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Module : 7th

Subject : Numerical Analysis

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Exam : Final Term

Q1) Apply both Euler's method and the improved Euler's method.

$$dy/dx = 2x; \quad y(0) = 1$$

Solution:- By Euler's Method

Given data:-

$$y(0) = 1, \quad h = 0.1 \quad x_0 = 0$$

By Formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h\,[2x_n]$$

Ist iteration:-

$$n = 0$$

$$y_1 = y_0 + h\,(2x_0)$$

$$y_1 = 1 + 0.1\,(2(0))$$

$$\boxed{y_1 = 1.0}$$

$$\rightarrow x_{n+1} = x_n + h$$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 0.1$$

$$\boxed{x_1 = 0.1}$$

2nd iteration:-

$$n = 1$$

$$y_2 = y_1 + h\,(2x_1)$$

$$y_2 = 1.0 + 0.1\,(2(0.1))$$

$\boxed{y_2 = 1.02}$

$x_{n+1} = x_n + h$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

3rd iteration

$n = 2$

$y_3 = y_2 + h(2x_2)$

$y_3 = 1.02 + 0.1(2(0.2))$

$\boxed{y_3 = 1.06}$

$x_{n+1} = x_n + h$

$x_3 = x_2 + 0.1$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

b) by Modified Euler Method

$dy/dx = 2x$

Given data:-

$y_0 = 1, \; x_0, 0, \; h = 0.1$

Formula

$y^*_{n+1} = y_n + h[f(x_n)]$

$y^*_{n+1} = y_n + h(2x_n)$ — ①

$y_{n+1} = y_n + h/2 \, [f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})]$

$$= y_n + h/2\,[2x_n + 2x_n]$$

$$= y_n + h/2\,[4x_n]$$

1st Iteration

$n = 0$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$\boxed{x_1 = 0.1}$

$y_1 = y_0 + h/2\,(4x_0)$

$y_1 = 1 + \frac{0.1}{2}\,(4(0))$

$y_1 = 1$

2nd Iteration

$n = 1$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

$y_2 = y_1 + h/2\,(4x_1)$

$y_2 = 1 + 0.1/2\,(4(0.1))$

$\boxed{y_2 = 1.02}$

3^{rd} Iteration:

$n = 2$

$x_3 = x_2 + h$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

$y_3 = y_2 + h/2\,(4x_2)$

$= 1.02 + \frac{0.1}{2}\,(4(0.2))$

$\boxed{y_3 = 1.06}$

Q2 Use the fourth order Runge kutta method to obtain a solution of $dy/dx = x^2 + x - y$ Subject to $y = 0$ when $x = 0$, for $0 \le x \le 0.6$ with $h = 0.2$ - work throughout to four decimal places.

Given data:-

$y = 0,\ x = 0 \quad h = 0.2 \quad 0 \le x \le 0.6$

$y_{n+1} = y_n + k$

"Ist Iteration"

$n = 0$

$y_1 = y_0 + k, \quad k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_n, y_n)$

$k_1 = h(x_0^2 - x_0 - y_0)$

$k_1 = 0.2(0^2 - 0 - 0)$

$\boxed{k_1 = 0}$

$k_2 = hf(x_n + h/2,\ y_n + h/2)$

$= 0.2f(x_0 + h/2,\ y_0 + h/2)$

$= 0.2f\left(0 + 0.2/2,\ 0 + \frac{0.2}{2}\right)$

$= 0.2f(0.1,\ 0.1)$

$= 0.2(0.1^2 + 0.1 - 0.1)$

$\boxed{k_2 = 0.0020}$

$k_3 = hf\left(x_n + h/2 \,,\, y_n + k_2/2\right)$

$= 0.2\, f\left(0 + 0.2/2 \,,\, 0 + 0.002/2\right)$

$= 0.2 f\left(0.1 \,,\, 0.001\right)$

$= 0.2\left(0.1^2 + 0.1 - 0.001\right)$

$\boxed{k_3 = 0.0218}$

$k_4 = hf\left(x_n + h \,,\, y_n + k_3\right)$

$= 0.2 f\left(0 + 0.2 \,,\, 0 + 0.0218\right)$

$= 0.2 f\left(0.2 \,,\, 0.0218\right)$

$= 0.2\left(0.2^2 + 0.2 - 0.0218\right)$

$\boxed{k_4 = 0.0436}$

$k = \frac{1}{6}\left(0 + 2(0.002) + 2(0.0218) + 0.0436\right.$

$k = 0.0152$

$y_1 = 0 + 0.0152$

$\boxed{y_1 = 0.0152}$ Ans

Q. No 3:-

Given Data:-

$a = 0$, $b = 10$, $n = 10$

$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$

Solution:-

x	0	1	2	3	4	5	6	7	8	9	10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

Using formula

$$f(x)\,dx = h/2\left[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) \ldots \ldots f(x_9)) + f(10)\right]$$

$$= \frac{1}{2}\left[10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 60.3 + 61.2) + 62.1\right]$$

$\boxed{= 412.9}$ Ans

Q No 4:-

$$\int_2^3 \ln(x^3+1)\,dx$$

Use 10 Strips.

Solution:-

$n = 10$

$h = \frac{3-2}{10} = 0.1$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now using formula

$$\int_a^b f(x)\,dx = h/3\left[f(x_0) + 4\left(f(x_1) + f(x_3)\ldots\ldots\right) + 2\left(f(x_2\ldots\ldots\right) + f(x_n)\right]$$

$$= \frac{0.1}{3}\left[0.693 + 4\left(0.846 + 1.162 + 1.476 + 1.777 + 2(1.003 + 1.320 + 1.628 + 1.922\right) + 2.062\right]$$

$$\boxed{= 1.184}$$ Ans