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Question # 1

There are total 5 machines and 5 employments are to be relegated and the related cost network is as per the following?

Solution :-

		MACHINES				
		A	B	C	D	E
J	1	6	12	3	11	15
O	2	4	2	7	1	10
B	3	8	11	10	7	11
S	4	16	19	122	23	21
	5	9	5	7	6	10

Step 1 - Row minimization :-

		Machines					
		#	A	B	C	D	
J	1	6	12	3	11	15	3
O	2	4	2	7	1	10	1
B	3	8	11	10	7	11	7
S	4	16	19	122	23	21	16
	5	9	5	7	6	10	5

(2)

→ Step # 2.

Row Subtraction:-

		Machines				
		A	B	C	D	E
J	1	3	9	0	8	12
D	2	3	1	6	0	9
B	3	1	4	3	0	4
S	4	0	3	106	6	5
	5	4	0	2	1	5

Step # 3.

Column Minimization:-

		Machines				
		A	B	C	D	E
J	1	3	9	0	8	12
O	2	3	1	6	0	9
B	3	1	4	3	0	4
S	4	0	3	106	6	5
	5	4	0	2	1	5

0 0 0 0 4

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→ Step # 4

Column Subtraction.

		Machines				
		A	B	C	D	E
J	1	3	9	0	8	8
O	2	3	1	6	0	5
B	3	1	4	3	0	0
S	4	0	3	106	6	1
	5	4	0	2	1	1

→ step # 5

Draw horizontal & vertical lines through circles -

		Machines					↑	↑
		A	B	C	D	E		
J	1 ←	3	⊙	⊙	8	8	→	
O	2 ←	3	1	6	⊙	5	→	
B	3 ←	1	4	3	0	⊙	→	
S	4 ←	⊙	3	106	6	1	→	
	5 ←	4	⊙	2	1	1	→	

↓ ↓ ↓ ↓ ↓

Hence $S = 5$

Optimal Solution -

(4)

→ Step # 6
Timing:

		Machines				
		A	B	C	D	E
J	1	6	12	3	11	15
O	2	4	2	7	1	10
B	3	8	11	10	7	11
S	4	16	19	122	23	21
	5	9	5	7	6	10

Jobs	operators	Time
1	C	3
2	D	1
3	E	11
4	A	16
5	B	5
		<u>36</u>

Total processing time = 36 cr. hr.

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Q2

Question # 2.

minimization $z = 2x_1 + 3x_2$

subj = $\frac{1}{2}x_1 + \frac{1}{4}x_2 \leq 4$

$x_1 + 3x_2 \geq 20$

$x_1 + x_2 \geq 10$

$x_1, x_2 \geq 0$

Solution:-

$\frac{1}{2}x_1 + \frac{1}{4}x_2 + S_1 = 4$

$x_1 + 3x_2 - S_2 + a_1 = 20$

$x_1 + x_2 + a_2 = 10$

Step # 2:-

Set the objective function equal to zero.

$z = -2x_1 - 3x_2$

$z = -2x_1 - 3x_2 - mA_1 - mA_2$

$z + 2x_1 + 3x_2 + mA_1 + mA_2 = 0$

$2x_1 + 3x_2 + mA_1 + mA_2 + z = 0$

Step # 3 = Create a simple table.

	x_1	x_2	S_1	S_2	a_1	a_2	z	
$\rightarrow R_1$	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
$\rightarrow R_2$	1	3	0	-1	1	0	0	20
$\rightarrow R_3$	1	1	0	0	0	1	0	10
$\rightarrow R_4$	2	3	0	0	m	m	1	0

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$R_4 + (-mR_3)$

$$\begin{array}{cccccccc} 2 & 3 & 0 & 0 & m & m & 1 & 0 \\ -m & -m & 0 & 0 & 0 & -m & 0 & -10m \\ \hline 2-m & 3-m & 0 & 0 & m & 0 & 1 & -10m \end{array}$$

x_1	x_2	S_1	S_2	a_1	a_2	Z	
$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
1	3	0	-1	1	0	0	20
1	1	0	0	0	1	0	10
$2-m$	$3-m$	0	0	m	0	1	$-10m$

$R_4 + (-mR_2)$

$$\begin{array}{cccccccc} 2-m & 3-m & 0 & 0 & m & 0 & 1 & -10m \\ -m & -3m & 0 & m & -m & 0 & 0 & -20m \\ \hline 2-2m & 3-4m & 0 & m & 0 & 0 & 1 & -30m \end{array}$$

	x_1	x_2	S_1	S_2	a_1	a_2	Z	
S_1	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
a_1	1	3	0	-1	1	0	0	20
a_2	1	1	0	0	0	1	0	10
Z	$2-2m$	$3-4m$	0	m	0	0	1	$-30m$

(3)

Step # 4 :-

Select the pivot column - E_1 Row

	x_1	x_2	S_1	S_2	a_1	a_2	Z	
S_1	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
a_1	1	3	0	-1	1	0	0	20
a_2	1	1	0	0	0	1	0	10
Z	$2-2m$	$3-4m$	0	m	0	0	1	$-30m$

Step # 5 :-

Select the pivot Row - E_1 column

	x_1	x_2	S_1	S_2	a_1	a_2	Z	
S_1	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	$4 \times \frac{1}{4} = 16$
a_1	1	3	0	-1	1	0	0	20 $\times \frac{1}{3} = 6.66$
a_2	1	1	0	0	0	1	0	10 $\times \frac{1}{1} = 10$
Z	$2-2m$	$3-4m$	0	m	0	0	1	$-30m$

Step # 6 :-

Select the pivot which is the entry in the pivot column E_1 pivot Row.

	x_1	x_2	S_1	S_2	a_1	a_2	Z	
S_1	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
a_1	1	3	0	-1	1	0	0	20
a_2	1	1	0	0	0	1	0	10
Z	$2-2m$	$3-4m$	0	m	0	0	1	$-30m$

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Step # 7:

Perform Row operations to make pivot equal to 1
 & the remaining elements in the pivot column equal to zero

Multiply R_2 by $\frac{1}{3}$

	x_1	x_2	S_1	S_2	a_1	a_2	Z	
S_1	2	1	4	0	0	0	0	16
a_1	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
a_2	1	1	0	0	0	1	0	10
Z	$2-2m$	$3-4m$	0	m	0	0	1	$-30m$

$R_1 + (-1R_2)$

$$\begin{array}{r}
 2 \quad 1 \quad 4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 16 \\
 -\frac{1}{3} \quad -1 \quad 0 \quad \frac{1}{3} \quad -\frac{1}{3} \quad 0 \quad 0 \quad 0 \quad -\frac{20}{3} \\
 \hline
 \frac{5}{3} \quad 0 \quad 4 \quad \frac{1}{3} \quad -\frac{1}{3} \quad 0 \quad 0 \quad 0 \quad \frac{28}{3}
 \end{array}$$

	x_1	x_2	S_1	S_2	a_1	a_2	Z	
S_1	$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$
a_1	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
a_2	1	1	0	0	0	1	0	10
Z	$2-2m$	$3-4m$	0	m	0	0	1	$-30m$

$R_3 + (-1R_2)$

$$\begin{array}{r}
 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 10 \\
 -\frac{1}{3} \quad -1 \quad 0 \quad \frac{1}{3} \quad -\frac{1}{3} \quad 0 \quad 0 \quad 0 \quad -\frac{20}{3} \\
 \hline
 \frac{2}{3} \quad 0 \quad 0 \quad \frac{1}{3} \quad -\frac{1}{3} \quad 1 \quad 0 \quad 0 \quad \frac{10}{3}
 \end{array}$$

QED # 3 =

(5)

	x_1	x_2	s_1	s_2	a_1	a_2	Z	
s_1	$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$
a_1	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
a_2	$\frac{2}{3}$	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{10}{3}$
Z	$2-2m$	$3-4m$	0	m	0	0	1	$-3m$

$$R_4 + (-3-4m R_2)$$

$$\begin{array}{r}
 2-2m \quad 3-4m \quad 0 \quad m \quad 0 \quad 0 \quad 1 \quad -3m \\
 -1+4m \quad -3+4m \quad 0 \quad 1-4m \quad 1+4m \quad 0 \quad 0 \quad -1+8m \\
 \hline
 1+2m \quad 0 \quad 0 \quad 1-3m \quad 1+4m \quad 0 \quad 1 \quad -10+5m
 \end{array}$$

Step # 8

	x_1	x_2	s_1	s_2	a_1	a_2	Z	
x_2	$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$
s_1	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
a_2	$\frac{2}{3}$	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{10}{3}$
Z	$1+2m$	0	0	$1-3m$	$1+4m$	0	1	$-10+5m$

Step # 8 :-

Repeat the identifying process by the most.

$$x_2 = \frac{28}{3}$$

$$s_1 = \frac{20}{3}$$

$$a_2 = \frac{10}{3}$$

$$Z = -10 + 5m$$

Ans :-

Question # 3

Use Vogel's approximation Method to obtain the initial feasible solution of:-

Origin	1	2	3	4	Supply
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

Solution :-

	1	2	3	4									
1	X 20	40	22	X 17	80	4	120	13	(13)	-	-	-	-
2	10	24	X 37	30	9	30	70	2	2	2	(17)	24	(24)
3	50	X	X	X	50	50	50	5	5	5	17	(32)	-
	32	37	20	15									
	60	40	30	110	240								
	10			30									

4	(15)	8	3
4	-	8	3
8	-	(11)	8
8	-	-	8
8	-	-	-
(24)	-	-	-

$$\begin{aligned}
 \text{Total Cost} &= 40 \times 22 + 80 \times 4 + 10 \times 24 + \\
 &\quad 30 \times 9 + 30 \times 7 + 50 \times 32 \\
 &= 880 + 320 + 240 + 270 + 210 + \\
 &\quad 1600 = \boxed{3520}
 \end{aligned}$$

$$\text{Total Cost} = \boxed{3520} \text{ Ans.}$$

