# QUESTION No.1

Solve the following objective type questions.

i. The order of matrix A is $m \times p$ and the ~~ott~~ order of matrix B is $p \times n$. Then the order of matrix AB is?

## SOLUTION:

The order of matrix is equal to the no. of rows multiply by no. of column.

Order of matrix = Rows x Columns

So,
$A = m \times p$ has "m" no. of rows and "p" no. of columns.

Similarly,
$B = p \times n$

Then it has "p" no. of rows and "n" no. of columns.

Also the no. of columns in A is equal to no. of rows in B so there matrix are comfortable for multiplication and there order will be

$$AB = m \times n.$$

$$[A]_{m \times \underline{p}} \times [B]_{\underline{p} \times n} = [AB]_{m \times n}$$

ii. The number of non-zero rows in Echelon form?

**Echelon form:**

A matrix is in echelon form if it has the shape resulting from a Gaussian elimination.

**Number of non-zero rows**

The number of non-zero rows in Echelon form is "1".

iii. If $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ is a singular matrix then $a = ?$

**SOLUTION:**

For singular matrix $|R| = 0$

So,

$$|B| = 1 \times a - 4 \times 2 = 0$$

$$= a - 8 = 0$$

So $\boxed{a = 8}$ **Ans.**

iv. If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

**SOLUTION:**

$$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= (2i)(-i) - (i)(i)$$

$$= -2i^2 - i^2$$

We know that $i^2 = -1$.

So,

$$|A| = -2(-1) - (-1)$$

$$|A| = 2 + 1$$

$$\boxed{|A| = 3} \Rightarrow \text{Ans.}$$

v. The matrix $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ is ?

**SOLUTION:**

If each element of a principal diagonal of a matrix is some non-zero scalar and all other are zero then it is called a <u>scalar matrix</u>.

vi. $\frac{dy}{dx} + 2xy = y$

**SOLUTION:**

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

$$dy = y(1-2x)dx$$

$$\frac{dy}{y} = (1-2x)\,dx$$

$$\int \frac{dy}{y} = \int (1-2x)\,dx \Rightarrow \ln y = x - \frac{2x^{1+1}}{1+1} + C$$

$$\ln y = x - \frac{2x^2}{2} + C$$

$$\ln y = x - x^2 + C.$$

vii. The order and degree of differential equation.

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is } = ?$$

**SOLUTION:**

The order of differential equation is the order of highest derivatives known as differential co-efficient and Degree is the power of highest derivatives So,

Order = 1

Degree = 3

viii. The order and degree of differential equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx}\right) \text{ is } ?$$

**SOLUTION:**

Order = 2

Degree = 1

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ix. The differential equation $2\frac{dy}{dx} + x^2y = 2x+3$, $y(0) = 5$ is?

**SOLUTION:**

$$2\frac{dy}{dx} + x^2y = 2x+3$$

$$2dy + x^2y = (2x+3)\,dx$$

$$2dy = (2x+3-x^2y)\,dx$$

$$\int 2dy = \int (2x+3-x^2y)\,dx$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + C$$

$$2y = x^2 + 3x - \frac{x^3y}{3} + C$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + C \quad \text{— (1)}$$

Put $x = 0$, $y = 5$

$$5 = 0 + 0 - 0 + C$$

$$5 = C$$

or

$$C = 5$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + 5$$

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X. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

**SOLUTION:**

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by $R_1$

$$|A| = +1\begin{vmatrix} b & b \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= +1(bc^2 - b^2c) - a(c^2 - b^2) + a^2(c - b)$$

$$\boxed{|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b}$$ Ans.

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QUESTION No.2

①

i. Express the determinent

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

As the product of factors which are linear in a, b, c

**SOLUTION:**

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by $R_1$

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$$

$$= abc(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b)$$

$$\Rightarrow \boxed{abc\left[bc(\cancel{a}c-b) - ac(c-a) + ab(b-a)\right]}$$ **Ans.**

ii. Find the Eigen value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$ (2)

**SOLUTION:**

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equation $\Rightarrow |A - \lambda I| = 0$ — (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by $R_1$

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \text{ — (B)}$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ expand by $R_1$

$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$

$\Rightarrow (3-\lambda)\left[(3-\lambda)(2-\lambda) - (-1)(-1)\right] + 1\left[(-1)(2-\lambda) - (-1)(-1)\right] - 1\left[(-1)(-1) - (-1)(3-\lambda)\right]$

$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (1+3-\lambda)$

$\Rightarrow (3-\lambda)(\lambda^2-5\lambda+5) + \cancel{\lambda+1}(-3+\lambda) - (4-\lambda)$

$\Rightarrow 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$

$\Rightarrow \boxed{-\lambda^3 + 8\lambda^2 - 18 + 8}$ — (a)

$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$

Expand by $C_1$

$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$

$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2+1) + 1(-2+\lambda-1)$

$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$

$\Rightarrow \boxed{-\lambda^2 + 6\lambda - 8}$ — (b)

$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix}$ (4)

Expand by $C_1$

$\Rightarrow -\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$

$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]$

$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5)$

$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$

$\Rightarrow \boxed{-\lambda^2+6\lambda-8}$ — (c)

Put (a) (b) and (c) in (B)

$\Rightarrow (2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$

$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^2-8\lambda^3+18\lambda^2-8\lambda$ •

$\Rightarrow$ • $-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda+6\lambda+6\lambda+16-16$

$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

By Synthetic division,

We get;

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$\lambda=0$

$\lambda-2=0 \Rightarrow \lambda=2$

$\lambda^2-8\lambda+16=0$

Again,

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} \text{ expand by } R_1$$ 5

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\left[(3-\lambda)(2-\lambda)-(-1)(-1)\right]-(-1)\left](3-\lambda)\right]$$

By factorization Method:

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)=0$$

$$\lambda=4, \lambda=4$$

$$\boxed{\lambda_1=0, \lambda_2=2, \lambda_3=4, \lambda_4=4}$$ Ans

# QUESTION No.3

The rate of change in the form of differential equation is given by $(x^2+3y^2)dx - 2xy\,dy = 0$. Find the general solution at $x=2$ and $y=6$.

**SOLUTION:**

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy$$

Divide both sides by $2xy\,dx$

$$\frac{(x^2+3y^2)\cancel{dx}}{2xy\,\cancel{dx}} = \frac{\cancel{2xy}\,dy}{\cancel{2xy}\,dx}$$

$$\frac{(x^2+3y^2)}{2xy} = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x \cdot \cancel{x}}{2\cancel{x}y} + \frac{3y \cdot \cancel{y}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{x}{2y} + \frac{3y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \quad \text{—} (1)$$

Let $y = Vx$ or $V = y/x$ or $\frac{1}{V} = \frac{x}{y}$

Differential with respect to "$x$" So,

$$dy = Vdx + xdv$$

Dividing b/s by "$dx$"

$$\frac{dy}{dx} = \frac{V\cancel{dx}}{\cancel{dx}} + \frac{xdv}{dx}$$

$$\frac{dy}{dx} = V + \frac{xdv}{dx} \quad \text{—} (a)$$

Put eq.(a) in eq.(1)

$$V + \frac{xdv}{dx} = \frac{1}{2}\left[\frac{1}{V} + 3V\right] \quad \therefore V = y/x \text{ and } x/y = 1/V$$

Multiplying b/s by "2"

$$2V + 2\frac{xdv}{dx} = \cancel{2} \times \frac{1}{\cancel{2}}\left[\frac{1}{V} + 3V\right]$$

$$2V + 2x\frac{dv}{dx} = \frac{1}{V} + 3V$$

Subtracting "2V" from both sides.

$$\cancel{2V} - \cancel{2V} + 2x\frac{dv}{dx} = \frac{1}{v} + 3V - 2V$$

$$2x\frac{dv}{dx} = \frac{1}{V} + V$$

$$2x\frac{dv}{dx} = \frac{1+V^2}{V}$$

Multiplying b/s by "dx"

We get

$$2x\,dV = \frac{1+V^2}{V}\,dx$$

Multiplying b/s by $\frac{V}{x(1+V^2)}$

we get,

$$\frac{2V}{1+V^2}\,dV = \frac{1}{x}\,dx$$

Taking $\int$ on b/s

$$\int\frac{2V}{1+V^2}\,dv + C = \int\frac{1}{x}\,dx$$

$$\ln|1+v^2| + \ln C = \ln x$$

$$\cancel{\ln}\, x = \cancel{\ln}\left(|1+v^2|\right)(|C|)$$

$$x = |1+v^2|\,|C|$$

Put $V = y/x$

$$1 + (y/x)^2 C = x$$

$$\frac{x^2 + y^2}{x^2} C = x$$

$$x^2 + y^2 C = x^3$$

As $x = 2$
$y = 6$

So,

$$(2)^2 + (6)^2 c = (2)^3$$

$$4 + 36c = 8$$

$$40C = 8$$

$$C = \frac{8}{40} = \frac{1}{5}$$

So,

$$x^2 + y^2 C = x$$

$$y^2 C = x^3 - x^2$$

$$y^2 = \frac{x^3 - x^2}{C}$$

$$y^2 = \frac{x^3 - x^2}{5}$$

$$y^2 = \frac{x^2(x-1)}{5}$$

$$y = \sqrt{\frac{x^2(x-1)}{5}}$$

$$\boxed{y = \pm x\sqrt{\frac{(x-1)}{5}}} \Rightarrow \text{Ans.}$$