INTRODUCTION TO STRUCTURAL DYNAMICS & EARTHQUAKE ENGINEERING

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Question No 1:

Given data:

- A beam in pulled in a downward direction = $\frac{1}{2}$ inch
- Ignore the self-weight of beam as well as damping effect.
- $E = 29000$ ksi
- $I = 150\ in^4$
- δ_{st} = Deflection due to 7715 lb Static load.

Required data:

- Natural time period of system = ?
- Develop and Solve equation of motion for vibrations = ?

Solution :

As we know that;

The general E.O.M for SDOF system is;

$$Ku + c\dot{u} + m\ddot{u} = p(t)$$

In our case system is undamped $(c = 0)$ undergoing free vibration $(P(t) = 0)$.

Hence;

The general EOM become;

$$Ku + m\ddot{u} = 0 \longrightarrow (1)$$

Here;

$$K = 3EI/L^3$$

$$K = \frac{3(29000)(150)}{(10 \times 12)^3}$$

$$K = 7.55 \text{ k/in}$$

Also;

$$K = 7.55 \text{ k/in} \times 1000 \times 12$$

$$K = 90625 \text{ lb/ft}$$

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Similarly;

$$m = \frac{7715}{32.2} = 239.6 \text{ Slug.}$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{239.6}}$$

$$\omega_n = 19.45 \text{ rad/sec}$$

- For Natural Time period of a system we have;

$$T_n = \frac{2\pi}{\omega_n} = \frac{2 \times \pi}{19.45}$$

$T_n = 0.323$ Sec

By substituting the corresponding values in eq(1) we get;

$$90625\,u + 239.6\,\ddot{u} = 0$$

where "k" is in lb/ft and "m" is in lb sec/ft^2

General Solution to the EOM for undamped fee vibration is;

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$u(0) = \frac{1}{2}'' = \frac{1}{24}\ ft$ and $\dot{u}(0) = 0$

$u(t) = \frac{1}{24}\cos(19.45t) + 0 = \frac{1}{24}\cos(19.45t)$

Eqquivalen

$$U(t) = \frac{1}{24}\cos(19.45t)$$

Equivalent Static force at any time "t" is

$$f_s(t) = k \cdot u(t) = \frac{90625\cos(19.45(t))}{24}$$

$$f_s(t) = 3776.04\cos(19.45t)$$

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Amplitude of dynamic displacement, u_o for undamped free vibration is;

$$u_o = \sqrt{(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2}$$

$$u_o = \sqrt{\left(1/24\right)^2}$$

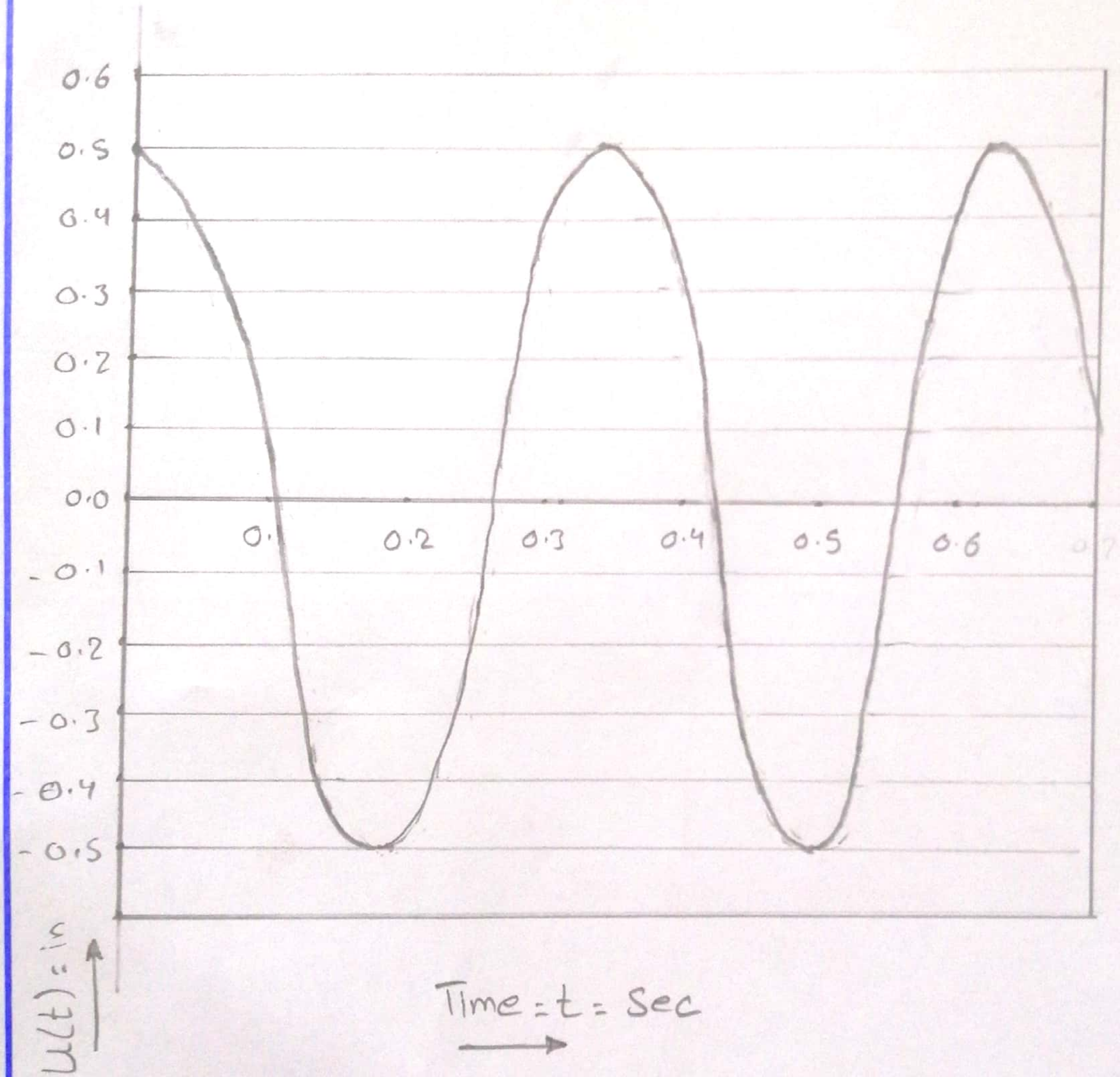
$u_0 = \frac{1}{24}$ ft

Amplitude of equivalent Static force, f_{S0};

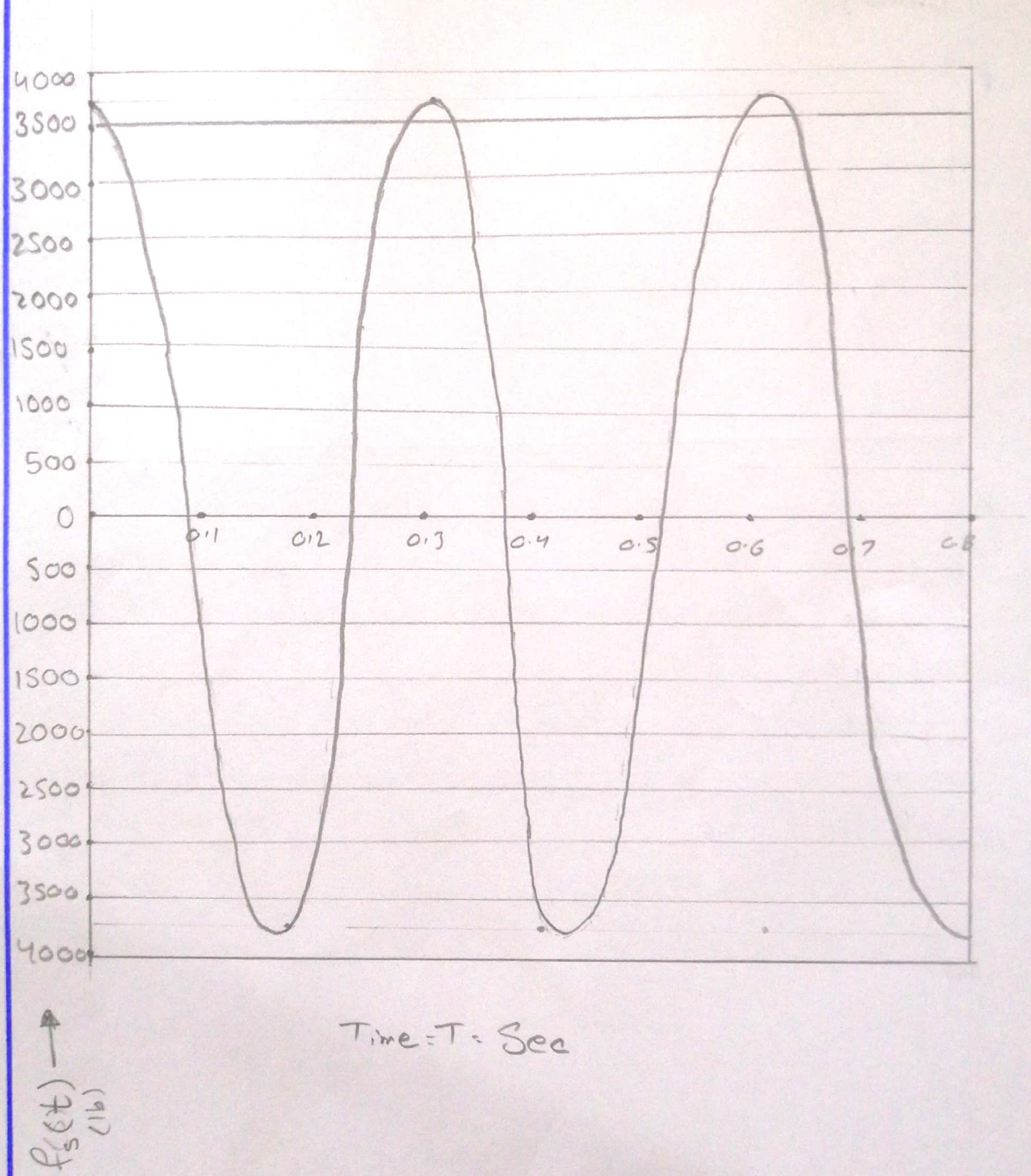
$k u_0 = 90625 \times \frac{1}{24} = 3776.04$ lb Ans

Graphs:

a- Showing Variation of displacement with time;



b. Showing Variation of Equivalent Static force with time:



Question No ~~3~~ 2:

Given data:

Damping ratio of reinforced concrete with considerable cracking = 3 - 5 %

- So we take $\xi = 3\,\%$

- Other data are taken from Question No 1;

Required data:

- Develop and Solve the equation of motion for vibration at free end = ?

- Develop an equation showing variation in Equivalent static forces with time = ?

Solution:

As we know that;

EOM (equation of motion) for damped fee vibration is;

$$ku + c\dot{u} + m\ddot{u} = 0 \longrightarrow (1)$$

As we know from question no 1 data

i.e

- $K = 90625 \ lb/ft$
- $m = 239.6 \ lb.sec^2/ft$
- $\omega_n = 19.45 \ rad/sec$

As we know that;

$$c = \zeta \times 2 m \omega_n$$

$$C = (0.03) \times 2 \times 239.6 \times 19.45$$

$$C = 279.61 \ lb.sec/ft$$

By putting values in eq (1) we get;

$$90625 u + 279.61 \dot{u} + 239.6 \ddot{u} = 0$$

Solution to the EOM for damped free vibration is;

$$u(t) = e^{-\zeta \omega_n t} \left[u(0) \cos(\omega_D t) + \frac{1}{\omega_D} [\dot{u}(0) + u(0) \zeta \omega_n] \cdot \sin(\omega_D t) \right]$$

Here;

$$\omega_D = 19.45 \ rad/sec$$

$$u(t) = e^{-(0.03\times 19.45\times t)}\left[\frac{1}{24}\times \cos(19.45t) + \frac{1}{19.45}\left(0 + \frac{1}{24}\times 0.03\times 19.45\right)\times \sin(19.45t)\right]$$

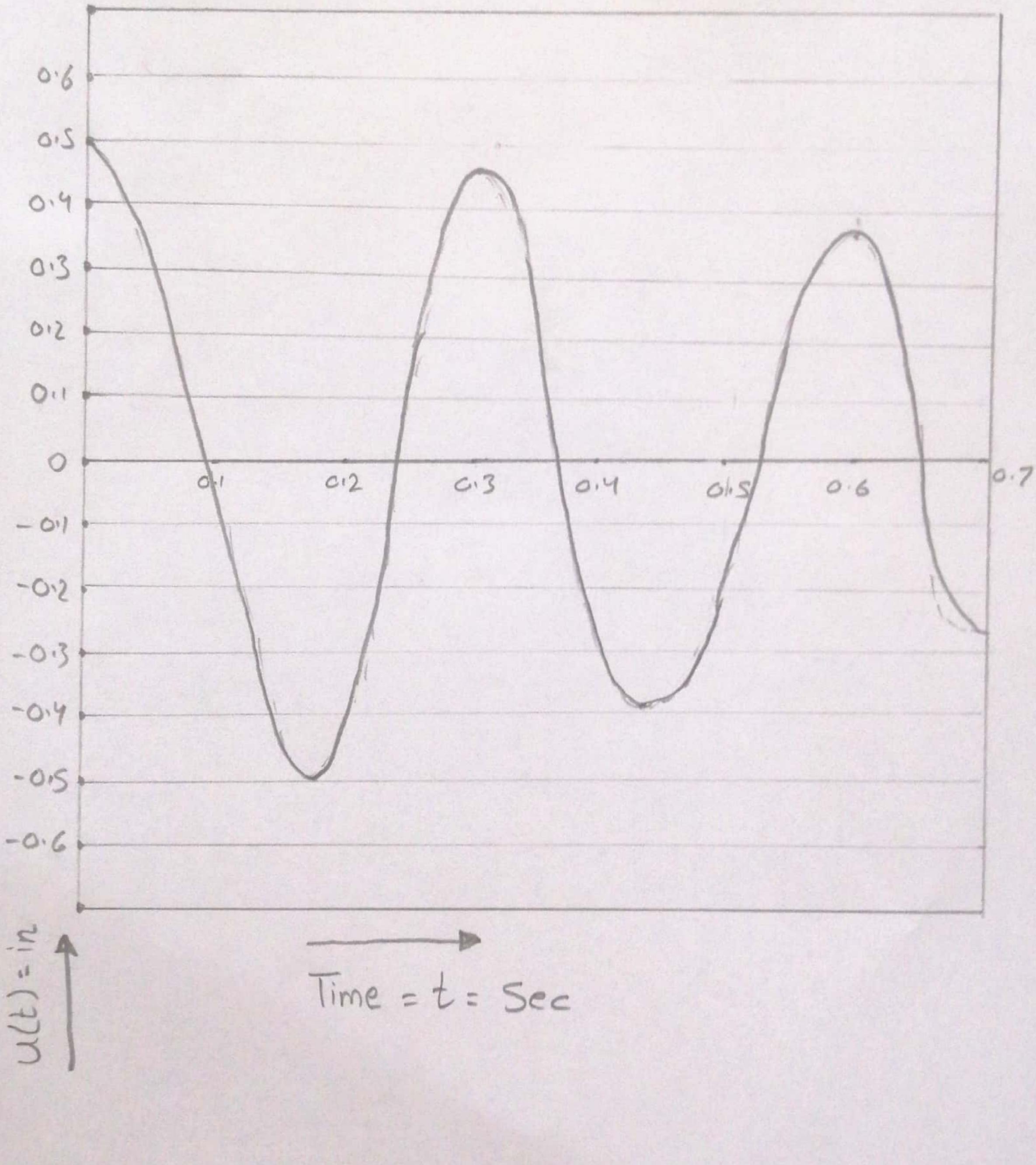
$$u(t) = e^{-0.584t}\left[0.042 \cos(19.45t) + 0.024 \sin(19.45t)\right]$$

$$f_s(t) = k.u(t) = 90625 \times u(t)$$

$$f_s(t) = e^{-0.584t}\left[3806.25 \cos(19.45t) + 2175 \sin(19.45t)\right]$$

Graph:

a. Shows variation of displacement with time;



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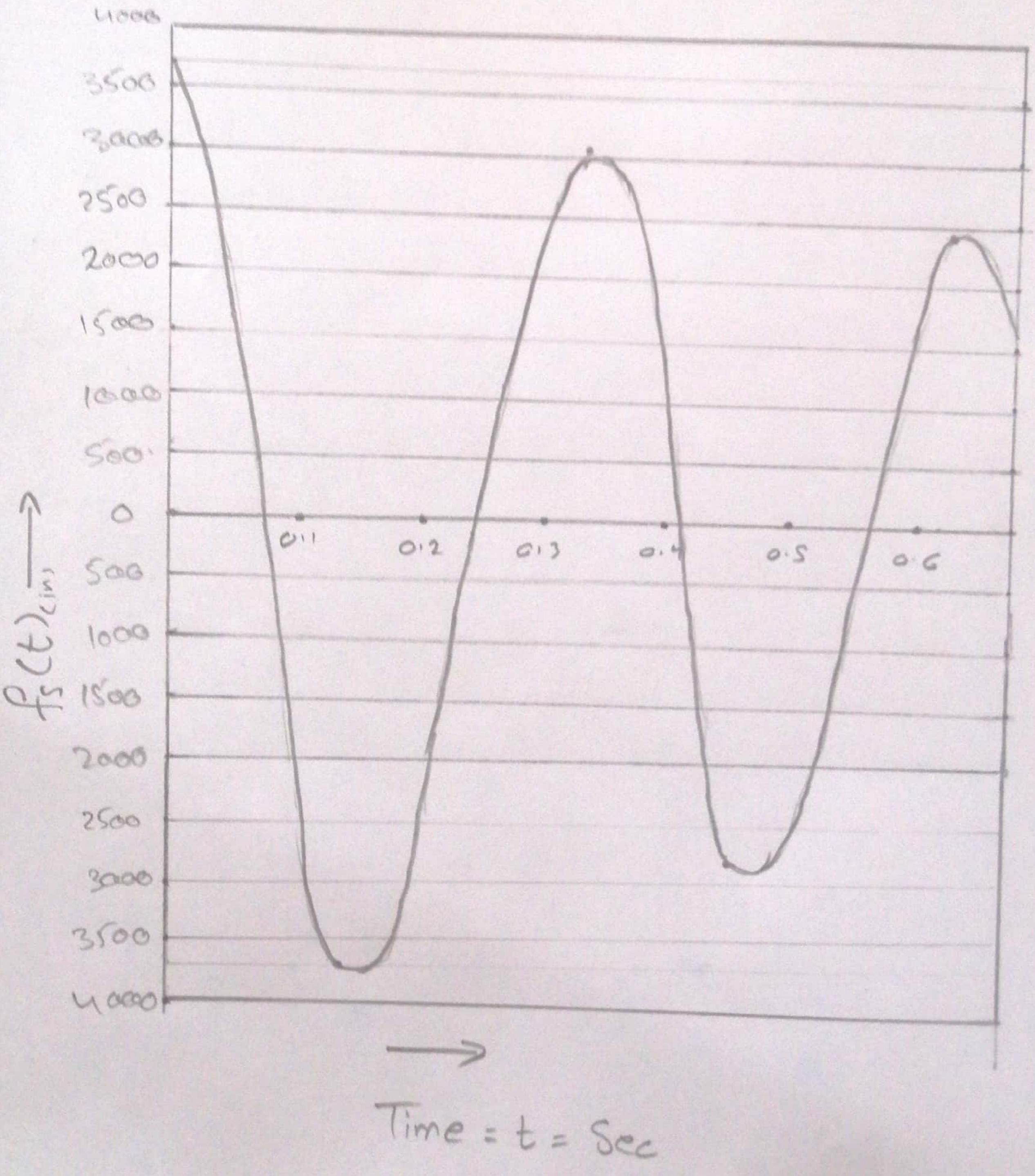
b- Showing Variation of Equivalent Static force with time:



Question No 3:

Given data:

- Applied Cable force = 60 kips
- Horizontal displacement of tank = $\frac{7715}{1000}$
  $= 7.715$ in
- Cycles = 7
- Cycle completion time = 3.57 sec
- Amplitude of displacement = 2.286 cm = 0.9 in

Required data:

Compute the following:

a. Damping ratios
b. Natural period of un-damped vibration
c. Stiffness of structures
d. Weight of tank
e. Damping coefficient
f. Number of cycles to reduce the displacement amplitude to 0.5"

Solution :

As given in question;

$u_1 = 7.715 \text{ in}$

After $J = \cancel{8}7$

$u_{J+1} = u_8 = 2.286 \text{ cm} = 0.9 \text{ in}$

a. ζ = Damping ratio = ?

$$J = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{J+1}}\right]$$

By putting values we get;

$$\cancel{8}\ 7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.715}{0.9}\right]$$

$$\zeta = \frac{1}{2\pi \times 7} \ln\left[\frac{7.715}{0.9}\right]$$

$\zeta = 0.0488 = 4.88\%$ Ans

$\zeta = 4.88\%$

b. Natural Period of undamped vibration:

According to question;

7 cycles of vibrations are completed in 3.57 sec

Time period to complete one cycle $= \frac{3.57}{7}$

$T_D = 0.51$ sec

As we know that;

$$\omega_D = \omega_n \sqrt{(1) - (\zeta)^2}$$

$$\frac{2\pi}{\omega_D} = 2\pi / \omega_n \sqrt{(1) - (\zeta)^2}$$

$$T_D = T_n / \sqrt{(1) - (\zeta)^2}$$

$$T_n = T_D \sqrt{(1) - (\zeta)^2}$$

By putting values we get;

$$T_n = 0.51 \sqrt{(1) - (0.0488)^2}$$

$$T_n = 0.5094$$

$$\mathbf{T_n = 0.51 \text{ sec}}$$

c. Stiffness of Structure = k =

As we know that;

$$K = \frac{60 \cos(60)}{7.715}$$

$$K = 3.88 \ K/in$$

$$K = 46560 \ lb/ft$$

d. Weight of tank = W =

As we know that

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{k/\left(\frac{W}{g}\right)} = \sqrt{\frac{k.g}{W}}$$

$$\omega_n^2 = \frac{k.g}{W}$$

$$W = \frac{k.g}{W_n^2} \longrightarrow (a)$$

Also; $W_n = \dfrac{2\pi}{T_n}$

eq (a) =>

$$W = \frac{k.g \times T_n^2}{4\pi^2}$$

By putting Values we get;

$$W = \frac{46560 \times 32.2 \times (0.51)^2}{4\pi^2} = 9877.5\ \text{lb}$$

$W = 9.878\ k$

e. Damping Coefficient = C =

As we know that;

$$\zeta = c/_{2m\omega_n}$$

$$c = \zeta \times 2m\omega_n = \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = \frac{\zeta \times 2 \times \pi \times m}{T_n}$$

By putting Values we get;

$$c = \frac{0.0488 \times 4 \times \pi \times (9877.5/_{32.2})}{0.51}$$

$c = 368.85\ \text{lb.sec}/ft$

$C = 368.85\ \text{lb.Sec}/ft$

f. Number of cycles to reduce the displacement amplitude to 0.5" = j =

As we know that;

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2 \times \pi \times 0.0488} \ln\left[\frac{7.715}{0.5}\right]$$

$j = 8.92$ say 9 cycles.

j = 9 cycles