Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	Pir Meher Ali Shah	**Total Marks:**	50

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks 7** **CLO 2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks 7** **CLO 2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks 6** **CLO 2**
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad |z| > |a|$$	**Marks 6** **CLO 2**
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition H(0) = 1 and $\left|H(\frac{\pi}{4})\right|^2 = \frac{1}{2}$.	**Marks 6** **CLO 3**

Name: Iqbal Hussain
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Q:1 (a)
Ans.

Sol: $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The characteristics equ is

$$\lambda^2 - 4\lambda + 4 = 0$$
$$\lambda = 2, 2$$

Hence

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

Particular Solution is
$y_p(n) = K(-1)^n u(n)$

Substituting this Solution into the difference equation we obtained

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

for

$n = 2$, $K(1+4+4) = 2$

$K = \frac{2}{9}$ Solution is

$$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n) \quad \text{—(i)}$$

From the initial Conditions, we obtain $y(0) = 1$ $y(1) = 2$ then

$$C_1 + 2/9 = 1$$
$$C_1 = 7/9$$
$$2C_1 + 2C_2 - 2/9 = 2$$
$$C_2 = 1/3$$

So put in equn (i)

$$\boxed{y(n) = \left[\frac{7}{9}2^n + \frac{1}{3}n2^n + \frac{2}{9}(-1)^n\right]u(n)}$$

Q:1 (b)

Ans:

Solution:

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

the characteristics equn is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = 1/2, 1/5 \text{ Hence}$$

$$y_h(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{5}^n \quad \text{— (i)}$$

with $x(n) = \delta n$ we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0$$

$$y(1) = 1.4$$

Hence $C_1 + C_2 = 2$ &

$$\frac{1}{2}C_1 + \frac{1}{5} = \frac{7}{5} = 1.4$$

$$C_1 + 2/5\, C_2 = \frac{14}{5}$$

so equation as

$$C_1 = 10/3 \qquad C_2 = -4/3$$

So put C_1 & C_2 in equn (i)

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$$

The step response as

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^{-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n\left(2^{n+1}-1\right)u(n) - \frac{1}{3}\left(\frac{1}{5}\right)^n\left(5^{n+1}-1\right)u(n)$$

Required Ans

Q:2 a

Ans

Solution:

$$x(Z) = \frac{1}{(1-2Z^{-1})(1-Z^{-1})^2}$$

By partial fraction method

$$\frac{1}{(1-2Z^{-1})(1-Z^{-1})} = \frac{A}{(1-2Z^{-1})} + \frac{B}{(1-Z^{-1})} + \frac{CZ^{-1}}{(1-Z^{-1})^2}$$

$$= \frac{A(1-Z^{-1})^2 + B(1-2Z^{-1}) + CZ^{-1}(1-2Z^{-1})}{(1-2Z^{-1})(1-Z^{-1})^2}$$

$$1 = A(1-Z^{-1})^2 + B(1-2Z^{-1})(1-Z^{-1}) + CZ^{-1}(1-2Z^{-1}) \quad \text{— (i)}$$

Put $Z = 1$

$$1 = A(1-1)^2 + B(1-2(1))(1-1) + C1(1-2(1))$$

$$1 = A(0) + B(-1)(0) + C(1)(-1)$$

$$1 = 0 + 0 - C$$

$$1 = -C$$

$$C = -1$$

Put $Z = 2$ in equ (i)

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$$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) + C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$$

$$1 = A\left(\frac{1}{2}\right)^2 + B(0)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$$

$$1 = \frac{A}{4} + B(0) + C(0)$$

$$1 = \frac{A}{4} + 0 + 0$$

$$A = 4$$

Put $z = 3$ in equ (i)

$$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$$

$$1 = A\left(\frac{4}{9}\right) + B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + C\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$$

$$1 = \frac{A4}{9} + \frac{2}{9}B + \frac{1}{9}C$$

$$1 + \frac{1}{9} - \frac{16}{9} = \frac{2}{9}B$$

Taking L.C.M

$$\frac{9+1-16}{9} = \frac{2}{9}B$$

$$\frac{-6}{9} \times \frac{9}{2} = B$$

$$B = -3$$

Hence $x[n] = 4(2)^n - 3 - n] u(n)$

Q:2 (b)

Ans:

Solution: $X(z) = \frac{1}{1-az^{-1}}$ $|z| > |a|$

we have

$$x(n) = \frac{1}{2\pi j}\oint_C \frac{z^{n-1}}{1-az^{-1}} dz$$

$$= \frac{1}{2\pi j}\oint_C \frac{z^n dz}{z-a}$$

Where C is a circle at radius greather than $|a|$. we shall evaluate this integral using $f(z) = z^n$.

(1) if $n < 0$, $f(z) = z^n$ has an nth-order pole at $z=0$ which is also inside C. thus there are contributions from both poles for $n=-1$ we have.

$$x(-1) = \frac{1}{2\pi j}\oint \frac{1}{z(z-a)} dz$$

$$= \left.\frac{1}{z-a}\right|_{z=0} + \left.\frac{1}{z}\right|_{z=a} = 0$$

(2) if $n \geq 0$, $f(z)$ has only zeros and hence no poles inside C. the only pole inside C is $z=a$ Hence

$$x(n) = f(z_0) = a^n$$
$$n \geq 0$$

if $n = -2$ we have

$$x(-2) = \frac{1}{2\pi j} \oint_c \frac{1}{z^2(z-a)}\, dz$$

$$= \frac{d}{dz}\left(\frac{1}{z-a}\right)\Bigg|_{z=0} + \frac{1}{z^2}\Bigg|_{z=a} = 0$$

By continuing in the same way we can show that $x(n) = 0$ for $n < 0$, thus

$$x(n) = a^n u(n)$$

Q:3 (a)

Ans: Solution: At $\omega = 0$ we have

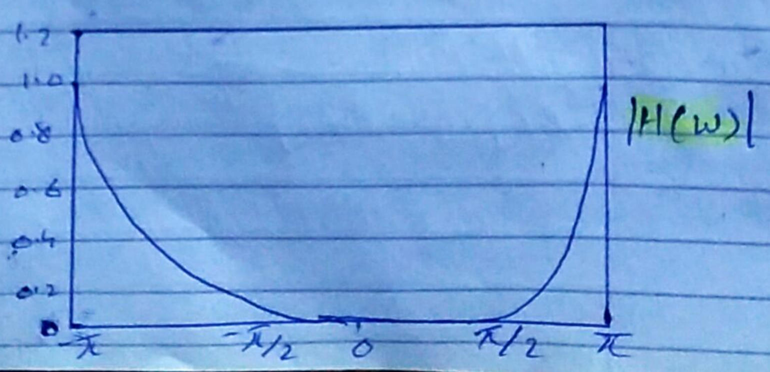
$$\cancel{H(0) = \frac{b_0}{(1-P_2)^2}}$$

$$H(z) = \frac{b_0}{(1-Pz^{-1})^2}$$

$$H(0) = \frac{b_0}{(1-P)^2} = 1$$

Hence

$$b_0 = (1-P)^2$$



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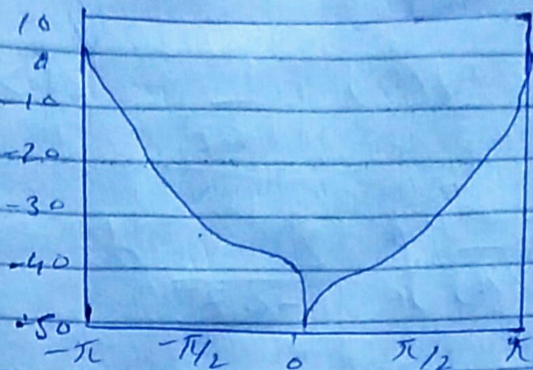
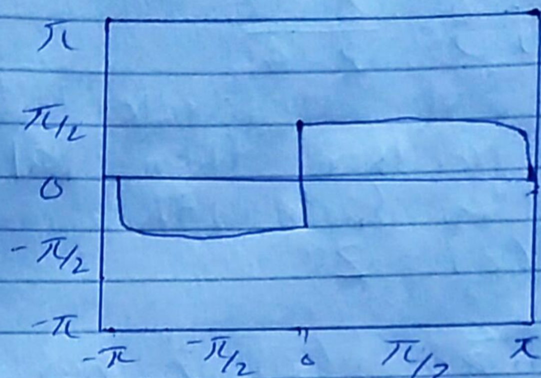




At $\omega = \pi/4$.

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4) + jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2} + jP/\sqrt{2})^2}$$

Hence $\boxed{\dfrac{(1-P)^2}{[(1-P/\sqrt{2})^2 + P^2/2]^2} = \dfrac{1}{2}}$

Required Answer.

Q:3 (B)

Ans:

Solution:

$$P_{1}, P_{2} = re^{4\pi/2}$$

& zeros at $z = 1$ & $z = -1$

consquently, the system function is

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2 - 1}{z^2 + r^2}$$

the gain factor is determaind by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$, thus we have

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

the value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^2+2r^2\cos(8\pi/9)}$$

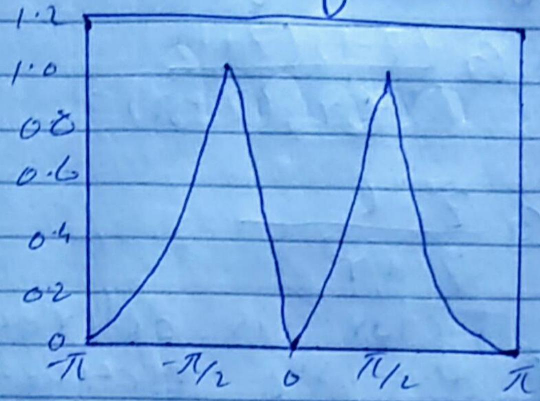
$$= \frac{1}{2}$$

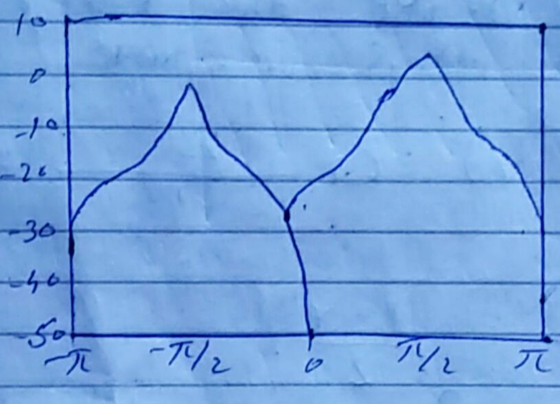
or equivalently.

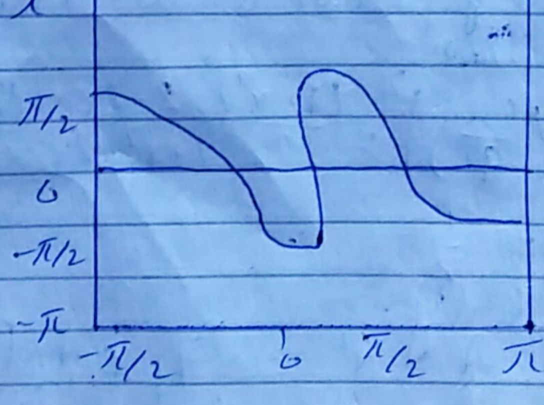
the value of $r^2 = 0.7$ satisfies this equation. therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1 - z^{-2}}{1 + 0.7z^{-2}}$$

its frequency response is illustrated







Q:4 (a)

Ans:

The fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

the magnitude & Phase of $X(\omega)$ are illustrated are the following figure for $L = 10$. the N-Point DFT of $(x(n)$ is simply $X(\omega)$ evaluated at the Set of N equallay spaced frequiencies

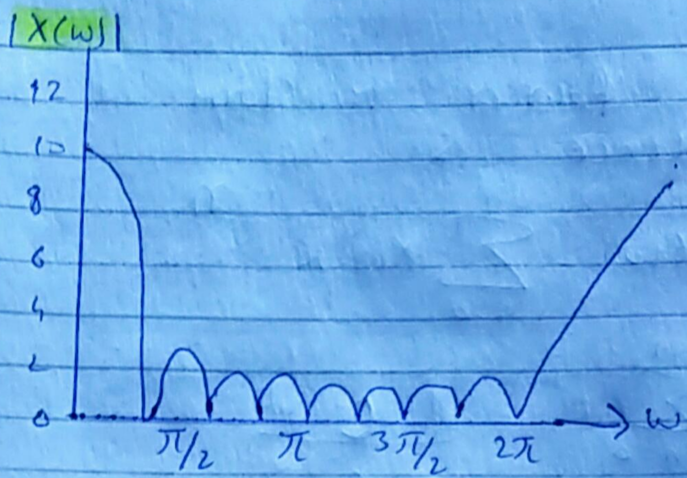
$$\omega_K = 2\pi K / N$$

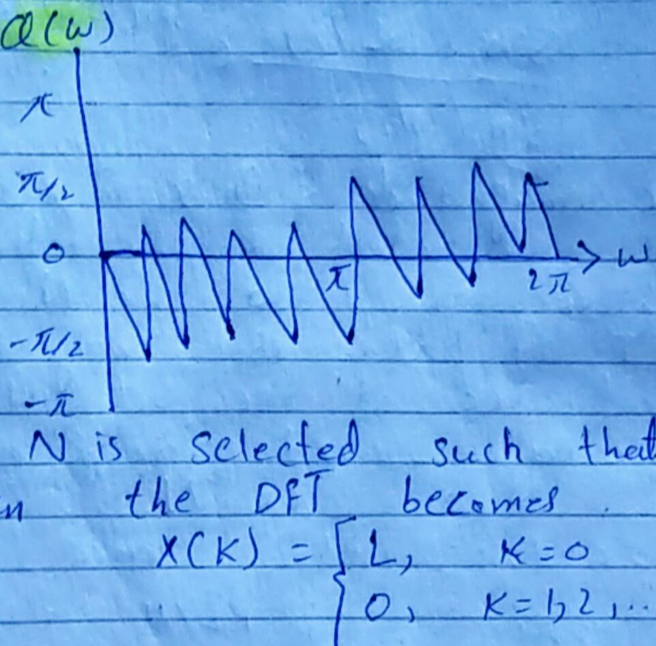
$K = 0, 1, \cdots, N-1$ Hence.

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}}$$

$K = 0, 1, \cdots, N-1$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





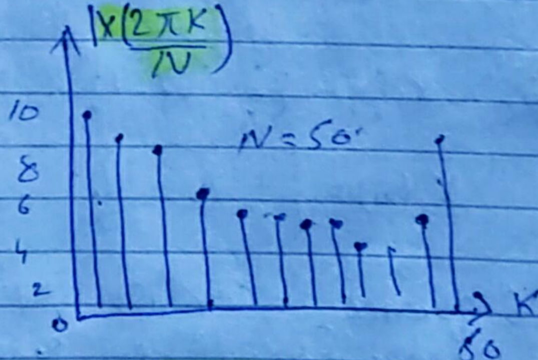
if N is selected such that $N = L$ then the DFT becomes

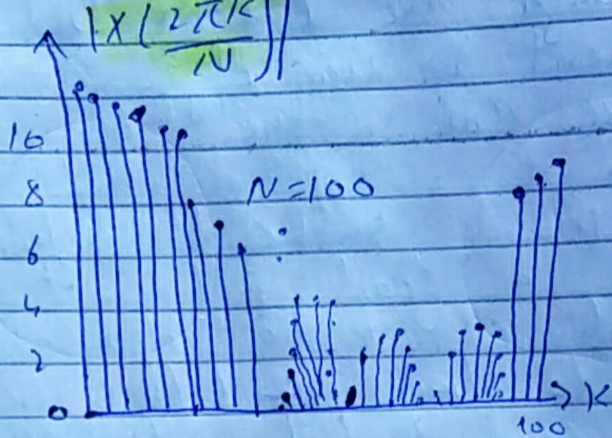
$$X(K) = \begin{cases} L, & K = 0 \\ 0, & K = 1, 2, \ldots L-1 \end{cases}$$

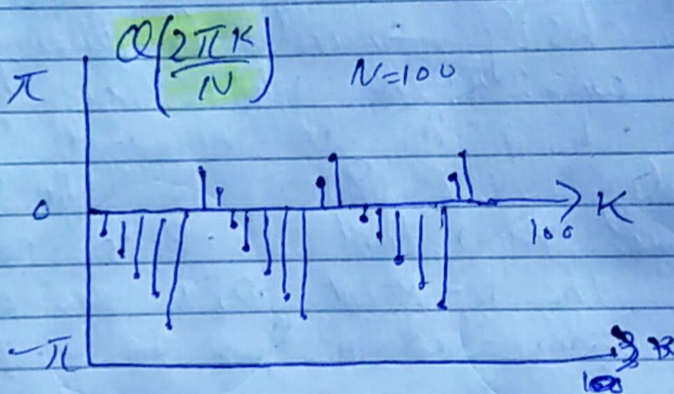
Thus there is only one nonzero value in the DFT. this is apparent from observation of $X(\omega)$, Since $X(\omega) = 0$ at the frequencies $\omega_K = 2\pi K / L$, $K \neq 0$ the reader should verify that $x(n)$ can be recovered from $X(K)$ by Performing an L-Point IDFT.

Although the L-Point DFT is sufficient to uniquely represent the Sequence $x(n)$ in the frequency domain, it is apparent that it does not provide sufficient detail to yield a good picture of the Spectral characteristics of $x(n)$ if we wish to have a better pic we must evaluate (interpolate) $X(\omega)$ at more closely spaced frequency say $\omega_k = 2\pi k/N$ where $N > L$ in effect. L points to N point by appending $N-L$ zeros to the sequence $x(n)$ that is zero padding then the N-point DFT provides finer interpolation than the L-point DFT.

$N = 50$, $L = 10$ & $N = 100$







Q:4 (b)

Ans:- **Solution:** $x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$

$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Each sequence consist of four nonzero points. For the purposes of illustrating the operations involved in circular convolution.

Now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$. Begining with $m=0$ we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)(x_2(-n))_N$$

$x_2((-n))_4$ is simply the sequence $x_2(n)$ folded & graped on a circle as illustrated in the graph

$$x_3(0) = 14$$

for $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

it is easily varified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ roted counterclockwise.

$$x_3(1) = 16$$

for $m=2$, we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2(2-n)_4$$

Now $x_2(2-n)_4$ is the folded sequence.

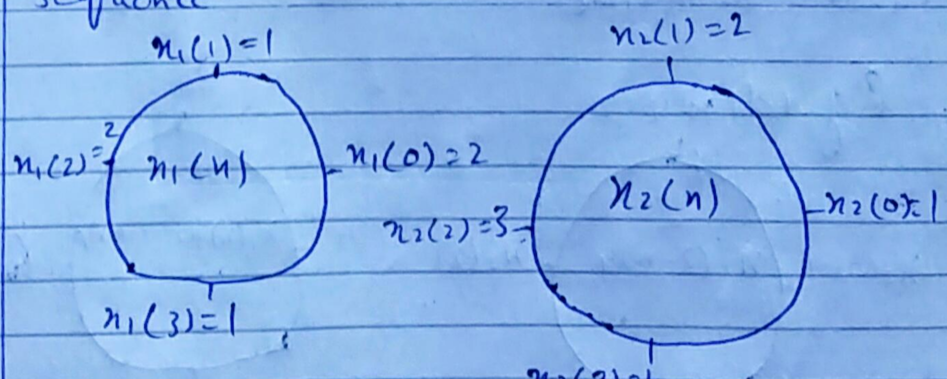
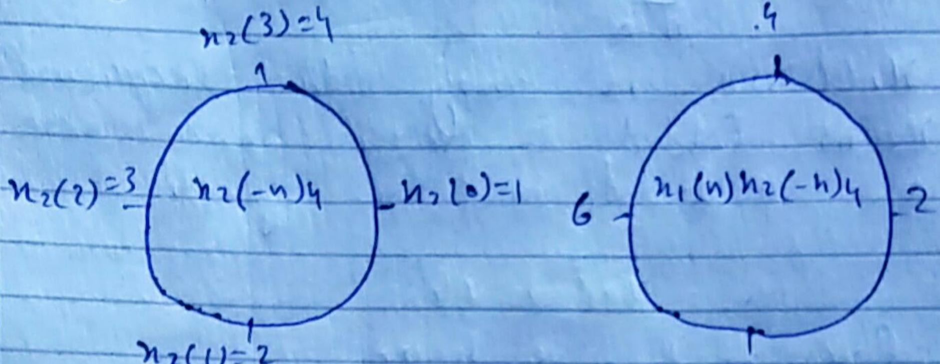


fig (a)

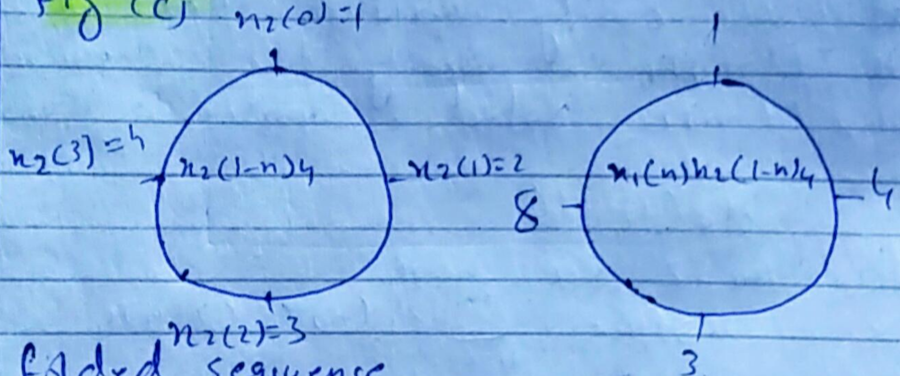
fig (b)



Folded Sequence

product Sequence

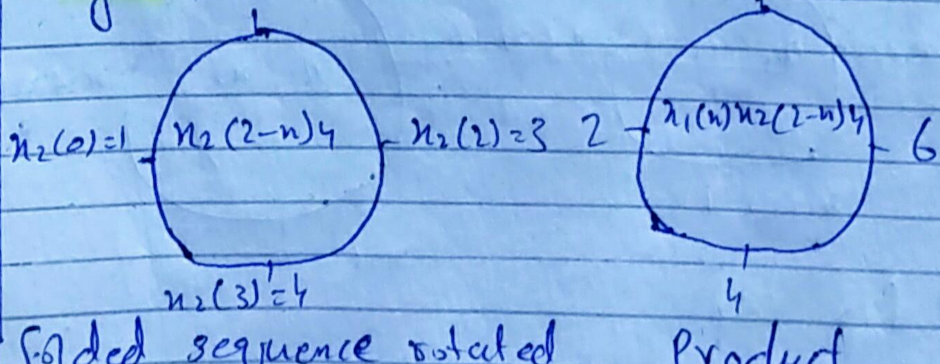
fig (c)



folded sequence rotated by one unit in time

Product Sequence.

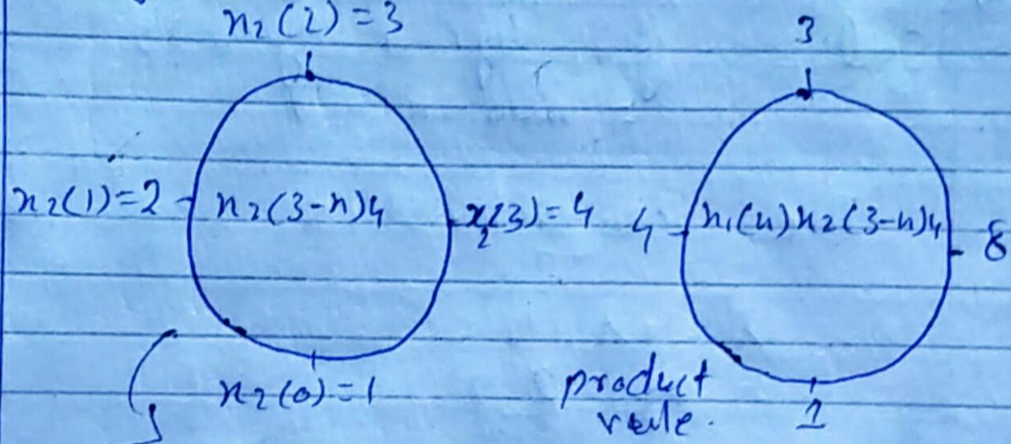
fig (d)



folded sequence rotated by two units in time

Product Sequence

fig (e)



Folded sequence rotated by three units in time