3) = (1\times 16) + (\alpha\cdot 8) + (\alpha^2\cdot 4) + (\alpha^3\cdot 2) + (\alpha^4\cdot 1)$$
$$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$$



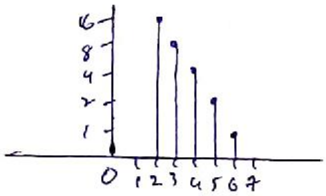
For $n_0 = 4$

$$y(4) = (\alpha\cdot 16) + (\alpha^2\cdot 8) + (\alpha^3\cdot 4) + (\alpha^4\cdot 2) + (\alpha^5\cdot 1)$$
$$= 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$$



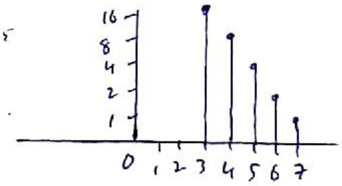
For $n_0 = 5$

$$y(5) = (\alpha^2\cdot 16) + (\alpha^3\cdot 8) + (\alpha^4\cdot 4) + (\alpha^5\cdot 2) + (\alpha^6\cdot 1)$$
$$= 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$$



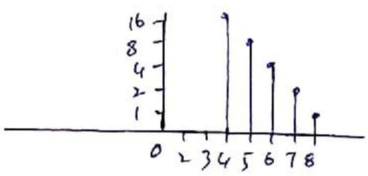
For $n_0 = 6$

$$y(6) = (\alpha^3\cdot 16) + (\alpha^4\cdot 8) + (\alpha^5\cdot 4) + (\alpha^6\cdot 2)$$
$$= 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$$



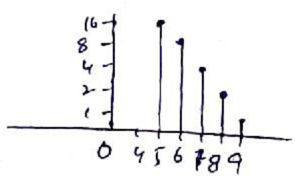
For $n_0 = 7$

$$y(7) = (\alpha^4\cdot 16) + (\alpha^5\cdot 8) + (\alpha^6\cdot 4)$$
$$= 16\alpha^4 + 8\alpha^5 + 4\alpha^6$$



For $n_0 = 8$

$$y(8) = (\alpha^5\cdot 16) + (\alpha^6\cdot 8)$$
$$= 16\alpha^5 + 8\alpha^6$$



For $n_0 = 9$

$$y(9) = \alpha^6\cdot 16$$
$$= 16\alpha^6$$

Ans (3) (i) $x(n) = \begin{cases} (\frac{1}{4})^n, & n \geq 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$

Solution:

$$x(n) = (\frac{1}{4})^n$$

Applying Z-transform

$$x(Z) = \sum_{n=-\infty}^{\infty} x[n] Z^{-n}$$

$$x_1(Z) = \sum_{n=-\infty}^{\infty} [\frac{1}{4}]^n Z^{-n}$$

$$x_1(Z) = \frac{1}{1-\frac{1}{4}Z^{-1}} \quad -(1) \qquad \because |ROC| = |Z| > \frac{1}{4}$$

Hence;

$$x_2(n) = (\frac{1}{3})^n$$

Applying Z-transform

$$x_2(Z) = \sum_{n=-\infty}^{\infty} (\frac{1}{3})^{-n} Z^{-n} \qquad \because n < 0$$

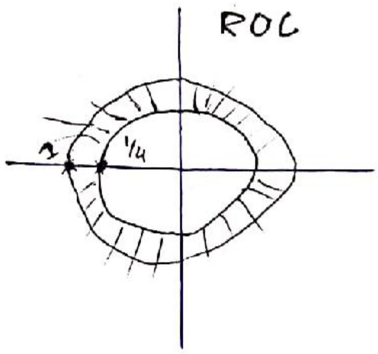
$$x_2(Z) = \frac{1}{1-\frac{1}{3}Z} \quad -(2) \qquad \because |ROC| = |Z| > 3$$

Now adding eqn (1) and (2)

$$x[Z] = x_1(Z) + x_2(Z)$$

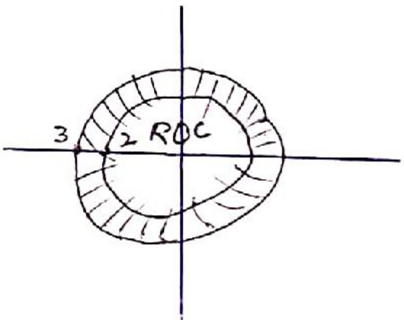
$$x[Z] = \frac{1}{1-\frac{1}{4}Z^{-1}} + \frac{1}{1-\frac{1}{3}Z}$$

$ROC = \frac{1}{4} < |Z| < 3$



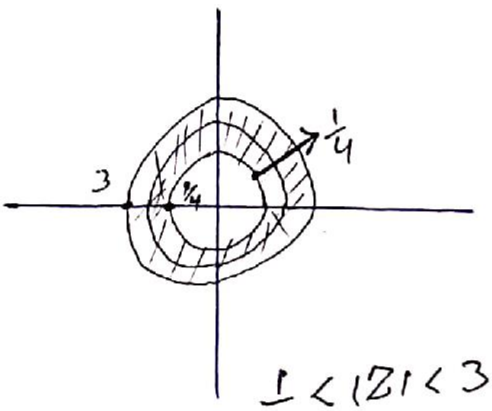
out side

$|Z| > \frac{1}{4}$



inside

$|Z| < 3$



in middle

$\frac{1}{4} < |Z| < 3$

Ans ③ (ii)

$$x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \geq 0 \\ 0 & , \text{elsewhere} \end{cases}$$

Solution:

$$x(n) = (\frac{1}{2})^n - 3^n$$

Now applying Z-transform

$$x(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[(\frac{1}{2})^n - 3^n \right] Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (\frac{1}{2})^n Z^{-n} - \sum_{n=-\infty}^{\infty} 3^n Z^{-n}$$

$$= \frac{Z}{Z-\frac{1}{2}} - \frac{Z}{Z-3} \quad \because \text{Taking L.C.M}$$

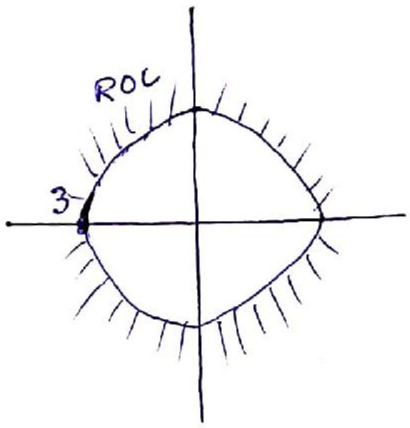
$$= \frac{Z(Z-3) - (Z-\frac{1}{2})Z}{(Z-\frac{1}{2})(Z-3)}$$

$$= \frac{\cancel{Z^2} - 3Z - \cancel{Z^2} + \frac{1}{2}Z}{(Z-\frac{1}{2})(Z-3)}$$

$$= \frac{\frac{-6+1}{2} Z}{(Z-\frac{1}{2})(Z-3)}$$

$$= \frac{-\frac{5}{2} Z}{(Z-\frac{1}{2})(Z-3)} \quad \because ROC = |Z| > 3$$

ROC =



out side
from 3

$|Z| > 3$