Iqra National University, Peshawar
Department of Electrical Engineering

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Final – Term Examination Summer 2020
Date:23/009/2020

Course Code: MTH305		**Course Title:** Numerical Analysis	
Prerequisite:		**Instructor:** Engr. Pir Meher Ali Shah	
Module: 3	**Program:** BEE	**Total Marks:** 50	**Time Allowed:**

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	(a)	Find the LU Factorization of the following matrix $$A=\begin{bmatrix}1 & 2 & -1\\ 2 & 1 & -2\\ -3 & 1 & 1\end{bmatrix}$$	Marks CLO1
	(b)	Find the unknown variables of the following equation using gaussian elimination methods $$\begin{aligned}x+2y-z&=3\\ 2x+y-2z&=3\\ -3x+y+z&=-6\end{aligned}$$	Marks 7 CLO1
	(c)	Apply Gaussian elimination with partial pivoting to solve the following system of equations $$\begin{aligned}x_1-x_2+3x_3&=-3\\ -x_1-2x_3&=1\\ 2x_1+2x_2+4x_3&=0\end{aligned}$$	Marks 7 CLO
Q2	(a)	Apply Gauss- Seidel Method to the following system $$\begin{bmatrix}3 & 1 & -1\\ 2 & 4 & 1\\ -1 & 2 & 5\end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix}=\begin{bmatrix}4\\ 1\\ 1\end{bmatrix}$$	Marks 6 CLO1
	(b)	Find the reduced QR factorization by applying Gram- Schmidt orthogonalization to the column of the following matrix $$A=\begin{bmatrix}1 & -4\\ 2 & 3\\ 2 & 2\end{bmatrix}$$	Marks 7 CLO2

	(c)	Let $x=[3 \quad 4]$ and $w=[5 \quad 0]$. Find a house holder reflector H that satisfies Hx=w	Marks 7 CLO
Q3	(a)	Find the Newton's method formula for the following equation $$x^3+x-1=0$$	Marks 5 CLO2
	(b)	Find the line that best fits the three data points (t,y)=(1,2),(-1,1) and (1,3) in the figure below 	Marks 5 CLO 2

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(a)

Q1:- Find the LU factorization of the following matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ -3 & 1 & 1 \end{bmatrix}$$

Sol:

$$= \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ -3 & 1 & 1 \end{bmatrix} \quad 2R_1 - R_2$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 0 \\ -3 & 1 & 1 \end{bmatrix} \quad -3R_1 - R_3$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 0 \\ 0 & 7 & -2 \end{bmatrix} \quad -7/3 R_2 - R_3$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix} = U$$

The lower triangular L is formed by putting one in its main diagonal

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and the multiplier in lower triangle.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -7/3 & 1 \end{bmatrix}$$

So

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -7/3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ -3 & 1 & 1 \end{bmatrix} = A$$

Q1(b) Find unknown variables of the following equation using gaussian elemination method

$$x + 2y - z = 3$$

$$2x + y - 2z = 3$$

$$-3x + y + z = -6$$

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Sol
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The augmented form of given equation is

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 1 & -2 & 3 \\ -3 & 1 & 1 & -6 \end{array}\right]$$

Two steps are needed to eliminate column 1

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 1 & -2 & 3 \\ -3 & 1 & 1 & -6 \end{array}\right] \; 2R_1 - R_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -3 & 0 & -3 \\ -3 & 1 & 1 & 6 \end{array}\right] \; -3R_1 - R_3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -3 & 0 & -3 \\ 0 & 7 & -2 & 3 \end{array}\right] \; -\tfrac{7}{3}R_2 - R_3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -3 & 0 & -3 \\ 0 & 0 & -2 & -4 \end{array}\right]$$

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from equations

$$-2z = -4$$

$$\boxed{z = 2} \rightarrow (i)$$

$$-3y = -3$$

So $y = \frac{-3}{-3}$

$$\boxed{y = 1} \rightarrow (ii)$$

$$x + 2y - z = 3$$

$$x + 2(1) - 2 = 3$$

$$x + \cancel{2} - \cancel{2} = 3$$

$$\boxed{x = 3} \rightarrow (iii)$$

Q(c):- Apply Guass Elimination with partial pivoting to solve the given equation.

$$x_1 - x_2 + 3x_3 = -3$$
$$-x_1 \quad - 2x_3 = 1$$
$$2x_1 + 2x_2 + 4x_3 = 0$$

Sol
The given equation in tableau form

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & -3 \\ -1 & 0 & -2 & 1 \\ 2 & 2 & 4 & 0 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 0 \\ -1 & 0 & -2 & 1 \\ 1 & -1 & 3 & -3 \end{array}\right] -\frac{1}{2}R_1 - R_2$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & -1 & 3 & -3 \end{array}\right] \frac{1}{2}R_1 - R_3$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -2 & 1 & -3 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 0 \\ 0 & -2 & 1 & -3 \\ 0 & 1 & 0 & 1 \end{array}\right] \quad -\frac{1}{2}R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 0 \\ 0 & -2 & 1 & -3 \\ 0 & 0 & 1/2 & -1/2 \end{array}\right]$$

The equavlian are now Simple to Solve

$$+1/2\, x_3 = -1/2$$

$$\boxed{x_3 = -1}$$

$$-2x_2 + x_3 = -3$$

$$-2x_2 + (-1) = -3$$

$$-2x_2 = -3 + 1$$

$$\boxed{x_2 = 1}$$

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$$2x_1 + 2x_2 + 4x_3 = 0$$

$$2x_1 + 2(1) - 4 = 0$$

$$2x_1 + 2 - 4 = 0$$

$$2x_1 = 2$$

$$\boxed{x_1 = 1}$$

The partial pivoting also solves the problem of zero ~~points~~ pivots.

Q2(a) Apply Gauss Seidal Method to the following system.

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 2 & 5 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

Sol

Applying Gauss Seidal Method

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 2 & 5 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

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The Given Seidal iteration is

$$u_{k+1} = \frac{4 - v_k + w_k}{3}$$

$$v_{k+1} = \frac{1 - 2u_{k+1} - w_k}{4}$$

$$w_{k+1} = \frac{1 + u_{k+1} - 2v_{k+1}}{5}$$

Starting with $x_0 = [u_0, v_0, w_0] = (0, 0, 0)$

$$\begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} \frac{4-0-0}{3} = 4/3 \\ \frac{1-8/3-0}{4} = -5/12 \\ \frac{1+4/3+5/6}{5} = 19/30 \end{bmatrix} \simeq \begin{bmatrix} 1.33 \\ -0.416 \\ 0.63 \end{bmatrix}$$

and

$$\begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{101}{60} \\ -3/4 \\ 251/300 \end{bmatrix} \simeq \begin{bmatrix} 1.683 \\ -0.75 \\ 0.836 \end{bmatrix}$$

The System is strictly dignally dominant, the iteration will converge to solution $[2, -1, 1]$

Q^2 (b) Find Reduced QR Factorization by applying Gram-Schmidt Orthognolizication to the column of the following matrix.

$$A = \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}$$

Solution:-

Set $y_1 = A_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ Then

$$r_{11} = \|y_1\|_2 = \sqrt{1^2 + 2^2 + 2^2}$$

$r_{11} = 3$ and the first unit vector is:

$$q_1 = \frac{y_1}{\|y_1\|_2} = \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$$

for finding q_2 Put ~~$y_2 = A_2$~~

$$y_2 = A_2 - q_1 q_1^T A_2$$

$$= \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix} 2 = \begin{bmatrix} -14/15 \\ 1/3 \\ 2/15 \end{bmatrix}$$

Since $r_{12} = q_1^T A_2 = 2$
and
$r_{22} = 5$ the
matrix form will be

$$A = \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1/3 & -14/15 \\ 2/3 & 1/3 \\ 2/3 & 2/15 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} = QR.$$

Q2(C):- Let $x = [3 \; 4]$ and
$w = [5 \; 0]$. find house holder
reflector H that satisfies $Hx = w$

Sol.

$$V = w - x$$

$$V = \begin{bmatrix} 5 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

and difinite projection matrix

$$P = \frac{VV^T}{V^TV} = \frac{1}{20}\begin{bmatrix} 4 & -8 \\ -8 & 16 \end{bmatrix} = \begin{bmatrix} 0.2 & -0.4 \\ -0.4 & 0.8 \end{bmatrix}$$

then

$$H = I - 2P$$

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & -0.8 \\ -0.8 & 1.6 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$$

Check that H moves x to w,

$$Hx = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$\boxed{Hx = w}$$

and

$$Hw = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\boxed{Hw = X}$$

Q3.(a) Find Newton's Method formula for the given equation

$$x^3 + x - 1 = 0$$

Sol #

$$f(x) = x^3 + x - 1$$

$$f'(x) = 3x^2 + 1$$

The formula is given by

$$x_{i+1} = x_i - \frac{x_i^3 + x_i - 1}{3x_i^2 + 1}$$

$$= \frac{2x_i^3 + 1}{3x_i^2 + 1}$$

Iterating this formula from initial guess $x_0 = -0.7$ yields

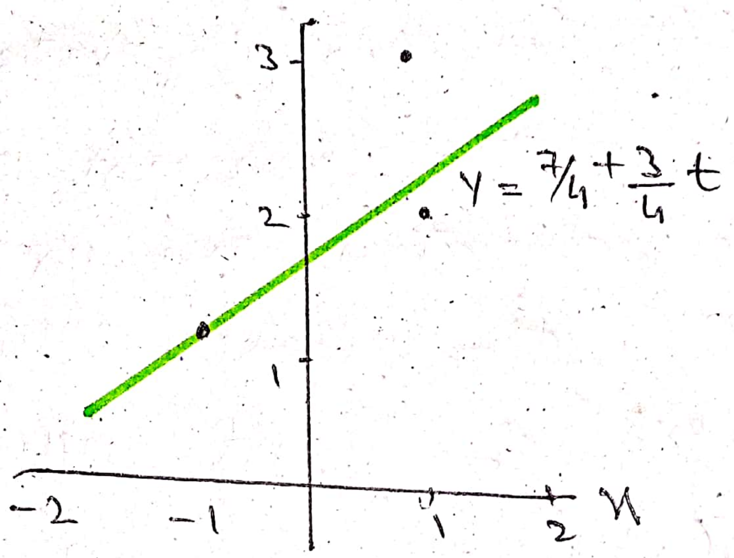
$$x_1 = \frac{2x_0^3 + 1}{3x_0^2 + 1} = \frac{2(-0.7)^3 + 1}{3(-0.7)^2 + 1}$$

$$x_1 \simeq 0.1271$$

$$x_2 = \frac{2x_1^3 + 1}{3x_1^2 + 1}$$

$$x_2 \simeq 0.9577$$

Q3 (b) Find the best line that fits the three data points $(t, y) = (1, 2), (-1, 1)$ & $(1, 3)$ in figure below



Sol

The model is $y = c_1 + c_2 t$ and the goal is to find the best c_1 and c_2. Substitutions of data points into model yields

$$c_1 + c_2(1) = 2$$

$$c_1 + c_2(-1) = 1$$

$$c_1 + c_2(1) = 3$$

Or in matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

This system has no solution (c_1, c_2) for two seperate reason

The points are not collinear the equation are in consistent.

So the best solution in term of least square is $(c_1, c_2) = (7/4, 3/4)$, Therefore the best line is

$$y = 7/4 + 3/4\, t$$

End of paper