CALCULUS EXAM

FINAL EXAM

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Calculus & Analytical Geometry
Final Paper (BS, SE)
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NOTE:- Attempt all questions:-

Q#01

(a) Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$ with respect to x.

Solution:-

$$y=\frac{3x^4-2x^3+5}{x^3+1}$$

Diff w.r.t x.

$$\frac{dy}{dx}=\frac{(x^3+1)\frac{d}{dx}(3x^4-2x^3+5)-(3x^4-2x^3+5)\frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$dy/dx=\frac{(x^3+1)(12x^3-6x^2)-(3x^4-2x^3+5)(3x^2)}{(x^3+1)^2}$$

$$dy/dx=\frac{12x^6-6x^5+12x^3-6x^2-9x^6+6x^5-15x^2}{(x^3+1)^2}$$

$$dy/dx=\frac{3x^6+12x^3-21x^2}{(x^3+1)^2}$$

$$=\frac{3x^2(x^4+4x-7)}{(x^3+1)^2}$$

(b) Differentiate $\frac{(x^3+1)^2}{x^3-1}$ with respect to x.

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Solution:-

$$y=\frac{(x^3+1)^2}{x^3-1}$$

Diff w.r.t x

$$dy/dx=\frac{(x^3-1)\frac{d}{dx}(x^3+1)^2-(x^3+1)^2\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

$$dy/dx=\frac{3x^2(x^3+1)[2(x^3-1)-(x^3+1)]}{(x^3-1)^2}$$

$$dy/dx=\frac{3x^2(x^3+1)[2x^3-2-x^3-1]}{(x^3-1)^2}$$

$$dy/dx=\frac{3x(x^3+1)(x^3-3)}{(x^3-1)^2}$$

Q#02

Find the integration of $\int \frac{1}{\sqrt{x^5}}dx$

(a)

Solution:-

$$\Rightarrow \int \frac{1}{\sqrt{x^5}}dx$$

$$\Rightarrow \int \frac{1}{(x^5)^{1/2}}dx$$

$$\Rightarrow \int x^{-5/2}dx$$

$$\Rightarrow \frac{x}{-5/2+1}+c$$

$$\Rightarrow \frac{x^{\frac{-5+2}{2}}}{\frac{-5+2}{2}}+c$$

$$\Rightarrow \frac{x^{-3/2}}{-3/2}+c$$

$= -\frac{2}{2} x^{-\frac{3}{2}} + C$

$= -\frac{2}{3} \frac{1}{x\sqrt{x}} + C$ Ans

(B) Find the integration of $\int \frac{1}{(8u+7)^8} du$

Solution:-

$\Rightarrow \int \frac{1}{(8u+7)^8} du$

$\Rightarrow \int (8u+7)^{-8} du$

Multiply and divide by 8.

$\Rightarrow \frac{1}{8} \int (8u+7)^{-8} du$

$\Rightarrow \frac{1}{8} \frac{(8u+7)^{-8+1}}{-8+1} + C$

$\Rightarrow \frac{1}{8} \frac{(8u+7)^{-7}}{-7} + C$

$\Rightarrow -\frac{1}{56} * (8u+7)^{-7} + C$

$\Rightarrow \frac{1}{56} \left(\frac{1}{(8u+7)^7} + C \right.$ Ans

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Q # 03

Find the integration of $\int \frac{-x+9}{2x^2-8x+6}\,dx$

Sol:-

$$I = \int \frac{-x+9}{2x^2-2x-6x+6}\,dx$$

$$\int \frac{-x+9}{2x(x-1)-6(x-1)}\,dx$$

$$\int \frac{-x+9}{(x-1)(2x-6)}\,dx$$

$$\frac{-x+9}{(x-1)(2x-6)} = \frac{A}{x-1} + \frac{B}{2x-6} \rightarrow (1)$$

Multiplying $(x-1)(2x-6)$ on both side

$$= A(2x-6) + B(x-1)$$

$$-x+9 = A(2x-6) + B(x-1) \rightarrow (2)$$

let

$$x-1 = 0$$

Put $x = 1$

$$-1+9 = A(2(1)-6) + B(1-1)$$

$$8 = A(-4) + 0$$

$$8 = -4A$$

$$\frac{8}{4} = A$$

$$\boxed{-2 = A}$$

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let $2x-6 = 0$

$$2x = 6$$

$$x = \frac{6}{2}$$

$$x = 3$$

$$-3+9 = A(2(3)-6) + B(3-1)$$

$$6 = A(6-6) + B(2)$$

$$6 = A(0) + 2B$$

$$6 = 0 + 2B$$

$$\frac{6}{2} = B$$

$$\boxed{3 = B}$$

Put value of A and B

$$\frac{-x+9}{(x-1)(2x-6)} = \frac{-2}{x-1} + \frac{3}{2x-6}$$

Taking integral on both sides

$$\int \frac{-x+9}{(x-1)(2x-6)} = \int \left(\frac{-2}{(x-1)} + \frac{3}{2x-6}\right) dx$$

$$\int \frac{-2}{x-1}\,dx + \int \frac{3}{2x-6}\,dx$$

$$-2\int \frac{1}{x-1}\,dx + 3\int \frac{1}{2x-6}\,dx$$

$$-2\ln(x-1) + \frac{3}{2}\int \frac{2}{2x-6}\,dx$$

$$-2\ln(x-1) + \frac{3}{2}\ln(2x-6) + C$$

$$\ln(x-1)^2 + \ln(2x-6)^{3/2} + C$$

Ans

(b) Find the integration of $\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)}\,dx$

Sol:-

$$\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)}\,dx$$

$$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{x+D}{(x^2+2x+3)}$$

$$4x^2+8x = Ax+B(x^2-2x+3)+(x+D)(x^2+1)$$

$$4x^2+8x = Ax^3+2Ax^2+3Ax+Bx^2+2Bx+3B+Cx^3+C+Dx^2+D$$

Coefficient Comparison

$D = A+C$	$4 = -2A+B$	$8 = 3A-2B$
$-C = A$	$8 = 4A+2B$	$8 = 3(0)-2B$
$-C = 0$	$8 = 2A-2B$	$8 = 2B$
$C = 0$	$0 = 2A$	$4 = B$

$$0 = A$$

$$0 = C+D+3B$$

$$0 = D+12 \quad \leftarrow \quad 0 = 0+D+3(4)$$

$$-12 = D$$

$$\int \frac{4x^2+8x}{(x^2+1)(x^2-2x-3)}\,dx = \int \frac{4}{x^2+1}\,dx + \int \frac{-1+2}{x^2+1}\,dx$$ Ans

Q# 4

Solve the following Matrix

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

Sol:-

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 5-3 & 1-(-1) \\ -3-2 & 1-2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$$ Ans

(b) $X + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ 2 & 0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5-0 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2+1 & -2-0 \\ -1-0 & 5-2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$ Ans

(c) $X + 2I \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I \Rightarrow X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow X \begin{bmatrix} 3-2 & -1-0 \\ 1-0 & 2-2 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$ Ans

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Q#05

(a) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Find $A^2 + BC$

$A^2 + BC$

$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$A^2 = A \cdot A$

$A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1\times1+4\times2 & 1\times4+4\times1 \\ 2\times1+1\times2 & 2\times4+1\times1 \end{bmatrix}$

$A^2 \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$

$BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$Bc = \begin{bmatrix} -3\times1+2\times0 & -3\times0+2\times2 \\ 4\times1+0\times0 & 4\times0+0\times2 \end{bmatrix}$

$BC = \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix}$

$BC = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$

$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$

$A^2 + BC = \begin{bmatrix} 9+(-3) & 8+4 \\ 4+4 & 9+0 \end{bmatrix}$

$A^2 + BC = \begin{bmatrix} 9-3 & 12 \\ 8 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$

Ans