Name :- Hammad Ali
ID :- 16086
Department:- Bs (Civil Engineering)
Section :- (A)
Subject :- Applied Calculas
Submitted :- Madam Shomaila Mazhar
Date :- 25.9.2020

Question 1

Sol

Cordinate of $P = (4,1,3)$

$$OP = 4i + 1j + 3k$$

OR $OQ = \vec{OQ} - \vec{Op}$

$$= (i + 2j + 4k) - (4i + 1j + 3k)$$

$$= -3i + 1j + 1k \quad \text{—— (1)}$$

Now that distance b/w P & $Q = |PQ|$

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \quad \text{—— (2)}$$

Let M be the point which divided PQ in ratio 1:3 Then by ratio theorom Position vector of $M = \vec{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1 + 3}$$

$$= \frac{12\hat{i} + 3\hat{j} + 9\hat{k}}{4} + \hat{i} + 2\hat{j} + 4\hat{k}$$

$$= \frac{13\hat{i} + 5\hat{j} + 13\hat{k}}{4} \quad \text{---} \quad (3)$$

Hence eq (1), (2) & (3) are the required Solution.

Q No 2

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+1 & 4x^3+10x+4 \\ & \cancel{4x^3} \quad \pm 2x^2 \\ \hline & -\cancel{2x^2}+10x+4 \\ & \mp 2x^2 \mp x \\ \hline & 11x+4 \end{array}$$

So

$$2x-1+\frac{11x+4}{2x^2+x}=\frac{4x^3+10x+4}{2x^2+x}$$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1+\int \frac{11x+4}{2x^2+x} \quad \text{—— (1)}$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \longrightarrow (2)$$

Now find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x}+\frac{B}{(2x+1)} \quad \text{—— (A)}$$

$$\frac{11x+4}{x(2x+1)} = \frac{A(2x+1)+Bx}{x(2x+1)}$$

$$11x+4 = A(2x+1)+Bx \quad \text{—— (3)}$$

Put $x=0$ in (3)

$\boxed{4 = A}$

Now put $x = -1/2$ in 3

$11(-1/2) + 4 = B(-1/2)$

$\frac{-11}{2} + 4 = \frac{-B}{2}$

$\frac{-11+8}{\cancel{2}} = \frac{-B}{\cancel{2}}$

$-3 = -B \Rightarrow \boxed{B = 3}$

Putting the values of A and B in A

$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$

Taking integral on both

$\int \frac{11x+4}{x(2x+1)} dx = \int \frac{4}{x} dx + \int \frac{3}{2x+1} dx$

$= 4\int \frac{1}{x} dx + 3\int \frac{1}{2x+1} dx$

$= 4\ln(x) + 3/2 \ln|2x+1|$

Putting these values in 2

$= x^2 - x + 4\ln|x| + 3/2 \ln(2x+1)$

Now put val in 1

$\int \frac{4x^3+10x+4}{2x+x} dx = x^2 - x + 4\ln(x) + 3/2 \ln|2x+1| + C$

Answ

Q No 3
a) $\int_0^2 x^2 e^x \, dx$

Now find

$= \int e x^2 e^x \, dx$

$= x^2 \int e^x dx - \int \left(\int e^x dx \, \frac{dx}{dx} x^2 \right) dx$

$= x^2 e^x - \int e^x (2x) \, dx$

$= x^2 e^x - 2\int x e^x \, dx$

$= x^2 e^x - 2\left[x \int e^x dx - \int \left(\int e^x dx \, \frac{d}{dx} x \right) dx \right]$

$= x^2 e^x - 2\left[x e^x - \int e^x dx \right]$

$= x^2 e^x - 2x e^x + 2e^x$

Now put limit

$= \left| x^2 e^x - 2x e^x + 2e^x \right|_0^2$

$= (2^2 e^2 \;\; 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0))$

$= (\cancel{4e^2} - \cancel{4e^2} - 2e^2 - 2)$

$= 2e^2 - 2$ Answer.

Q 3b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

First find integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = ? \rightarrow (1)$$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$2dy = \frac{1}{\sqrt{x}}\,dx \quad \text{Put in (1)}$$

$$\int \sin(y)\,(2dy) = 2\int \sin(y)\,dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

$$= \text{put } y = \sqrt{x}$$

$$= -2\cos\sqrt{x}$$

Put limits

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2\left(\cos\sqrt{2} - \cos 1\right)$$

$$= -2\cos\sqrt{2} + 2\cos(1) \text{ Ans.}$$

Q4

The laplace equation in 3.d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow (A)$$

So $u(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$u(x,y,z) = (x^2+y^2+z^2)$

$u(x,y,z) = (x^2+y^2+z^2)$

$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$

$$\frac{\partial u}{\partial x} = \frac{-1}{2}(x^2+y^2+z^2)(2x)$$

$$\frac{\partial u}{\partial x} = -x(x^2-y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x(-3/x)(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-1/2}\right]$$

$$= 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \quad (1)$$

Now

$$\frac{\partial y}{\partial y} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(-3/\cancel{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}\left(\cancel{2}y\right) + \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2\left(x^2+y^2+z^2\right)^{-5/2} + \left(x^2+y^2+z^2\right)^{-3/2} \longrightarrow (2)$$

$$\frac{\partial u}{\partial z} = \frac{-1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}\left(\cancel{2}z\right)$$

$$\frac{\partial u}{\partial z} = -z\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2\left(x^2+y^2+z^2\right) - \left(x^2+y^2+z^2\right)^{-3/2} \longrightarrow (3)$$

Putting (1) (2) and (3) in (A)

$$3\cancel{x}^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} + 3y^2\left(x^2+y^2+z^2\right)^{-5/2}$$

$$-\left(x^2+y+z^2\right)^{-3/2} + 3z^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}\left[3x^2 - \left(x^2+y^2+z^2\right) + 3y^2 - \left(x^2+y^2+z^2\right) + 3z^2 - \left(x^2+y^2+z^2\right)\right]$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}\left[\cancel{3x^2 - x^2 - y^2} - z^2 + 3y^2 - \cancel{x^2} - \cancel{y^2} - z^2 + 3z^2 - \cancel{x^2} - \cancel{y^2} - z\right]$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}(0) = 0$$

So the given is solution of Laplace equation is satisfied.