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QUESTION NO 18

ANSWER:

Solution:

$$\begin{bmatrix} 1 & 1D3 & 3 & 0 & 5 \\ 0 & 1 & -1D10st & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 1D3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -5 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

By using elementary row operation.

$$= \begin{bmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -5 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad R_1 - 3R_3$$

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$$= \begin{bmatrix} 1 & 0 & 0 & -3 & 11 \\ 0 & 1 & -5 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad R_2 + 5R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & -3 & 11 \\ 0 & 1 & 0 & 0 & -23 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad R_1 + 3R_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 11 \\ 0 & 1 & 0 & 0 & -23 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 11 \\ -23 \\ -6 \\ 0 \end{bmatrix} \quad \text{Answer.}$$

5
7
-6
103

3R₃

QUESTION NUMBER 2

PART A3

Solution:

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

By Using Elementary row operation

$$= R_3 - 2R_2 \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Similarly for

$$= R_3 + 2R_2 \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

So $R_3 - 2R_2$ is inverse row operation

PART B:

$$a) \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

"YES" It is echelon form of matrix, because each pivot value below and above zero entry exists.

$$b) \begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

"NO" It is not an echelon form of matrix because in third row no pivot value exist.

$$c) \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

"NO" Its not reduce echelon matrix because each pivot value must be equal to 1.

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a)

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

"No" It is not in reduced row echelon form because in 2nd row there is no pivot value.

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QUESTION NUMBER 33

ANSWERS

PART A3

The echelon form of a matrix isn't unique, which means there are infinite answers possible when you perform row reduction.

Reduced row echelon form is at the other end of spectrum, it's unique, which means row reduction on a matrix will produce the same answer.

e.g:

$$\begin{array}{cccc|cccc} 1 & 5 & -5 & 1.5 & 1 & 0 & 0 & 3 \\ 0 & 1 & 1.4 & -6 & 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{array}$$

↓
Row echelon form

↓
Reduced row echelon form

PART - B

$$\begin{bmatrix} 1 & 102 & 8 \\ 2 & 8 & -1 \\ -103 & 0 & 0 \\ 1 & -4 & 10 \end{bmatrix} \text{First-last}$$

$$= R_2 - 2R_1 \quad \left[\begin{array}{ccc|ccc} 1 & 6 & 8 & 1 & 0 & 0 \\ 2 & 8 & -1 & 0 & 1 & 0 \\ -0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -4 & 15 & 0 & 0 & 0 \end{array} \right]$$

$$= R_4 - R_1 \quad \left[\begin{array}{ccc|ccc} 1 & 6 & 8 & 1 & 0 & 0 \\ 0 & -4 & -17 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -4 & 15 & 0 & 0 & 0 \end{array} \right]$$

$$= R_2 - 4R_1 \quad \left[\begin{array}{ccc|ccc} 1 & 6 & 8 & 1 & 0 & 0 \\ 0 & -4 & -17 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 7 & -1 & 0 & 0 \end{array} \right]$$

$$= R_4 - 2R_2 \quad \left[\begin{array}{ccc|ccc} 1 & 6 & 8 & 1 & 0 & 0 \\ 0 & 1 & -17/4 & 1/2 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 7 & -1 & 0 & 0 \end{array} \right]$$

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$$= \frac{R_4}{2} \left[\begin{array}{ccc|ccc} 1 & 6 & 8 & 1 & 0 & 0 \\ 0 & 1 & +17/4 & 1/2 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -3/2 & -2 & -1/2 & 0 \end{array} \right]$$

$$= \frac{R_4}{-3} \left[\begin{array}{ccc|ccc} 1 & 6 & 8 & 1 & 0 & 0 \\ 0 & 1 & 17/4 & 1/2 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -3 & -1 & -1 & 0 \end{array} \right]$$

$$= R_1 - 8R_4 \left[\begin{array}{ccc|ccc} 1 & 6 & 8 & 1 & 0 & 0 \\ 0 & 1 & 17/4 & 1/2 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array} \right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 6 & 0 & -8 & -8 & 0 \\ 0 & 1 & 17/4 & 1/2 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array} \right]$$

$$1D2 = 6$$

$$-1D3 = 0$$

$$1D \text{ First-last} = 1.5$$

Answer.