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Section : C

Subject : Differential Eq-

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ANS # 01

Fourier Series

$$f_{(t)} = 1+t \quad , -\pi \le t \le \pi$$

Sol;

formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos x + \sum_{n=1}^{\infty} b_n \cdot \sin x \quad (A)$$

Here;

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$= \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2}\right)\right)$$

$$= \frac{1}{2\pi}\left(2\pi + 2\left(\frac{\pi^2}{2}\right)\right)$$

$$\boxed{a_0 = \frac{1}{2\pi}\left(2\pi + \pi^2\right)}$$

Now

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt \cdot dt$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)\cos nt \cdot dt$$

$$= \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n}\Bigg|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

$$= -1/\pi n^2 \left(\cos n\pi - \cos n(-\pi) \right)$$

$$= -1/\pi n^2 \left(-1-(-1) \right)$$

$$= -1/\pi n^2 \left(-1+1 \right)$$

$$= \frac{-1}{\pi n^2} (0)$$

$$\boxed{a_n = 0}$$

Now;

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) . \sin nt . dt$$

$$b_n = 1/\pi \int_{-\pi}^{\pi} (1+t) \sin nt . dt$$

$$= \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi} \sin nt - \int\left(\sin nt \cdot \frac{d}{dt}(1+t)\,dt\right)\right)$$

$$= \frac{1}{\pi}\left(\left((1+t)\frac{(-\cos nt)}{n}\right)\Big|_{-\pi}^{\pi} - \int\left(-\frac{\cos nt}{n}(1)\right)\right)$$

$$= \frac{1}{\pi}\left(-\frac{(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} \left(\cancel{\frac{\sin nt}{n^2}}\Big|_{-\pi}^{\pi}\right)^{0}\right)$$

$$= \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi)\right)$$

$$= \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos n\pi} + \pi\cos n\pi\right)$$

$$= -\frac{1}{n\pi}\left(2\pi\cos n\pi\right) \qquad \left(\cos n\pi = \frac{(-1)^{n+1}}{n}\right)$$

$$b_n = -\frac{2}{n}(-1)^{n+t}$$

By putting values in eq "A"

$$f(t) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + \sum_{n=1}^{\infty} \frac{2(1)^{n+1}}{n} \sin nt$$

ANS 02

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values = ?

Step = 1

We have

$$(A - \lambda I)x = 0$$

A = given matrix

Step 02

We have; a characteristic equation is:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 1-\lambda \end{vmatrix} = 0$$

Step 03

$$\lambda^3 - \left|\text{Some of diagnol element}\right| \lambda^2 + \left|\text{Sum of diagnol minor}\right| \lambda - |A| = 0 \quad \text{— B}$$

Sum of diagnal element $= 1+1+2 = 4$

Sum of diagnal minar $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq (B)

(p.t.o)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{—— (c)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(2-8) - 0 + 1(6-0)$$

$$= -6 + 6$$

$$= 0$$

By putting values in eq (c)

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

Using quadratic formulae

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

$$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2}\right)$$

Solution

ANS 03"

$$5x + 4z + 2m = 3$$

$$x - y + 2z + m = 1$$

$$4x + y + 2z = 1$$

$$x + y + z + m = 0$$

Sol (A)

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$R_4 - R_2 \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -1 & -3 \\ 0 & 2 & 1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 1/5 & 1/5 \end{array}\right] \quad 5 \times R_3 \text{ and } 5 \times R_4$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3 \text{ and } 5R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2 \times 5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

Page#12

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/ \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad R_2 \times -5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

(13)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 0 & \frac{31}{21} \\ 0 & 0 & 1 & 0 & -\frac{11}{21} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & 0 & \frac{31}{21} \\ 0 & 0 & 1 & 0 & -\frac{11}{21} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right]$$

$$(x, y, z, m) = \left(\frac{3}{4}, \frac{31}{21}, -\frac{11}{21}, \frac{1}{3}\right)$$

$$x = \frac{3}{4}$$

$$y = \frac{31}{21}$$

$$z = -\frac{11}{21}$$

$$m = \frac{1}{3}$$

ANS 04"

verify that

$$u(x,t) = \sin(x+2t)$$

one dimension equation

Solootion

$$u(x,t) = \sin(x+2t)$$

Diffrientiate w.r.t x partially

$$\partial u/\partial x = \frac{\partial}{\partial x}\sin(x+2t)$$

$$\partial u/\partial x = \cos(x+2t)\,\partial/\partial x\,(x+2t)$$

$$\partial u/\partial x = \cos(x+2t)(1+0)$$

$$\partial u/\partial x = \cos(x+2t$$

$$\partial^2 u / \partial x^2 = \partial / \partial x \cos(x+2t)$$

$$\partial^2 u / \partial x^2 = -\sin(x+2t)(1+0)$$

$$\boxed{\partial^2 u / \partial x^2 = -\sin(x+2t)}$$

and $\quad u(x,t) = \sin(x+2t$

Diff w.r.t "t"

$$\cancel{\partial u / \partial t} \quad \partial u / \partial t = \partial / \partial t \sin(x+2t)$$

$$\partial u / \partial t = \cos(x+2t)(0+2)$$

$$\partial u / \partial t = 2\cos(x+2t)$$

$$\boxed{\partial^2 u / \partial t^2 = -4\sin(x+2t)}$$

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(16)

we know that one dimension

$$\partial^2 u/\partial t^2 = c^2 \partial^2 u/\partial x^2$$

$$\cancel{-4\sin(x+2t) = c^2 \partial^2 u/\partial x^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

orbitary Constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2\sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

it well be verified

orbitary Constant $c = 2$

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