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Subject : Applied Mathematics II

Exam : Summer 2020

Semester : B-tech (8th semester)

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Q(1) Find the solution of ~~the~~ the following?

(a) $\int (x^2 \cdot e^x)\,dx.$

Given:-

$$\int (x^2 \cdot e^x)\,dx.$$

Required:-

Solve by integration.

Solution:-

$$f(x) = x^2.$$

$$g'(x) = e^x.$$

$$f(x) = x^2.$$

$$f'(x) = 2x,$$

$$g'(x) = e^x.$$

$$g(x) = e^x.$$

$$x^2 e^x - \int 2x e^x \, dx.$$ (Page : 2)

$$\int 2x e^x \, dx.$$

$$2 \int x e^x \, dx.$$

$$2 \left( x e^x - e^x \right).$$

$$x^2 e^x - 2(x e^x - e^x).$$

Distribute the -ive sign to the terms inside the parentheses.

$$-2(x e^x - e^x).$$

by using distributive property.

$$-(2x e^x - 2e^x).$$

$$-2x e^x + 2e^x.$$

add constant of integration.

Result:-

$$\boxed{x^2 e^x - 2x e^x + 2e^x + C.}$$

Q(1) 

(b) $(1 + 3t)t^3 \, dt.$

Given:-

$(1+3t)t^3 \, dt.$

Required:-

Solution by integration.

Solution:-

Ist lets rearrange the expression.

$(3t+1)t^3 \, dt.$

$3t(t)^3 + 1(t)^3 \, dt.$

$\int (3t^4 + t^3) \, dt.$

Split up the integration part.

$\int 3t^4 \, dt + \int t^3 \, dt.$

by integration.

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$$\frac{3t^5}{5} + \frac{t^4}{4}$$

add constant of integration :-

Result:-

$$\boxed{\frac{3t^5}{5} + \frac{t^4}{4} + C}$$

Q 1

(c) $\int (e^4 - e^3)\,du$.

Given:-

$$\int (e^4 - e^3)\,du$$

Required:-

Solve by integration.

Solution:-

$$\int (e^4 - e^3)\,dn$$

Split into parts

$$\int e^x \, dx - \int e^{3x} \, dx.$$

by integration

$$e^x - e^3 x.$$

adding constant to integratin.

Result:-

$$\boxed{e^x - e^3 x + C.}$$

Qs # 2 :- Find the taylor series.

Given :-

$$f(x) = e^{-6x}, \quad x = -4.$$

Required :-

Taylor Series.

Solution :-

$f(x) = e^{-6x}$ about $x = -4.$

# Using Taylor Series Formula:.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} \cdots \quad (1)$$

We know that:-

$$f(x) = e^{-6x} \quad , \quad x = -4.$$

Now.

$$f(-4) = e^{-6(-4)} = e^{24}.$$

$$f'(-4) = -6e^{-6(-4)} = -6e^{24}.$$

$$f''(-4) = (-6)^2 e^{-6(-4)} = 36e^{24}.$$

$$f'''(-4) = (-6)^3 e^{-6(-4)} = -216e^{24}.$$

putting values in eq (1).

$$f(x) = e^{24} - 6e^{24}(x+4) + \frac{36e^{24}(x+4)^2}{2!} - \frac{216e^{24}(x+4)^3}{3!} \cdots\cdots$$

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Now

$$f(x) = e^{-6x}.$$

So

$$f(x) = e^{-6x} = \sum_{n=0}^{\infty} (-6)^n \frac{e^{2n}(x+4)^n}{n!}$$

Result:-

$$\boxed{f(x) = \sum_{n=0}^{\infty} (-6)^n \frac{e^{2n}(x+4)^n}{n!}}$$

Q # 3 :- (Page : 8)

(a) $F(y) = x \sin x$.

**Given:-**

$$F(y) = x \sin x.$$

**Required:-**

$$F'(y) = ?$$

**Solution:-**

$$F(y) = x \sin x.$$

using dot product of diff.

$$F'(y) = x \, d/dx (\sin x) + \sin x \, d/dx (x).$$

$$F'(y) = x \cos x + \sin x (1).$$

**Result:-**

$$\boxed{F'(y) = x \cos x + \sin x.}$$

Q# 3:-

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(b) $F(y) = x^2 \cos x.$

Given:-

$$F(y) = x^2 \cos x.$$

Required:-

$$F'(y) = ?$$

Solution:-

$$F(y) = x^2 \cos x.$$

$$F'(y) = x^2 \, d/dx \, (\cos x) + \cos x \, d/dx \, (x)^2.$$

$$F'(y) = x^2 . (-\sin x) + \cos x \, (2x).$$

$$= -x^2 \sin x + \cos x \, (2x).$$

$$F'(y) = 2x \cos x - x^2 \sin x.$$

Result:-

$$\boxed{F'(y) = 2x \cos x - x^2 \sin x.}$$

Q # 3:- (page# 10)

(c) $F(t) = z(2z-2)^2$.

Given:-

$$F(t) = z(2z-2)^2.$$

Required:-

$$F'(t) = ?$$

Solution:-

$$F(t) = z(2z-2)^2$$
$$F(t) = z(4z^2+4-4z).$$

diff w.r.t 'z'.

$$F'(t) = d/dt\,(4z^3 + 4z - 4z^2).$$

$$F'(t) = \frac{d}{dz}(4z^3) + \frac{d}{dz}(4z) - \frac{d}{dz}(4z^2).$$

$$= 4(3z^2) + 4(1) - 4(2z).$$

Result:-

$$\boxed{F'(t) = 12z^2 + 4 - 8z.}$$

END