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Solution:-

The Sample Space S for this Problem is ;

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

Let A = { the Sum is 7 }

B = { the Sum is even

C = { the Sum is greather than 8 }

D = { The two dice had the Same outcome }

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So

$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$B = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

$C = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}$

$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$A \cap B = \phi$

$A \cap C = \phi$

$A \cap D = \phi$

Now

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = \frac{18}{36} = \frac{1}{2}$$

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$$P(C) = \frac{10}{36} = \frac{5}{18}$$

$$P(D) = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cap B) = 0$$

$$P(A \cap C) = 0$$

$$P(A \cap D) = 0$$

Hence

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{1/2} =$$

$$P(A/C) \cancel{=} = \frac{P(A \cap C)}{P(C)} = \frac{0}{5/18} =$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{0}{1/6} =$$

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Q 2

Ans:- When we are rolling two dice, there are 36 different Combination Counting these up. there are 15 Possibilities less than 7: (1,1), (1,2), (1,3) (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2) (3,3), (4,1), (4,2), (5,1). The probability of getting less than a 7 is

$$\frac{15}{36} = \frac{5}{12}.$$

There are 6 possible Combinations of getting a 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) which gives a probability of

$$\frac{6}{36} = \frac{1}{6}$$

This means that 21 Possibilities account for getting less than or equal to 7, So there are 15 remaining possibilities of getting more than 7. This is the Same as the probability of getting less than 7. So the probability must be $\frac{5}{12}$ as well. In calculating this, we must assume that each Combination is equally likely to roll as any other and therefore the dice are fair, or else the Calculating don't work.

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Q 3

Ans Solution

Given data

$P = \frac{2}{3}$, $n = 8$

$q = 1 - P$

$q = 1 - \frac{2}{3}$

$q = \frac{3-2}{3}$

$q = \frac{1}{3}$

Let "X" denoted the number of games won by A, then

(i) $P(X=4) = \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$

$= \frac{8!}{4!(8-4)!}\left(\frac{16}{81}\right)\left(\frac{1}{81}\right)$

$= \frac{8\cdot7\cdot6\cdot5\cdot4!}{4!\cdot4!}\cdot\left(\frac{16}{6561}\right)$

$= \frac{8\cdot7\cdot\cancel{6}\cdot5}{\cancel{24}\,\cancel{4}\,1}\cdot\frac{\cancel{16}\,4}{6561}$

(6)

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$$= \frac{1120}{6561}$$

$$= 0.1707$$

(ii) $P(x \geq 4) = 1 - P(x < 4)$

$$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[\left(\frac{1}{3}\right)^{8} + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{7} + 28\left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{6} + 56\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5}\right]$$

$$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

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(iii) $P(3 \le X \le 6) = \sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$

$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4} +$

$\binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3} + \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$

$= \frac{8}{(3)^8}\left[56 + 140 + 224 + 224\right]$

$= \frac{8}{6561}\left[56 + 140 + 224 + 224\right]$

$= \frac{8 \times 644}{6561}$

$= \frac{5152}{6561}$

$= 0.7852$

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Q. 4

Ans. Proof:

Since the C_i's form a partition of the sample space, we can apply the law of total probability for $A \cap B$.

$$P(A \cap B) = \sum_{i=1}^{m} P(A \cap B / C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i) P(B/C_i) P(C_i)$$

$\because$ (A and B are conditionally independant)

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i) P(B) P(C_i)$$

$\because$ (B is independant of all C_i's)

$$P(A \cap B) = P(B) \sum_{i=1}^{m} P(A/C_i) P(C_i)$$

$$P(A \cap B) = P(B) P(A)$$

$\because$ (Law of total probability)

Hence A and B are independant

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Q 5:-

Ans:- The Probability function for a binomial random variable is

$$b(x ; n, P) = \binom{n}{x} p^x q^{n-x}$$

$$b(x ; n, P) = \binom{n}{x} p^x (1-p)^{n-x} \because q = 1-p$$

This is the probability of having x successes in a series of n independent trails when the probability of success in any of the trails is p. if X is a random variable with this probability distribution.

$$E(X) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

Since $x=0$ term vanishes, Let $y = x-1$ and $m = n-1$, Subbing $x = y+1$ and $n = m+1$ into the last Sum,

$\boxed{E(x) = np}$

Similarly, but this time using

$y = x-2$ and $m = n-2$

$$E(x(x-1)) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= n(n-1)p^2 (p + (1-p))^m$$

$$= n(n-1)p^2$$

So the variance of X is

$$E(X^2) - E(X)^2 = E(X(X-1)) + E(X) - E(X)^2 = n(n-1)p^2 + np - (np)^2$$

$$= \boxed{np(1-p)}$$

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Q6
Ans. Binomial Distribution :-

Many experiment consist of repeated independent trails each trial having two possible out comes, e.g.

The two possible outcome of trial may head and trial Success and failure, True and False e-t-c.

The formula of binomial distribution is

$$P(X=x)\ f(x) = {}^{n}C_{x}\ P^{x}\ q^{n-x}$$

Binomail Frequency Distibution :-

If the binomail probability distibution is multiplied by N, than the number of experiments or sets the resulting distribution is known as the bio-nomail frequency distibution.

The formula of bionomail frequency distribution is

$$N \binom{n}{x} P^{x} q^{n-x}$$

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Q 7
Ans

Coefficient of variation

For Data set A

$CV = \frac{\sigma}{\mu} \times 100$

$CV = \frac{3}{4} \times 100$

$\boxed{CV = 6.7}$

For Data Set B

$CV = \frac{11}{60} \times 100$

$\boxed{CV = 18.3}$

For Data Set C

$CV = \frac{5}{50} \times 100$

$\boxed{CV = 10}$

For Data Set D

$CV = \frac{15}{25} \times 100$

$\boxed{CV = 60}$

The end