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Semester : 11th
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Subject : Differential equation
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Question no 1

Q Solve the initial value problem

$$dy/dx = e^{y-t}\sec(y)(1+t^2) \quad y(0)=0$$

Sol: $dy/dt = e^{y-t}\sec(y)(1+t^2)$

$y(0)=0$ so $x=0$ $y=0$

$$dy = e^{y}\cdot e^{-t}\sec(y)\ (1+t^2)\,dt$$

$$\frac{1}{e^y\cdot\sec(y)}dy = (1+t^2)e^{-t}dt$$

As $\cos(y) = \frac{1}{\sec(y)}$

$$\int e^{-y}\cos y\,dy = \int(1+t^2)e^{-t}dt$$

using Integration by parts

$$e^{-y}\int\cos y\,dx - \int\left(\int\cos y\cdot d/dy\, e^{-y}\right) =$$

$$(1+t^2)\int e^{-t} - \int\left(\int e^{-t}\cdot\frac{d}{dt}(1+t^2)\right) \cdots eq\ 1$$

L.H.S

$$e^{-y}\int\cos y\,dx - \int\left(\int\cos y\cdot d/dy\, e^{-y}\right)$$

$$e^{-y}\sin y - \int(\sin y\cdot e^{-y}(-1))$$

$e^{-y}\sin y + \int(\sin y \cdot e^{-y})$

$e^{-y}\sin y + \int(e^{-y}\sin y)$

Again using integration by parts

$e^{-y}\sin y + e^{-y}(-\cos y) - \int(\int \sin y \, \frac{d}{dy} e^{-y})$

$e^{-y}\sin y + e^{-y}(-\cos y) - \int(-\cos y \, \frac{e^{-y}}{-1})$

$e^{-y}\sin y - e^{-y}\cos y - \int(\cos y \, e^{-y})$

since $\int(\cos y \, e^{-y}) = L{\cdot}H{\cdot}S$

Since it is again same to the first one so L.H.S will become

$$L{\cdot}H{\cdot}S = e^{-y}(\sin y - \cos y) - L{\cdot}H{\cdot}S$$

$$2\,L{\cdot}H{\cdot}S = e^{-y}(\sin y - \cos y)$$

$$L{\cdot}H{\cdot}S = \frac{e^{-y}(\sin y - \cos y)}{2}$$

Now taking R.H.S

$= \int (1+t^2) e^{-t} dt$

$= (1+t^2)\int e^{-t} - \int(\int e^{-t} \cdot \frac{d}{dt}(1+t^2))$

$= (1+t^2) e^{-t} + \int(-e^{-t}(2t))$

$= -(1+t^2)e^{-t} + \int((2t)e^{-t})$

again using integration by parts

$= -(1+t^2)e^{-t} + (2t\int e^{-t} - \int(\int e^{-t} \frac{d}{dt} 2t))$

$= -(1+t^2)e^{-t} + (-2te^{-t} - \int(-e^{-t} 2))$

$= -(1+t^2)e^{-t} + (-2te^{-t} + \int(2e^{-t}))$

$= -(1+t^2)e^{-t} + (-2te^{-t} - 2e^{-t}) + c$

$-(1+t^2)e^{-t} - 2te^{-t} - 2e^{-t} + C$

$= -e^{-t} - e^{-t}t^2 - 2te^{-t} - 2e^{-t} + c$

$= -(t^2+2t+3)e^{-t} + C = R.H.S$

Now take L.H.S = R.H.S

$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t} + C$

we know that

$t=0 \quad y=0$

Put it above

$\Rightarrow t(0-1) = -3 + C$

$c = 5/2$

Put value of c

$\frac{e^{-y}}{2}(\sin y - \cos y) =$
$-(x^2+2(+3)e^{-t} + 5/2$

Ans

Question no 2

Q $(\sqrt{x+y}+\sqrt{x-y})\,dx-(\sqrt{x+y}-\sqrt{x-y})\,dy=0$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \quad \ldots (1)$$

This is homogenous differential eq in x & y To solve this

Put $y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

Thus eq (1) becomes

$$v+x\cdot\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}+\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$v+x\frac{dv}{dx}=\frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}v}$$

$$v+x\frac{dv}{dx}=\frac{1+\sqrt{1-v^2}}{v}$$

Day: M T W T F S S

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

taking Integral on b/s

$$\int \frac{x\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

put $1+\sqrt{1-v^2} = t$

$$= \tfrac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int \frac{-dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1-v^2}) = \ln cx$$

$$\ln(1+\sqrt{1-v^2}) = -\ln cx$$

$$\cancel{\ln}(1+\sqrt{1-v^2}) = \cancel{\ln}(cx)^{-1}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$1+\frac{\sqrt{x^2-y^2}}{x^2} = \frac{1}{cx}$$

$$x+\sqrt{x^2-y^2} = \frac{1}{c}$$

Taking reciprocal

$$\boxed{c = \frac{1}{x+\sqrt{x^2+y^2}}} \text{ Ans}$$

It is the required solution

Day: M T W T F S S

Qustion no 3

Q $(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$

Sol $(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$

$\Rightarrow f(D)y = f(x)$

As it is non-homogenous linear equation so solution will be

$y = y_c + y_p \cdots (i)$

complementary solution y_c

$D^4 - D^2 = 0 \Rightarrow D^2(D^2+1)=0$

either $D^2=0 \Rightarrow \boxed{D=0}$

$D^2+1=0 \Rightarrow D^2 = -1$

$D = \sqrt{-1} = \boxed{D=i}$ or $D = \boxed{0+i}$

Roots are real & complex

$y_c = c_1 e^{0x} + c_2 e^{0x}(c_2\cos x + c_3 \sin x)$

$y_c = c_1 + c_2\cos x + c_3 \sin x$

$y_p = \frac{1}{f(D)} F(x)$

$$y_p = \frac{1}{D^4+D^2}(3x^2 + 4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$f(D) = D^4 + D^2$

at $D=0 \rightarrow f(D) = 0$

so $f'(D) = 4D^3 + 2D$

Now also for $D=0 \Rightarrow f'(D) = 0$

again differentiatig

so $f'(D) = 4D^3 + 2D$

Now also for $D=0 \Rightarrow f'(D) = 0$

again differentiating

$f''(D) = 12D + 2$

so for $D = 0$

$f''(0) = 12(0) + 2 = 2$

replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_p = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \cdot 2\cos x.$$

putting $D=0$ in all

$$y_p = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2\sin x}{2} - \frac{2x^2\cos x}{2}$$

$= \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$

So puting in equation (i)

$y = C_1 + C_2 \cos x + C_3 \cos x + \frac{3}{2} x^4 + 2x^2 \sin x$
$- x^2 \cos x$

$$\boxed{y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \frac{3}{2}x^4}$$