Quiz

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Section "A"

subject Applied calculus

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Q 1

$$\int_0^1 \frac{4t^3 - 2t^2 + 3t - 1}{2t^2 + 1}\, dt$$

Solution:

$$= \int_0^1 \frac{4t^3 + 3t^2 - 2t^2 - 1}{2t^2 + 1}\, dt$$

$$= \int_0^1 \frac{t(4t^2 + 3) - (2t^2 + 1)}{2t^2 + 1}\, dt$$

$$= \int_0^1 \frac{t(4t^2 + 3)}{2t^2 + 4}\, dt - \int_0^1 \frac{2t^2 + 1}{2t^2 + 1}\, dt$$

$$= \int_0^1 \frac{t(4t^2 + 3)}{2t^2 + 1}\, dt - \int_0^1 1\, dt$$

$$= \int_0^1 \frac{t(4t^2 + 3)}{2t^2 + 1}\, dt - t\Big|_0^1$$

$$= \int_0^1 \frac{t(4t^2 + 3)}{2t^2 + 1}\, dt - [1 - 0]$$

$$= \int_0^1 \frac{t(4t^2 + 3)}{2t^2 + 1} - 1 \quad \text{---} \quad (1)$$

2 (6) (2) 1

Now let

$$2t^2+1=y$$

As $t \to 1$ i.e $y=3$

$t \to 0$ i.e $y=1$

$\Rightarrow 2t^2+1=y$

$2t^2=y-1$

$4t^2=2y-2$

$4t^2+3=2y-2+3$

$4t^2+3=2y+1$

Now diff

$$4t=\frac{dy}{dt}$$

$$\Rightarrow \int_1^3 \frac{t(2y+1)}{4} = \frac{dy}{4t} - 1$$

$$= \int_1^3 \frac{2y+1}{4y}\, dy - 1$$

$$= \frac{1}{4}\left[\int_1^3 \frac{2y}{y}\, dy + \int_1^3 \frac{1}{y}\, dy\right] - 1$$

$$= \frac{1}{4}\left[\int_1^3 2\, dy + \int_1^3 \frac{1}{y}\, dy\right] - 1$$

$$= \frac{1}{4}\left[2y\,|_1^3 + \ln y\,|_1^3\right] - 1$$

$$= \frac{1}{4}\left[2(3) - 2(1) + \ln(3) - \ln(1)\right] - 1$$

$$= \frac{1}{4}\left[6 - 2 + 1.0986\right] - 1$$

$$= \frac{1}{4}\left[5.0986\right] - 1$$

$$= 1.27465 - 1$$

$$= 0.2746 \quad \text{Ans.}$$

Q2)

$$\int_1^3 t \sin t^2 \, dt$$

Sol:

let $t^2 = y$

D.W.R.T - "t"

$$2t = \frac{dy}{dt}$$

$$dt = \frac{dy}{2t}$$

$t^2 = y$
$(3)^2 = y$
$9 = y$

Now

As $t \to 3$ then $y = 9$

As $t \to 2$ then $y = 4$

So $$\int_2^3 t \sin t^2 \, dt = \int_4^9 t \sin y \frac{dy}{dt}.$$

$$= \frac{1}{2} \int_4^9 \sin y \, dy$$

$$= \frac{1}{2} - \cos \Big|_4^9$$

$$= \frac{1}{2} [\cos(4) - \cos(9)]$$

$$= \frac{1}{2} [0.998 - 0.987]$$

$$= \frac{0.998 - 0.987}{2} = 0.0055 \text{ Ans.}$$