HURRAIRAH KABIR

7758

SECTION B

INTRO TO STRUCTURAL DYNAMICS & EARTHQUAKE ENGG

FINAL ASSIGNMENT

Question No 1:-

The general E.O.M for SDOF system is

$$Ku + c\dot{u} + m\ddot{u} = P(t)$$

In our case system is undamped ($c=0$) undergoing Free vibration ($P(t)=0$).

Hence general EOM become $ku + m\ddot{u} = 0$ ---(1)

$$K = 3EI/L^3$$

$$= \frac{3 \times 29000 \times 150}{(10 \times 12)^3}$$

$$= 7.55 \text{ k/in} \Rightarrow K = 7.55208$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use Fundamental units like lb, ft sec or kg m, sec.

$$K = 7.55 \text{ K/in} = 90625 \text{ lb/ft}$$

$$m = 7758/32.2 = 240.9 \text{ slug}$$

$$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{240.9}}$$

$$\omega_n = 19.42 \text{ rad/sec}$$

$$T_n = \frac{2\pi}{\omega_n} \Rightarrow \frac{2\pi}{19.42} = 0.323 \text{ sec}$$

put m & k in eq (1)

$$90625u + 240.9\ddot{u} = 0$$

where k is in lb/ft & m is in lbsec/ft^2 (slug)

⇒ General solution to EOM for a undamped free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$u(0) = \frac{1}{2''} = \frac{1}{24}$ ft & $\dot{u}(0) = 0$

$$u(t) = \frac{1}{24} \times \cos(19.425t) + 0 = \left(\frac{1}{24}\right) \times \cos(19.42t)$$

Equivalent static force at any time 't' is

$$F_s(t) = k \cdot u(t) = \frac{90625 \times \cos(19.42t)}{24}$$

$$= 3776 \cos(19.42t)$$

Amplitude of dynamic displacement, u_o for undamped free vibration is

$$u_o = \sqrt{[u(0)]^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2}$$

$= \sqrt{\left(\left(\frac{1}{24}\right)^2 + 0\right.}$

$= \frac{1}{24} ft$

Amplitude of equivalent static force, F_{so}

$K_{u_0} = 90625 \times \frac{1}{24}$

$= 3776.$

Question No 2

ξ (Damping ratio) of reinforced concrete with considerable cracking = 3-5%
= 3%

Using Data of beam given in Question # 1

Required:-

- Develop & solve the equation showing variation in Equivalent static force with time.
- Draw graph to show the variation of displacement with time & the variation of equivalent static force with time.

Sol:-

E.O.M for damped free variation is

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{--- (1)}$$

from question 1

$K = 90625$ lb/ft & $m = 240.9$ lb sec²/ft
$\omega_n = 19.42$ rad/sec

$$C = \xi \times 2m\omega_n$$

$$C = (0.03) \times 2(240.9 \times 19.42)$$

$$C = 280.69 \text{ lb.sec/ft}$$

Put values in eq (1)

$$90625u + 280.69 + 240.9\ddot{u} =$$

Solution to the EOM for damped free vibration is

$$u(t) = e^{-\zeta \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left(\dot{u}(0) + u(0)\zeta\omega_n\right)\sin \omega_D t\right]$$

$\omega_D = 19.42$ rad/sec

$$u(t) = e^{-0.03 \times 19.42 t}\left[\frac{1}{24} \times \cos(19.42t) + \frac{1}{19.42} \times \left[0 + \frac{1}{24} \times 0.03 \times 19.42\right] \times \sin(19.42t)\right]$$

$$u(t) = e^{-0.582t}\left(0.041 \times \cos(19.42t) + 0.00125 \times \sin(19.42t)\right)$$

$$F_s(t) = k.u(t) \Rightarrow 90625 \times u(t)$$

$$F_s(t) = e^{-0.582t}\left((90625 \times 0.041)\cos(19.42t) + (90625 \times 0.00125) + \sin(19.42t)\right)$$

$$F_s(t) = e^{-0.582t}\left(3715.62\cos(19.42t) + 113.28\sin(19.42)t\right)$$

Question No 3

Given Data:-

Force = 60 kips

Displacement of tank = $\frac{TD}{1000}$ = $\left(\frac{7758}{1000}\right)''$ = 7.758"

time taken to complete 7 cycles = 3.57 sec

amplitude of displacement = 2.286 cm = 0.9"

Required Data:-

1) Damping ratio.
2) Natural period of undamped vibration
3) Stiffness of structures
4) Weight of tank.
(5) Damping co-efficient
(6) No. of cycles to reduce the displacement amplitude to 0.5"

Sol:-

→ Displacement of tank, u_1 = 7.758"
→ After 7 cycles i-e after $j=7$, $u_{j+1} = u_8 = 0.9"$

a) Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u+1}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.758}{0.9}\right]$$

$$\zeta = 0.048 = 4.88\%$$

b) Natural period of undamped vibration = T_n = ?

As, The 7 cycles of vibrations are completed in 3.57 sec

⇒ Time required to complete on cycle, T_D

$$= \frac{3.57}{7} = 0.51 \text{ sec}$$

Now,

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1-\zeta^2})}$$

$$= T_D = \frac{T_n}{(1-\zeta)^2}$$

$$= T_n = T_D \times \sqrt{1-\zeta^2}$$

$$= 0.51 \times \sqrt{1-(0.0488)^2}$$

$$= 0.5094$$

$$= 0.51 \text{ sec}$$

$T_n = 0.51$ sec

c) Stiffness of Structure, k = ?

$$K = \frac{60 \times W \times 60}{7.758} = 3.91 \text{ k/in}$$

$K = 3.91$ k/in = 46920 lb/ft

d) Weight of Tank, w=?

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{\frac{w}{g}}} = \sqrt{\frac{k.g}{w}}$$

$$\Rightarrow \omega_n^2 = \frac{k \times g}{w}$$

$$w = \frac{k \times g}{\omega_n^2}$$

Also $\omega_n = \frac{2\pi}{T_n}$

$$w = \frac{kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$= kg \times \frac{T_n^2}{4\pi^2}$$

$$w = \left[\frac{46920\,lb}{ft} \times \frac{32.2\,ft}{sec^2}\right] \times \frac{(0.51\,sec)^2}{4\pi^2}$$

$$= 9953.93\,lb = 9.95\,k$$

c) Damping Co-efficient, C=?

It is known that,

$$\xi = \frac{c}{2m\omega_n}$$

$$\Rightarrow C = \xi \times 2m\omega_n$$

$$= \xi \times 2m \times \frac{2\pi}{T_n}$$

$$= \frac{0.0488 \times 4 \times \pi \times (9953.93/32.2)}{0.51}$$

$$C = 371.71\ lb\,sec/ft$$

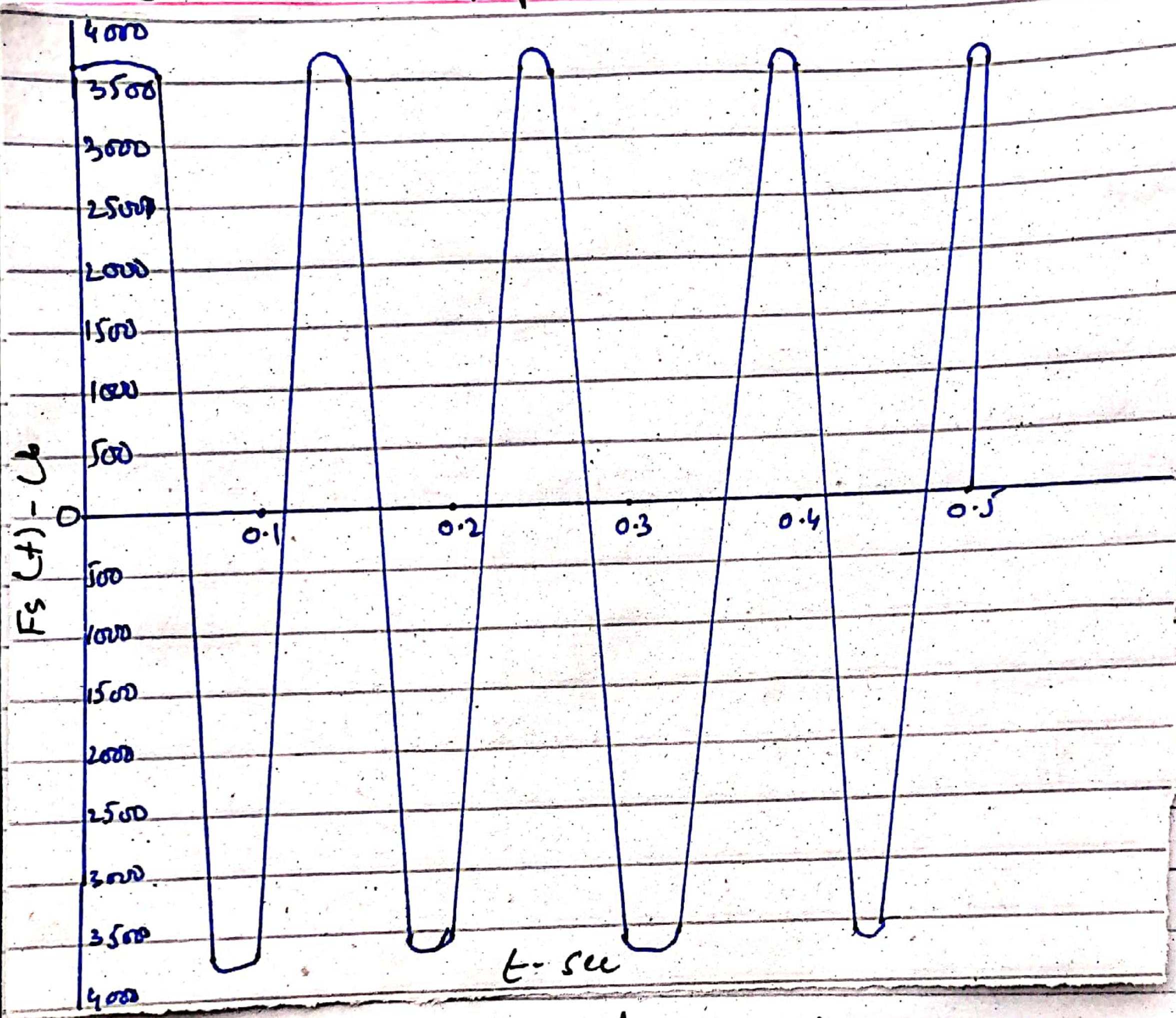
8) No. of cycles to reduce the displacement amplitude to 0.5", j = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{U_1}{U_{j+1}}\right]$$

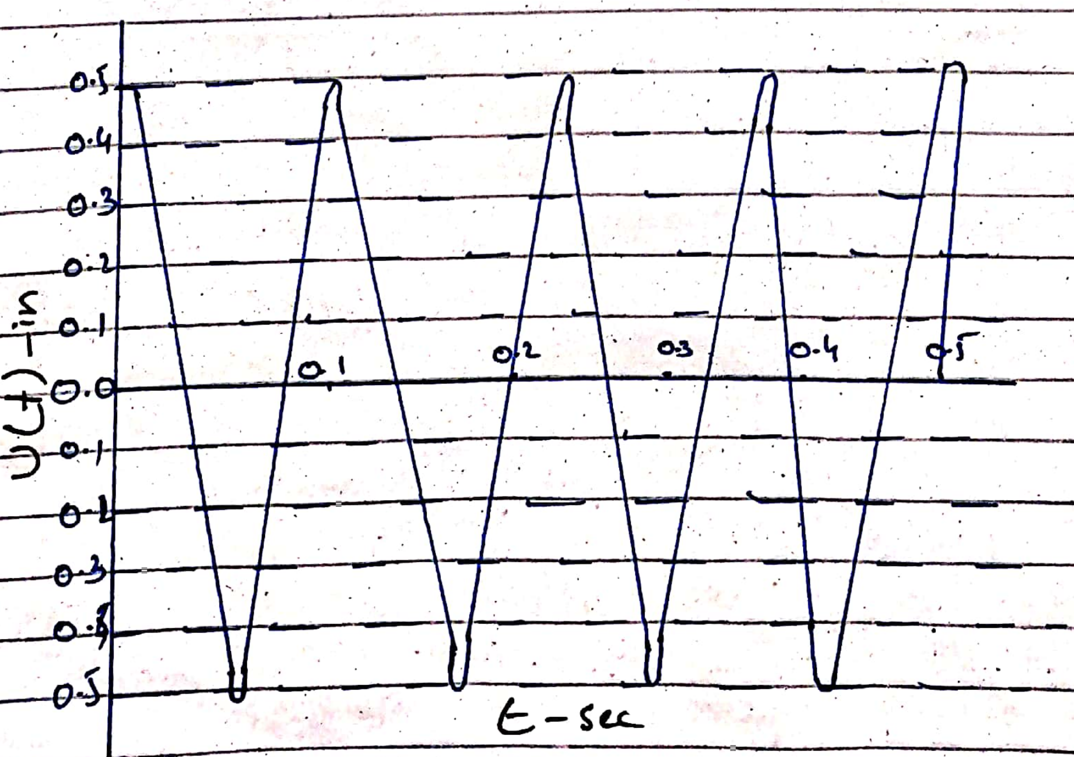
$$\Rightarrow j = \frac{1}{2\pi \times 0.0488} \ln\left[\frac{7.758}{0.5}\right]$$

$j = 8.91$ or 9 cycles

Question 1 Graphs.

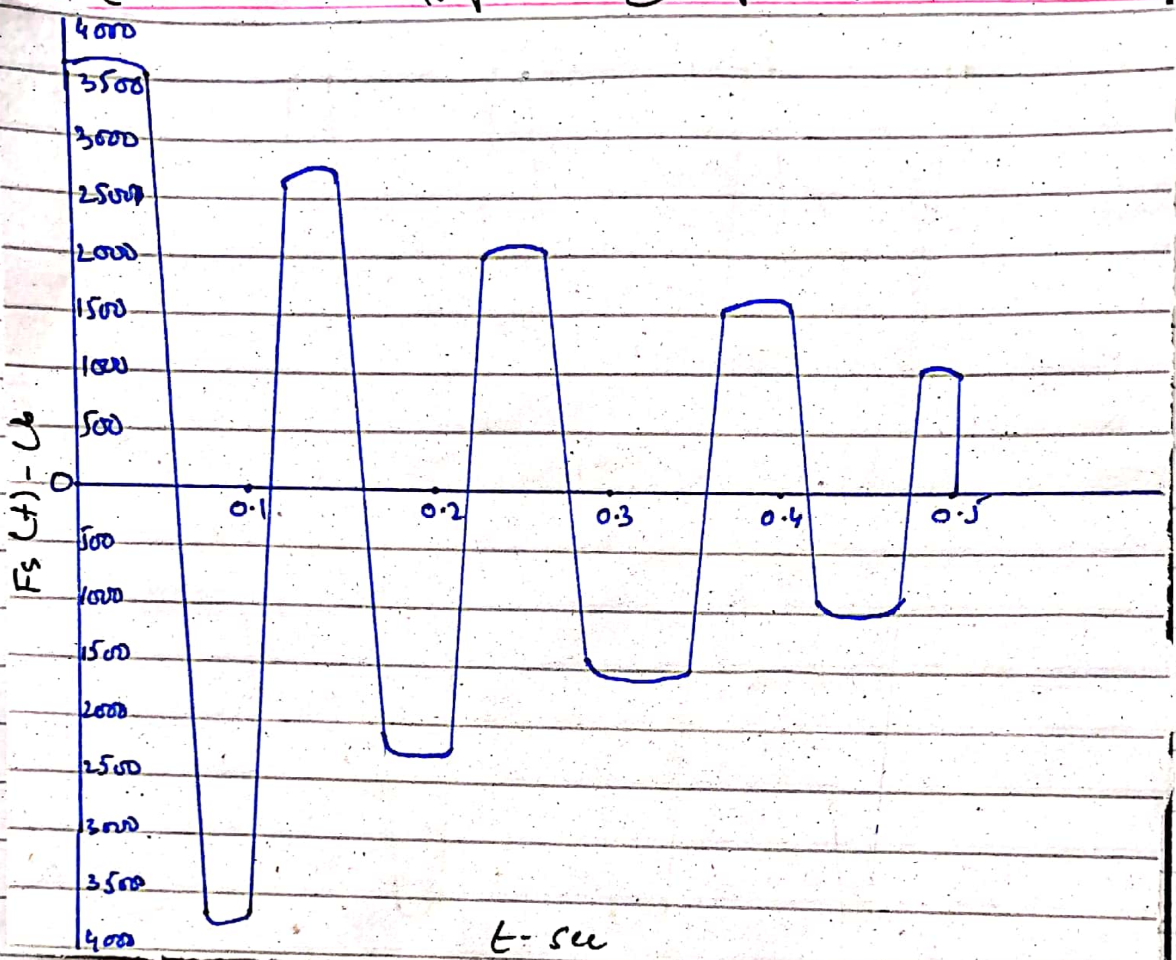


Variation of equivalent static forces with time.

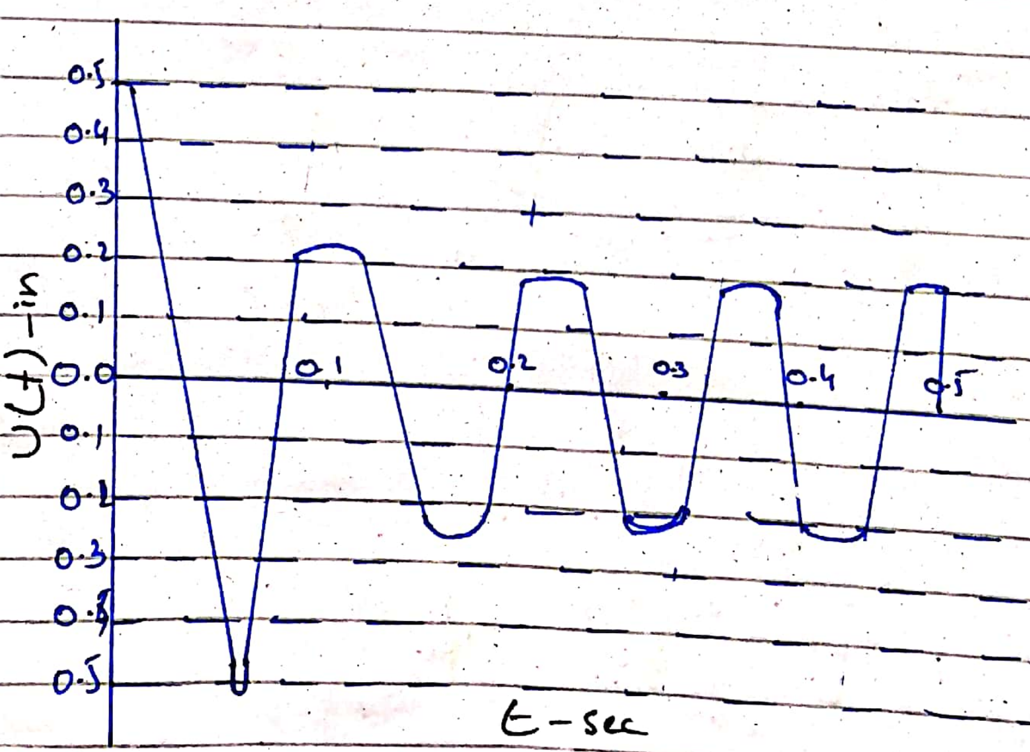


Variation of displacement with time

Question 2 Graphs. Damped Free vibration



Variation of equivalent static forces with time.



Variation of displacement with time