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Paper : lineer Algebra

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Depermnt : BSCS 2

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Q1

Sol

Here $A = (1,5,8), B = (5,8,6), C = (8,6,0)$

The vector A, B, C are linearly dependent, if their determinant is zero. i.e. $|D| = 0$

$$|D| = \begin{vmatrix} 1 & 5 & 8 \\ 5 & 8 & 6 \\ 8 & 6 & 0 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} 8 & 6 \\ 6 & 0 \end{vmatrix} - 5 \times \begin{vmatrix} 5 & 6 \\ 8 & 0 \end{vmatrix} + 8 \times \begin{vmatrix} 5 & 8 \\ 8 & 6 \end{vmatrix}$$

$$= 1 \times (8 \times 0) - 6 \times 6) - 5 \times (5 \times 0 - 6 \times 8) + 8 \times (5 \times 6 - 8)$$

$$= 1 \times (0 - 36) - 5 \times 5 \times (0 - 48) + 8 \times (30 - 64)$$

$$= 1 \times (-36) - 5 \times (-48)\ 8 \times (-34)$$

$$= -36 + 240 - 272$$

$$= 68 \neq 0$$

Since $|D| \neq 0$, So vectors A, B, C are linearly independent.

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Q2

P(b)

- $T(U+V) = T(U) + T(V)$
- $T(cU) = cT(U)$.

Sol

(a) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \to \begin{bmatrix} x - y \\ x + y \\ 2x \end{bmatrix}$

(b) $W\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \to \begin{bmatrix} x + y \\ y + 2 \end{bmatrix}$

$= T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \to \begin{bmatrix} x + y \\ x + y \\ 2x \end{bmatrix}$

$= T\left(\begin{bmatrix} 4 \\ 15 \end{bmatrix}\right) \to \begin{bmatrix} 4 - 15 \\ 4 + 15 \\ 2(4) \end{bmatrix} = \begin{bmatrix} 11 \\ 19 \\ 8 \end{bmatrix}$

$T(U+V) = T(U) + T(V)$

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$U \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad V = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} a_1 + b_1 \\ a_2 \quad b_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} a_1 + b_1 - (a_2 + b \\ a_1 + b_1 + a_2 + b_2 \\ 2(a_1 + b_1) \end{bmatrix}$$

$$= \begin{bmatrix} a_1 + b_1 - a_2 + b_2 \\ a_1 + b_1 + b_1 + b_2 \\ 2a_1 + 2ab_1 \end{bmatrix} \stackrel{?}{=} T(U+V)$$

$$T(U) + T(V)$$

$$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) + T\left(\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} a_1 - a_2 \\ a_1 + a_2 \\ 2a_1 \end{bmatrix} + \begin{bmatrix} b_1 - b_2 \\ b_1 + b_2 \\ 2b_1 \end{bmatrix}$$

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(4)

$$\begin{bmatrix} a_1 - a_2 + b_1 - b_2 \\ a_1 + a_2 + b_1 + b_2 \\ 2a_1 + 2b_1 \end{bmatrix} = T(U+V)$$

$$T(U+V) = T(U)$$

$$a\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$$

$$T(U+V) = T(U) + T(V)$$

$$T(cU) = cT(U)$$

$$U = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$T\left(c\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} c \cdot a_1 \\ c \cdot a_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} ca_1 - ca_2 \\ ca_1 + ca_2 \\ 2 \cdot c \cdot a \end{bmatrix}$$

$$c \cdot T\left(\begin{bmatrix} a_1 \\ a_2 \\ 2a \end{bmatrix}\right)$$

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$$\begin{bmatrix} c\,(a_1 - a_2) \\ c\,(a_1 + a_2) \\ c\,(2a_1) \end{bmatrix} = c \cdot T(v)$$

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Q3

Sol

The four main ingredinents are (i) a Set V of vectors. (ii) number field K (usually K = R) (iii) a rule vectors addition and (iv) way to muliply vectors by a number to Products a new (sclar mulitplication. There are, of course, ten rules that four ingredients must obey.

(a)

This is not a vector Space. Notice distsibutity of sclar muliplication require $2U = (1+1)U = U+U$ for any vector a but

$$2 \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & b \\ 2c & d \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 2c & 2d \end{pmatrix}$$

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$$k \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

(b)

This is a vector space. ȝ Vk
Although the question does not ask to, it is a useful exercise to verify, that all vectorspace) (ten space rules are satisfied.

x ———— x ———— x ————

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Q4 Determinants let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is is a 2×2 martrix

P (a) for which values of det M does and M have an invers?

$$= M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= M^{-1} = \text{Adj} M \cdot \frac{1}{|M|}$$

$$= |M| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= \cancel{|M| = \begin{bmatrix} \; \end{bmatrix} ad - bc}$$

$$= M = \begin{bmatrix} a & b \\ -c & d \end{bmatrix}$$

$$= \text{Adj} M = \begin{bmatrix} d & b \\ -c & a \end{bmatrix}$$

$$= M = \begin{bmatrix} d & b \\ -c & a \end{bmatrix}$$

$$= M^{-1} = \frac{\begin{bmatrix} d & b \\ -c & a \end{bmatrix}}{ad - bc}$$

$$\boxed{M^{-1} = [ab + bc] \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

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(9)

Q4

P(b) write down all 2x2 bit maxtrices with determinant
" " " " "

Sol

$$\Rightarrow D = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow |D| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow |D| = 1 - 0$$

$$\Rightarrow |D| = 1$$

Q4

P(a)

Sol

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Q4

P(c) write down 2x2 bit matrices with determinant 0

Sol

$$= B = \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= |B| = \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= |B| = |4 \times 1| \cancel{\ } - |2 \times 2|$$

$$= |B| = |4 \times 1| - |2 \times 2|$$

$$= |B| = 4 - 4$$

$$= |B| = 0$$

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Q4.

P(d) Compute det A for below 3x3 matrix

Sol

find |A|

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 5 & 8 & 5 \\ 6 & 1 & 0 \end{vmatrix}$$

Sol

$$A = \begin{vmatrix} 1 & 1 & 1 \\ 5 & 8 & 5 \\ 6 & 1 & 0 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} 8 & 5 \\ 1 & 0 \end{vmatrix} - 1 \times \begin{vmatrix} 5 & 5 \\ 6 & 0 \end{vmatrix} + 1 \begin{vmatrix} 5 & 8 \\ 6 & 1 \end{vmatrix}$$

$= 1 \times (8 \times 0 - 5 \times 1) - 1 \times (5 \times 0 - 5 \times 6) + 1 \times (5 \times 1 - 8 \times 6)$

$= 1 \times (0 - 5) - 1 \times (0 - 30) + 1 \times (5 - 48)$

$= 1 \times (-5) - 1 \times (-30) + \times (-43)$

$= -5 + 30 - 43$

$= -18$

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