Course: Discrete Structure
Program: BS (SE)
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PROGRAM: **BS,SE 2**

SECTION: **A**

COURSE: **DISCRETE STRUCTURE**

Note: Attempt all questions. Use examples and diagrams where necessary.

Q.1

a)Explain the concept of Biconditional statement.

Ans:

Biconditional Statement

A biconditional statement is a combination of a conditional statement and its converse written in the *if and only if* form.

Two line segments are congruent *if and only if* they are of equal length.

It is a combination of two conditional statements, “if two line segments are congruent then they are of equal length” and “if two line segments are of equal length then they are congruent”.

A biconditional is true if and only if both the conditionals are true.

Bi-conditionals are represented by the symbol $\leftrightarrow\leftrightarrow$ or $\Leftrightarrow\Leftrightarrow$. $p\leftrightarrow qp\leftrightarrow q$ means that $p\rightarrow qp\rightarrow q$ and $q\rightarrow pq\rightarrow p$. That is, $p\leftrightarrow q=(p\rightarrow q)\wedge(q\rightarrow p)p\leftrightarrow q=(p\rightarrow q)\wedge(q\rightarrow p)$.

Example:

Write the two conditional statements associated with the bi-conditional statement below.

A rectangle is a square if and only if the adjacent sides are congruent.

The associated conditional statements are:

a) If the adjacent sides of a rectangle are congruent then it is a square.

b) If a rectangle is a square then the adjacent sides are congruent.

[redacted]

b)

Let p, q, and r represent the following statements:

p: Sam had pizza last night.

q: Chris finished her homework.

r: Pat watched the news this morning

Give a formula (using appropriate symbols) for each of these statements.

i. Sam had pizza last night if and only if Chris finished her homework.
ii. Pat watched the news this morning iff Sam did not have pizza last night.
iii. Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.
iv. In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework

Ans:

In logic, own symbols are used in order to be able to represent the relations between propositions in a general and independent way to the proposition, in order to be able to find the relationship process that operates in the communicated message, the propositional logic.

For this purpose there are, among others, the following logical operators: conjunction (and) $\wedge$, disjunction (or) $\vee$, denial (not) $\neg$, conditional (if - then) $\Rightarrow$ and double conditional (if and only if, iff) $\Leftrightarrow$.

So for this case we have:

a) Sam had pizza last night if and only if Chris finished her homework.

$p\Leftrightarrow q$

b) Pat watched the news this morning iff Sam did not have pizza last night.

$r\Leftrightarrow\neg p$

c) Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.

$r\Leftrightarrow(q\wedge\neg p)$

d) In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework.

$r\Leftrightarrow(p\wedge q)$

e) $q \Leftrightarrow r$

Chris finished his homework if and only if Pat watched the news this morning

f) $p \Leftrightarrow (q \wedge r)$

Sam had pizza last night if and only if Chris finished his homework and Pat watched the news this morning

g) $(\neg p) \Leftrightarrow (q \vee r)$

Sam didn't have pizza last night if and only if Chris finished his homework or Pat watched the news this morning

h) $r \Leftrightarrow (p \vee q)$

Pat watched the news this morning if Sam had pizza last night or Chris finished his homework

Q.2

a) Lets p, q, r represent the following statements:

p: it is hot today.

q: it is sunny

r: it is raining

Express in words the statements using Bicondtional statement represented by the following formulas:

i. $q \leftrightarrow p$

i. $p \leftrightarrow (q \wedge r)$

i. $p \leftrightarrow (q \vee r)$

v. $r \leftrightarrow (p \vee q)$

Ans:

I. $q \leftrightarrow p$
it is sunny if and only if hot day.

II. $p \leftrightarrow (q \wedge r)$
it is hot day if and only if is sunny and if is raining.

III. $p \leftrightarrow (q \vee r)$
it is hot day if and only if it is sunny either if is raining.

IV. $r \leftrightarrow (p \vee q)$

it is raining if and only if it is day either it is sunny.

Q.3

a) explain Argument with proper examples. Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments. (Note: Examples and truth table should not belongs to your book or slides)

Ans:

Argument

Argument is a list of statements (premises or assumptions or hypotheses) followed by a statement (conclusion)

P1 Premise

P2 Premise

. . . .

Pn Premise

____

∴ C Conclusion

Example:

An interesting teacher keeps me awake.

I stay awake in Discrete Mathematics class. Therefore, my Discrete Mathematics teacher is interesting.

Valid & Invalid Argument

Argument is **valid** if the conclusion is true when all the premises are true **or** if conjunction of its premises imply conclusion.

$(P1 \wedge P2 \wedge P3 \wedge \ldots \wedge Pn) \rightarrow C$ is a tautology.

Argument is **invalid** if the conclusion is false when all the premises are true **or** if conjunction of its premises does not imply conclusion.

$(P1 \wedge P2 \wedge P3 \wedge \ldots \wedge Pn) \rightarrow C$ is a Contradiction.

A **valid argument** may have: true premises and a true conclusion
or false premises and a false conclusion
or false premises and a true conclusion
but it cannot have all true premises and yet a false conclusion

Arguments may either valid or invalid; and statements may either true or false

__Argument with Example..__

We have already encountered a few basic concepts related to propositions. They include **true** (*T*), **false** (*F*), **tautology**, **contradiction**, **and** *($\wedge$)*, **or** *($\vee$)*, **not** *($\sim$)* as well as **implication** *($\rightarrow$)*.

If we further define an **equivalence connective** $\leftrightarrow$ for any two propositions *p* and *q* by $(p \rightarrow q) \wedge (q \rightarrow p)$, denoted by $p \leftrightarrow q$, then the usual ''order of precedence'' is

1.connectives within parentheses, innermost parentheses first

2.$\sim$

3.$\wedge$, $\vee$

4.$\rightarrow$

5.$\leftrightarrow$

We note that $\wedge$ and $\vee$ are both left associative. From time to time you may also find that some people actually place a higher precedence for $\wedge$ than for $\vee$. To avoid possible confusion we shall always insert the parentheses at the appropriate places.

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	
T	T	T	T	T	T	critical rows
T	T	F	T	F	F	
T	F	T	T	T	T	critical rows
T	F	F	T	F	T	
F	T	T	T	T	T	critical rows
F	T	F	T	T	F	
F	F	T	F	T	T	
F	F	F	F	T	T	not a critical row

Valid and Invalid Argument:

An **argument** is a set of initial statements, called **premises**, followed by a **conclusion.**

An argument is **valid** if and only if in every case where all the premises are true, the conclusion is true. Otherwise, the argument is **invalid.**

1. The validity of the following argument is confirmed by the critical rows of the truth table as shown below.

$p \vee (q \wedge r)$

$\sim p$

$\therefore\ q \wedge r$

p	q	r	$p \vee (q \wedge r)$	$\sim p$	$q \wedge r$
T	T	T	T	F	T
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	F	T	F
$p \vee (q \wedge r)$ and $\sim p$ are the premises, while $q \wedge r$ is the conclusion. The critical row is highlighted in blue.					

2. An invalid argument form can likewise be demonstrated by truth tables.

$p \vee (q \wedge r)$

~(p ∧ q)

∴ *R*

p	*q*	*r*	*p ∨ (q ∧ r)*	*~(p ∧ q)*	*r*
T	T	T	T	F	T
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	F	T	T
F	F	F	F	T	F

Q.4

a) Explain the concept of Union, also explain membership table for union by giving proper example of truth table.

Ans:

<u>Union.</u>

Let A and B are two sets

A and B are subsets of a universal set U

The union of A and B is the set of all elements in U that belong to A or to B or to both

It is denoted A ∪ B

A ∪ B = {x ∈U | x ∈A or x ∈ B}

Union is commutative: A ∪ B = B ∪ A

A ⊆ A ∪ B and B ⊆ A ∪ B

Example: Let U = {a, b, c, d, e, f, g}
A = {a, c, e, g}, B = {d, e, f, g}
Then A ∪ B = {a, c, d, e, f, g} .

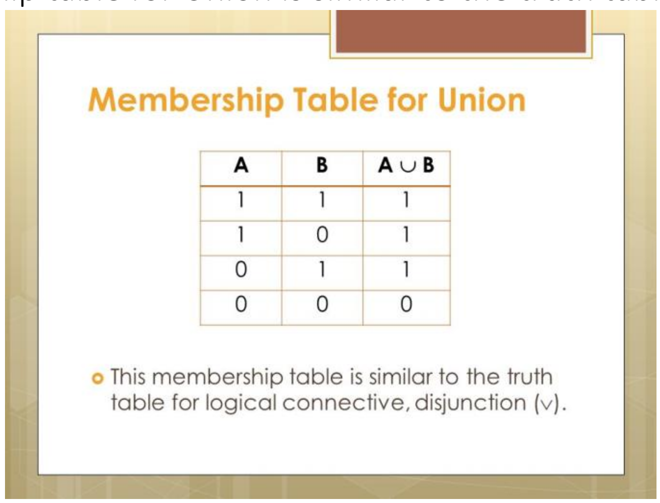
Membership Table for Union

The Membership table for the union of sets A and B is given below

The truth table for disjunction of two statements P and Q is given below

In the membership table of Union replace, 1 by T and 0 by F then the table is same as of disjunction

So membership table for Union is similar to the truth table for disjunction(v).


Membership Table for Union

A	B	A ∪ B
1	1	1
1	0	1
0	1	1
0	0	0

p	q	~p	~p ∨ q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

b) Explain the concept of Intersection, also explain membership table for Intersection by using proper example of truth table. (Note: Examples and truth table should not belongs to your book or slides)

Ans:

Intersection :

Let A and B are two sets

A and B are subsets of a universal set U

The intersection of A and B is the set of all elements in U that belong to both A and B

It is denoted A ∩ B

$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$

Intersection is commutative: $A \cap B = B \cap A$

$A \cap B \subseteq A$ and $A \cap B \subseteq B$

If A and B are disjoint, then $A \cap B = \phi$

Example: Let U = {a, b, c, d, e, f, g}

A = {a, c, e, g}, B = {d, e, f, g}

Then A ∩ B = {e, g}

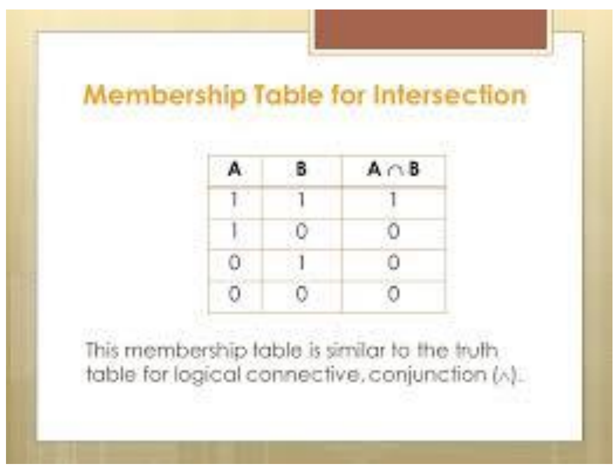
Membership Table For Intersection

The Membership table for intersection of sets A and B is given below

The truth table for conjunction of two statements P and Q is given below

In the membership table of Intersection, replace 1 by T and 0 by F then the table is same as of conjunction

So membership table for Intersection is similar to the truth table for conjunction (∧)


Membership Table for Intersection

A	B	A ∩ B
1	1	1
1	0	0
0	1	0
0	0	0

This membership table is similar to the truth table for logical connective, conjunction (∧).

p	q	~p	~p ∨ q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Q.5

a) Explain the concept of Venn diagram with examples.

Ans:

Venn diagram:

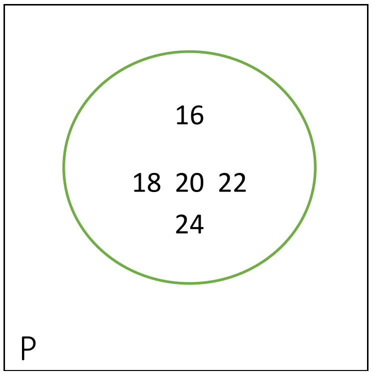
A Venn diagram is an illustration of the relationships between and among sets, groups of objects that share something in common. Usually, Venn diagrams are used to depict set intersections (denoted by an upside-down letter U). This type of diagram is used in scientific and engineering presentations, in theoretical mathematics, in computer applications, and in statistics.

The drawing is an example of a Venn diagram that shows the relationship among three overlapping sets X, Y, and Z. The intersection relation is defined as the equivalent of the logic AND. An element is a member of the intersection of two sets if and only if that element is a member of both sets. Venn diagrams are generally drawn within a large rectangle that denotes the *universe*, the set of all elements under consideration.

b)Given the set *P* is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set *P* and indicate all the elements of set *P* in the Venn diagram.

Ans:

P→The set of even integer between 15 and 25.
P={16,18,20,22,24}

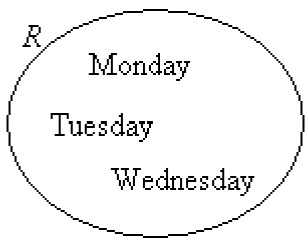


c) Draw and label a Venn diagram to represent the set

R = {Monday, Tuesday, Wednesday}.

Ans:

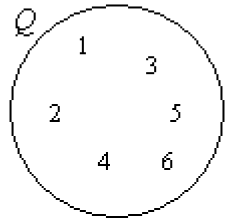
Draw a circle or oval. Label it *R* . Put the elements in *R*.



d) Given the set $Q = \{x : 2x - 3 < 11, x \text{ is a positive integer}\}$. Draw and label a Venn diagram to represent the set *Q*.

Ans:

Since an equation is given, we need to first solve for *x*.
$2x - 3 < 11 \Rightarrow 2x < 14 \Rightarrow x < 7$



So, $Q = \{1, 2, 3, 4, 5, 6\}$

Draw a circle or oval. Label it Q .

Put the elements in Q.