Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Total Marks:** 30

Student Details

Name: SHER DARAZ KHAN **Student ID:** 13976

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \left\{2, \underset{\uparrow}{1}, -2, 3, -4\right\} \quad , h[n] = \left\{\underset{\uparrow}{3}, 1, 2, 1, 4\right\}$$	Marks 5 CLO 2
	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} a^{n+1}, & -3 \leq n \leq 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^n, & 0 \leq n \leq 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

Q 1

Part (a)

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Sampling rate required to avoid aliasing:

Minimum Sampling rate:

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

$F_1 = 50\ Hz$

$F_2 = 100\ Hz$

$F_s = 100\ Hz$

F_s is maximum than F_1

$F_1 = 50$ is minimum sampling rate to avoid aliasing.

$f_b = 2 f_{max}$

F_2 is max, $F_2 = 100\ Hz$

$F_b = 2 \times 100$

$F_b = 200$

(ii) We have

$F_s = 100\ Hz$

So $f_1' = f_1/F_s = 50/100 = 0.5\ Hz$

F_2 become

$f_2' = F_s/F_s = 100/100 = 1\ Hz$

So $w_1' = 2\pi f_1'$

$w_1' = 2\pi \times 0.5 = \pi$

&

$w_2' = 2\pi f_2'$

$w_2' = 2\pi \times 1 = 2\pi$

$x[n] = 3\cos 100\pi t + 4\sin 200\pi t$

$x[n] = 3\cos \pi t + 4\sin 2\pi t$

(iii) We can costruct the orignal signal & also frequency component. Since at 50 Hz & 100 Hz are present in the sampled signal.

The ~~sampled~~ signal we can recover is

$y_a(t) = 3\cos\pi t + 4\sin 2\pi t$

$x_a(t)$, original signal is different from its, due to Low sampling rate used, distortion of the orignal analog signal was caused by the aliasing effect.

Q 1

Part (b)

$$x[n] = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This signal is sampled at the rate

$f_z = 2Hz$

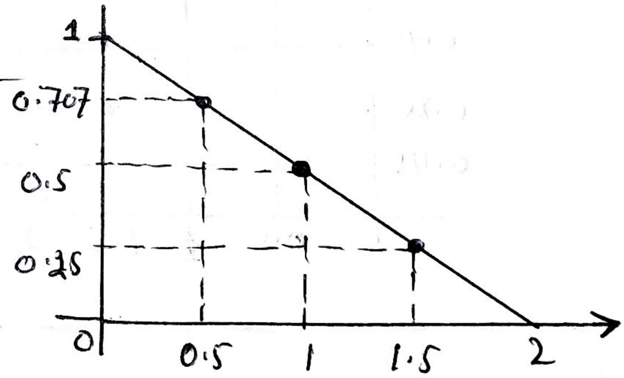
As we know that

$F_s = 1/T \Rightarrow T = 1/2$

$T = 0.5$ sec

(i) Draw Sampled Signal :

xn	0.5^n
0	1
0.5	0.707
1	0.5
1.5	0.25



T = 0.5 sec

(ii) The sampled of the signal are intended to carry 3 bits per samples determine the quantization level & quantization resolution to quantized the sampled signal achived in part 1

So

$n = 3$ bit / sample

first we have to find quantization level

$L = 2^n$

$L = 2^3$

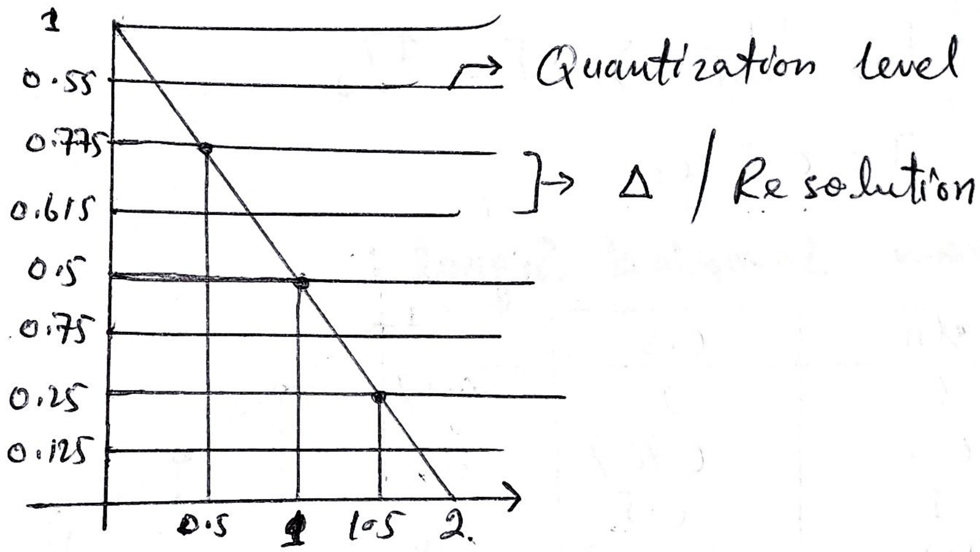
$L = 8$

Quantization resolution (Δ)

$$= \frac{x_{max} - x_{min}}{L}$$

$$\Delta = \frac{1-0}{8} = 1/8 = 0.142$$

$$\boxed{\Delta = 0.142}$$



(iii)

n	Discrete t. Signal	Rounding	Truncation	$e_q[n] = x_q[n] = x[n]$
0	1	1	1	0
1	0.87	0.9	0.8	-0.0
2	0.75	0.8	0.7	0.0
3	0.65	0.7	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.37	0.4	0.3	-0.1
6	0.25	0.3	0.2	-0.1
7	0.12	0.1	0.1	0.0

Q2

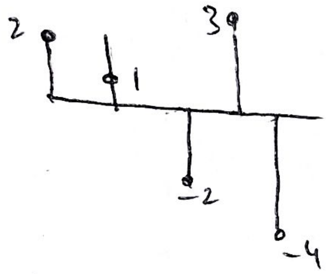
Part (a)

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}, \quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

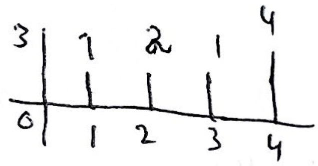
Solution:

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}, \quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

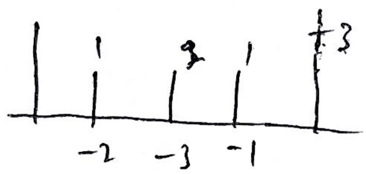
$$Y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h(n-k)$$



h (k)



h (−k) folded signal



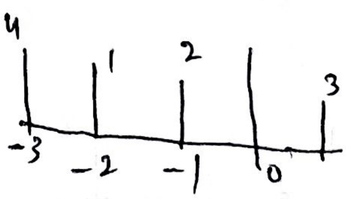
$$Y(0) = \sum_{k=1}^{0} x(-1)\, h(-1) + x(0)\, h(0)$$

$$Y(0) = (2)(1) + (1)(3)$$

$$= 2 + 3 = 5$$

for $n = 1$

$h(1-k)$



$Y(1) = \sum_{k=-1}^{1} x(n)\, h(1-k)$

$= x(-1)\, h(-1) + x(0)\, h(0) + 0\, x(1)\, h(1)$

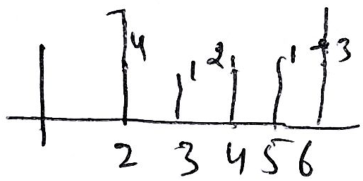
$+ (x)\, h(1) + x(2) + 0\, x(3)\, h(3)$

$Y(2) = (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$

$= 8 + 1 - 4 + 3 - 12 = -4$

$n = 3$

$h(3-k)$



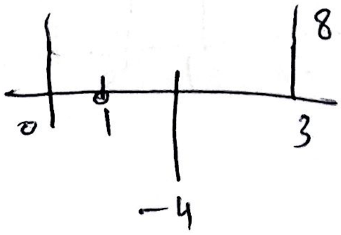
$Y(3) = \sum_{k=2}^{3} x(k)\, h(n-k)$

$= x(2)\, h(2) + x(3)\, h(3)$ (3)

$= (3)(4) + (-4)(1) =$

$= 12 + (-4)$

$= 12 - 4 = 8$



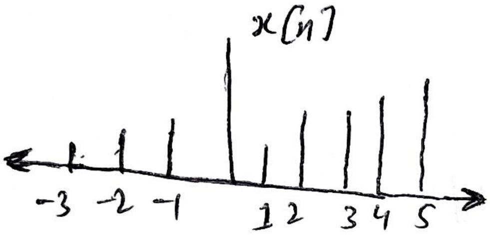
Q2

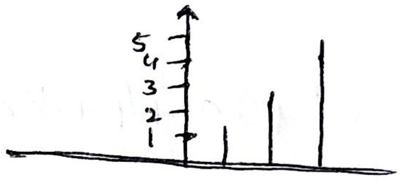
Part (b)

$$x[n] = \begin{cases} a^{n+1}, & -3 \le 5 \\ 0, & \text{else where} \end{cases}$$

$$h[n] = \begin{cases} 2^{n}, & 0 \le x \le 4 \\ 0, & \text{else where} \end{cases}$$

Solution:

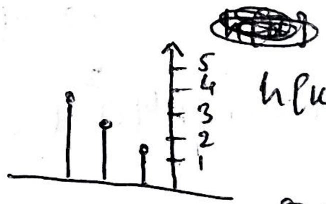




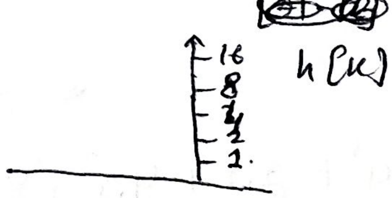
$$x[n] = \{a^{-1}, a^{-1}, 1, \underset{\uparrow}{a}, a^{2}, a^{3}, a^{4}, a^{5}, a^{6}\}$$

$$h(n) = \{\underset{\uparrow}{1}, 2, 4, 8, 16\}$$

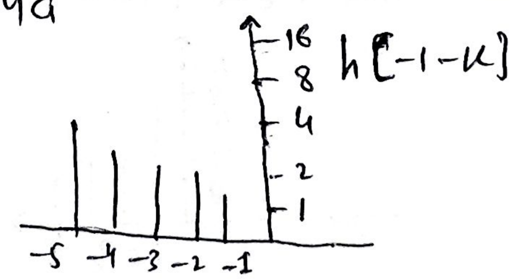
$$y[n_0] = \sum_{k=-\infty}^{\infty} x[k]\, h[n_0 - k]$$



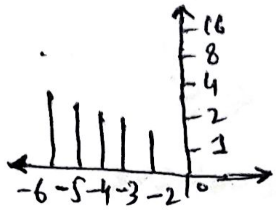
$$y[0] = a - 2 + 4a^{-1} + 8a^{-2}$$



$$y[-1] = 1 + 2a^{-1} + 4a^{-2}$$

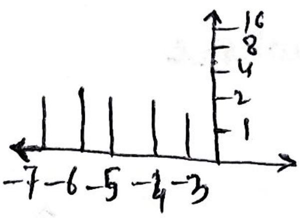


$y[-2] = 2a^{-2} + a^{-1}$



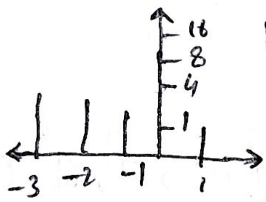
$h[-2-k]$

$y[-3] = a^{-2}$



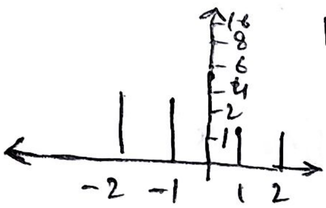
$h[-3-k]$

$y[1] = a^{2} + 2a + 4 + 8a^{-1} + 16a^{-2}$



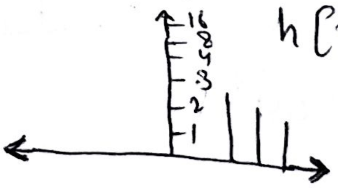
$h[-4-k]$

$y[2] = a^{3} + 2a^{2} + 4a + 8 + 16a^{-1}$



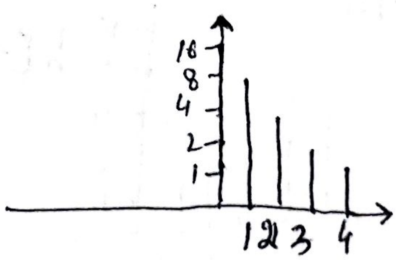
$h[2-k]$

$y[3] = a^{4} + 2a^{3} + 4a^{2} + 8a + 16$

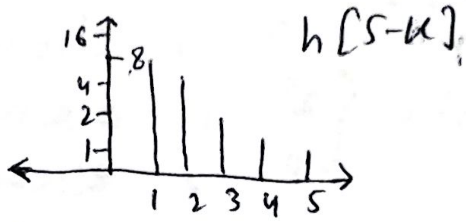


$h[3-k$

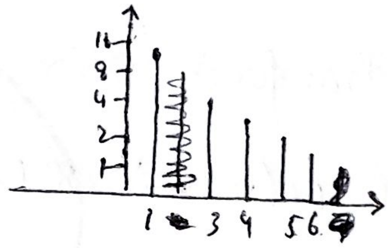
$y[4] = a^{5} + 2a^{4} + 4a^{3} + 8a^{2} + 16a$



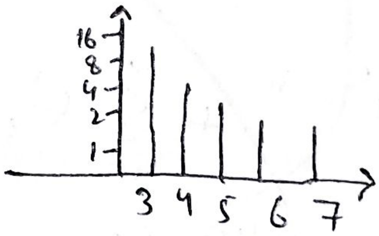
$y[5] = 16a^2 + 8a^3 + 4a^4 + 2a^5 + a^6$



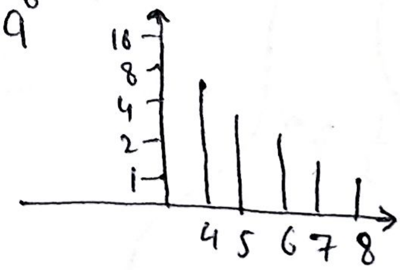
$y[6] = 16a^3 + 8a^4 + 4a^5 + 2a^6$



$y[7] = 16a^4 + 8a^5 + 4a^6$



$y[8] = 16a^5 + 8a^6$



$y[9] = 16\,a^6$

Q 3

Part (a)

(i) $x(n) = \begin{cases} (\frac{1}{4})^n , & n \geq 0 \\ (\frac{1}{3})^{-n} , & n < 0 \end{cases}$

Solution:

The z-Transform pair is

$x(n) = a^n u(n) \Leftrightarrow X(z) \frac{1}{1-\alpha z^{-1}}$ ROC $|z|$

Put values of the above equation we get;

$$X(z) = \sum_{n=0}^{\infty} (\tfrac{1}{4})^n z^{-n} + \sum_{n=0}^{\infty} (\tfrac{1}{3})^n z^{-u} - 1$$

Using geometric series

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$$

taking LCM:

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - (1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - (1-\frac{1}{3}z - \frac{1}{4}z^{-1} + \frac{1}{12}z^{-1} \cdot z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - 1 + \frac{1}{3}z + \frac{1}{4}z^{-1} + \frac{1}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1+\frac{1}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

Taking LCM:

$$\frac{1}{1} + \frac{1}{12} = \frac{12+1}{12} = \frac{13}{12}$$

$$= \frac{13/12}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

So ROC is $\frac{1}{4} < |z| < 3$

(ii) $x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \geq 0 \\ 0, & \text{else where} \end{cases}$

Solution :

$x(n) = \begin{cases} (\frac{1}{2})^n - 3 & , n \geq 0 \\ 0 & , \text{else where} \end{cases}$

So is z- Transform

$$Xz = \sum_{n=0}^{\infty} (\tfrac{1}{2})^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

$$\frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

Taking LCM:

$$= \frac{\cancel{1} - 3z^{-1} - \cancel{1} - \frac{1}{2} z^{-1}}{(1 - \frac{1}{2} z^{-1})(1 - 3z^{-1})}$$

$$= \frac{-5}{2} z^{-1} / (1 - \tfrac{1}{2} z^{-1})(1 - 3z^{-1})$$

So the ROC is $|z| > 3$

THE END