

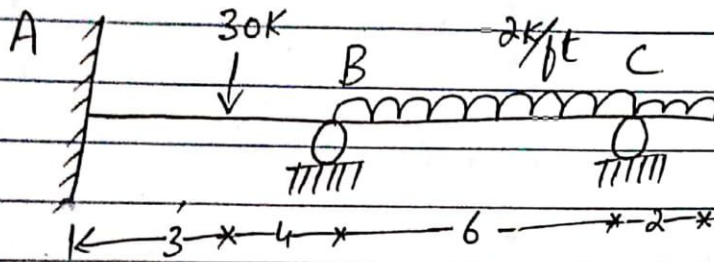
NAME=INAM UL HASSAN

ID=7693

STRUCTURE ANALYSIS 2 FINAL TERM SUMMER EXAME

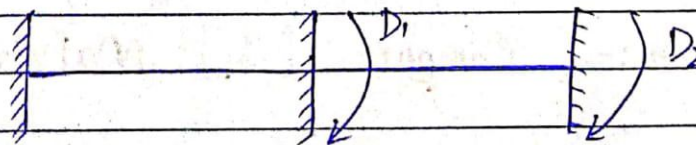
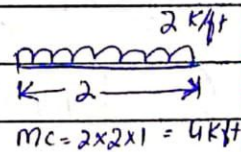
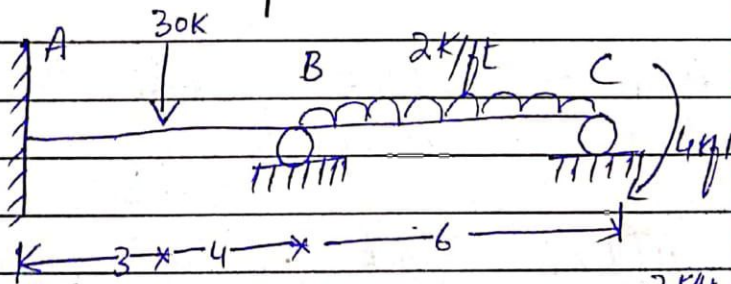
Q1 Problem No 1

Analyze the beam shown in fig 1 by stiffness method. Assume EI is constant.



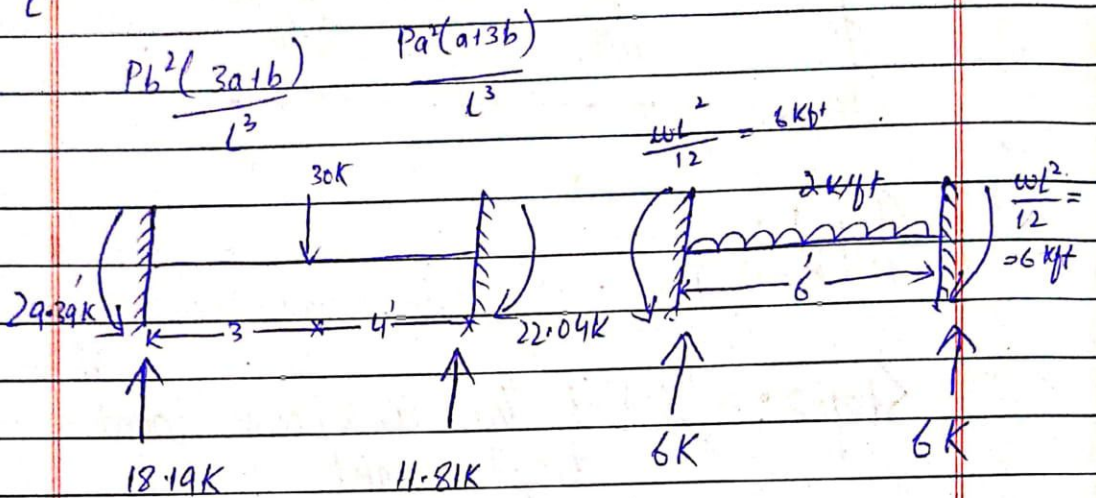
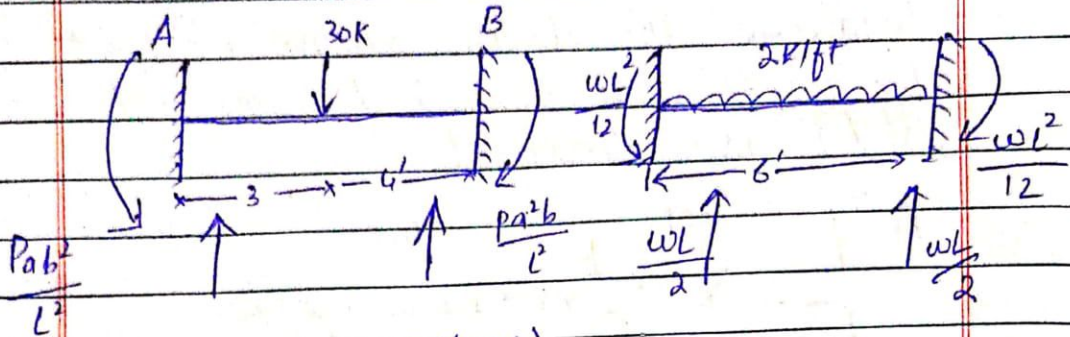
Step 1:
 $K.T = 2^{\circ}$ (Neglecting Axial Effects)

Step 2:- Select The unknown joint displacement



$$[D] = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad [AD] = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step 3: Compute [ADL] Matrix



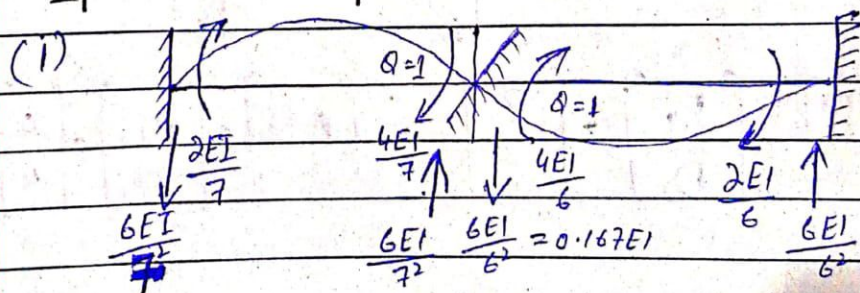
$$ADL_1 = 22.04k - 6k$$

$$ADL_1 = 16.04k$$

$$ADL_2 = 6k$$

$$\text{So } [ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} 16.04 \\ 6 \end{bmatrix}$$

Step 4:- Compute [S] Matrix

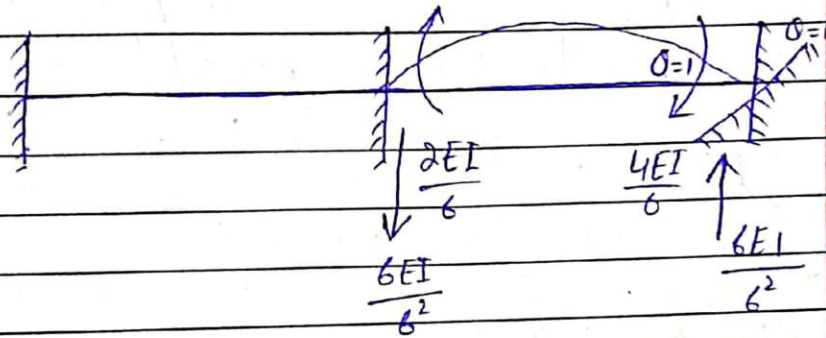


When $D_1 = 1$, $D_2 = 0$

$$S_{11} = \frac{4EI}{7} + \frac{4EI}{6} = 1.238 EI$$

$$S_{21} = \frac{2EI}{6} = 0.333 EI$$

(ii) When $D_1 = 0$, $D_2 = 1$



$$S_{12} = \frac{2EI}{6} = 0.333 EI$$

$$S_{22} = \frac{4EI}{6} = 0.667 EI$$

Stiffness matrix $[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

$$S = \begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI$$

Step 5:- Compute the values of D_1 and D_2

$$[AD] = [ADL] + [S][D]$$

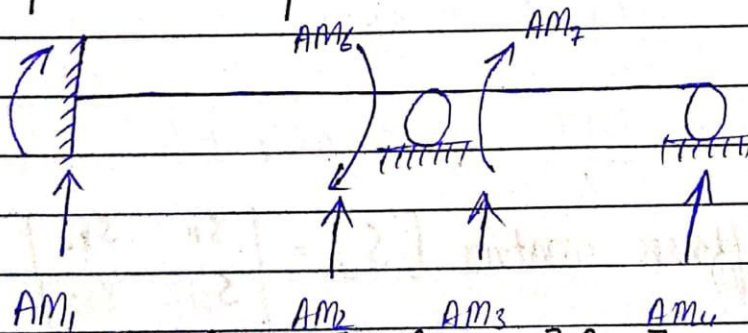
$$[D] = [S]^{-1} [AD - ADL]$$

$$[D] = \left(\begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI \right)^{-1} \begin{bmatrix} 0 - (16.04) \\ 4 - (6) \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 0.933 & -0.466 \\ -0.466 & 1.732 \end{bmatrix} \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$[D] = \frac{1}{EI} \begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

Step 6:- Compute member end Actions



$$[AM] = [AML] + [AMD][D]$$

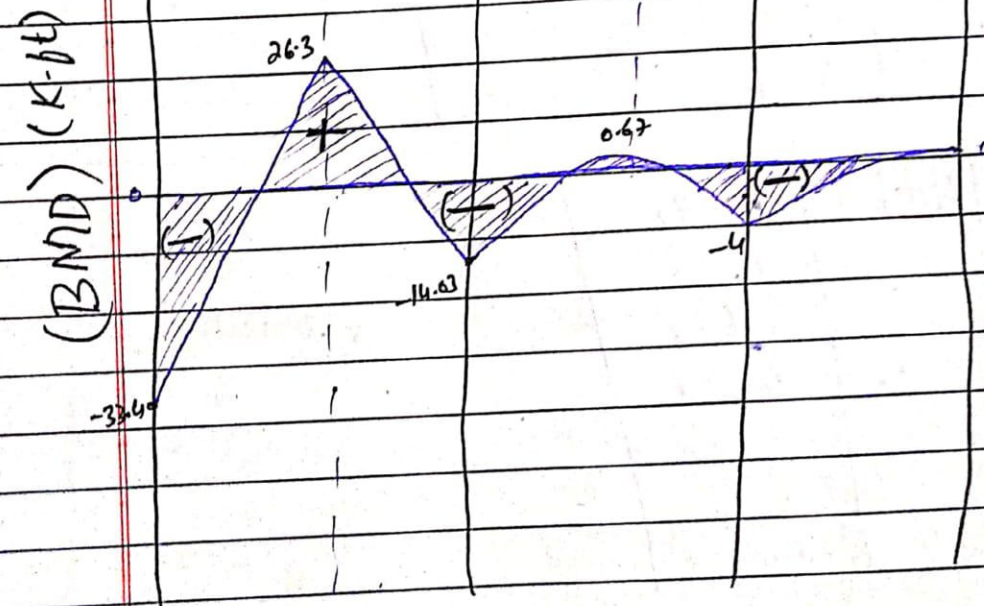
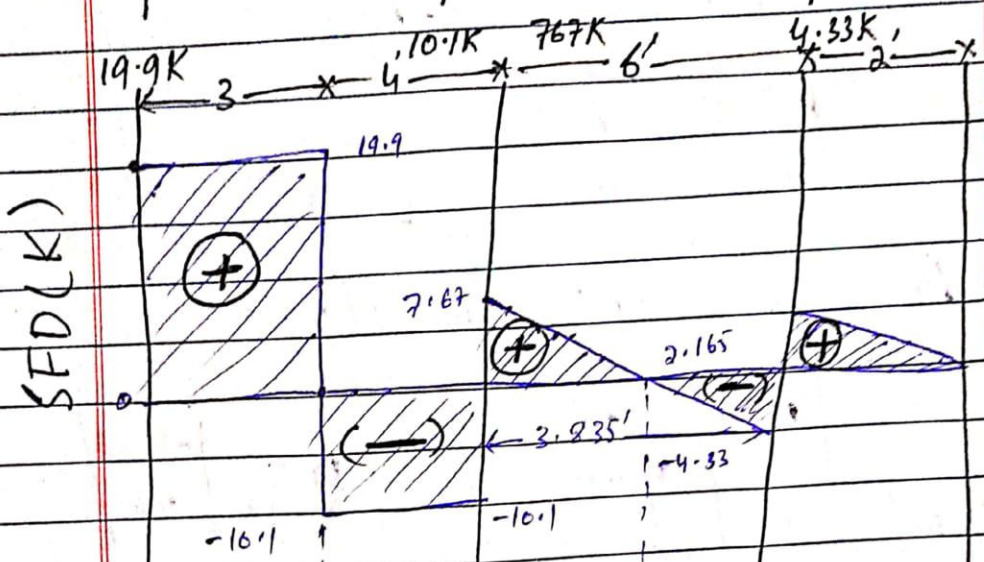
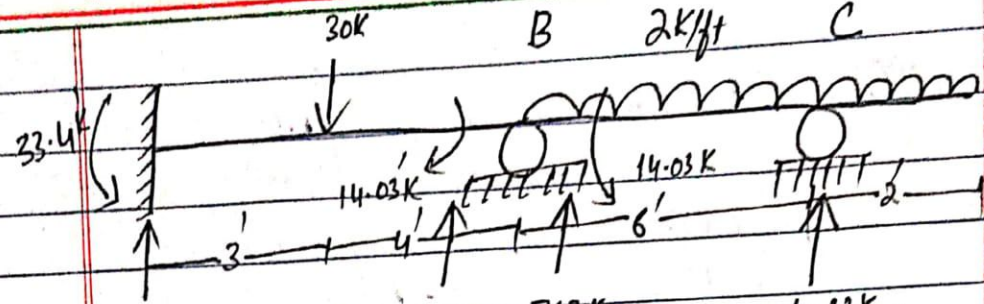
$$[AML]; (AML) = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 18.19 \text{ K} \\ 11.81 \text{ K} \\ 6 \text{ K} \\ 6 \text{ K} \\ -29.39 \text{ K} \\ 22.04 \text{ K} \\ -6 \text{ K} \end{bmatrix}$$

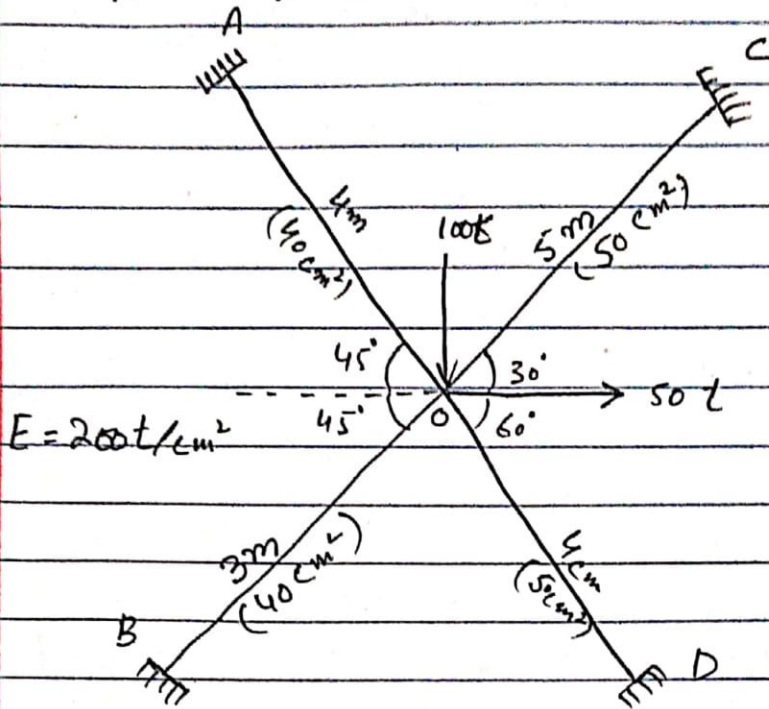
$$[AMD] = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \\ AMD_{71} & AMD_{72} \end{bmatrix} = EI \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & -0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.333 \end{bmatrix}$$

Now; $[AM] = [AML] + [AMD][D]$

$$[AM] = \begin{bmatrix} 18.19 \\ 11.81 \\ 6 \\ 6 \\ -24.39 \\ 22.04 \\ -6 \end{bmatrix} + \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & 0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.333 \end{bmatrix} \begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

$$[AM] = \begin{bmatrix} 19.9K \\ 10.1K \\ 7.67K \\ 4.33K \\ -33.4K \\ 14.03K \\ -14.03K \end{bmatrix}$$



Problem No 2Sol

For A :-

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow P = 2.828 \text{ m}$$

$$\cos 45^\circ = \frac{b}{4}$$

$$\Rightarrow b = 2.828 \text{ m}$$

For B :-

$$\sin 45^\circ = \frac{P}{h}$$

$$\Rightarrow P = 2.12 \text{ m}$$

$$\cos 45^\circ = \frac{b}{h}$$

$$b = 2.12 \text{ m}$$

For C :-

$$\frac{x}{h} = \sin 30^\circ = \frac{P}{h=5}$$

$$\Rightarrow P = 2.5 \text{ m}$$

$$\cos 30^\circ = \frac{b}{5}$$

$$\Rightarrow b = 4.33 \text{ m}$$

Now

$$EA(A) = 2000 \times 40 = 80,000 \text{ t}$$

$$EA(B) = 2000 \times 40 = 80,000 \text{ t}$$

$$EA(C) = 2000 \times 50 = 100,000 \text{ t}$$

$$EA(D) = 2000 \times 50 = 100,000 \text{ t}$$

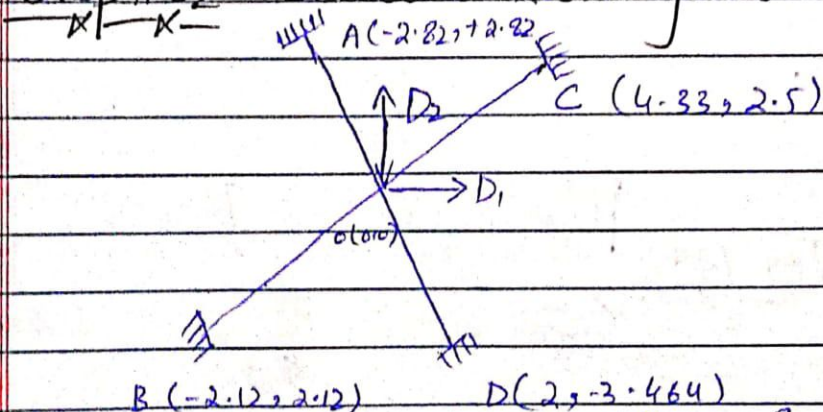
Step # 01 :- K.I

$$\frac{x}{h} = \frac{x}{h}$$

$$K.I = 2j - r$$

$$= 2(5) - 8 = 2^\circ$$

Step # 02 Select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step # 03 [AMD] 4×2 and [S] 2×2

$$(i) D_1 = 1, D_2 = 0$$

$$AMD = \frac{EA}{L^2} (x_k - x_j)$$

$$AMD_{11} = \frac{800,000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{800,000}{(300)^2} \times (0 + 212) = 188.44$$

$$AMD_{31} = \frac{1,000,000}{(500)^2} \times (0 - 433) = -173.2$$

$$AMD_{41} = \frac{1,000,000}{(400)^2} \times (0 - 200) = -125$$

$$\text{Now } S_{11} = \sum_{121}^m \frac{EA}{L^3} (x_k - x_j)^2$$

$$= \frac{800,000}{(400)^3} (282)^2 + \frac{800,000}{(300)^3} \times (212)^2 + \frac{1,000,000}{(500)^3} \times (-433)^2$$

$$+ \frac{1,000,000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$S_{11} = 445.063$$

$$S_{12} = S_{21} = \sum_{i,j} \frac{EA}{L^3} x (X_k - X_j) (Y_k - Y_j)$$

$$= \frac{80,000}{(400)^3} x (282)(-282) + \frac{80,000}{(300)^3} x (212)(212)$$

$$+ \frac{100,000}{(500)^3} x (-433)(0-250) + \frac{100,000}{(400)^3} x (-200)(0+346)$$

$$S_{12} = S_{21} = 12,237$$

(ii) $D_1 = 0$, $D_1 = 1K$

$$AMD = \frac{EA}{L^2} (Y_k - Y_j)$$

$$AMD_{12} = \frac{80,000}{(400)^2} (-282) = -141$$

$$AMD_{22} = \frac{80,000}{(300)^2} (212) = 188.44$$

$$AMD_{32} = \frac{100,000}{(500)^2} (-250) = -100$$

$$AMD_{42} = \frac{100,000}{(400)^2} (346) = 216.25$$

$$\text{Now } S_{22} = \sum_{i,j} \frac{EA}{L^3} (Y_k - Y_j)^2$$

$$= \frac{80,000}{(400)^3} (-282)^2 + \frac{80,000}{(300)^3} (212)^2 + \frac{100,000}{500^3} (-250)^2$$

$$+ \frac{100,000}{(400)^3} (346)^2 = S_{22} = 469.62$$

Checked By: Parents: Excellent Good **BABAR** PAPER PRODUCTS

Step # 04

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

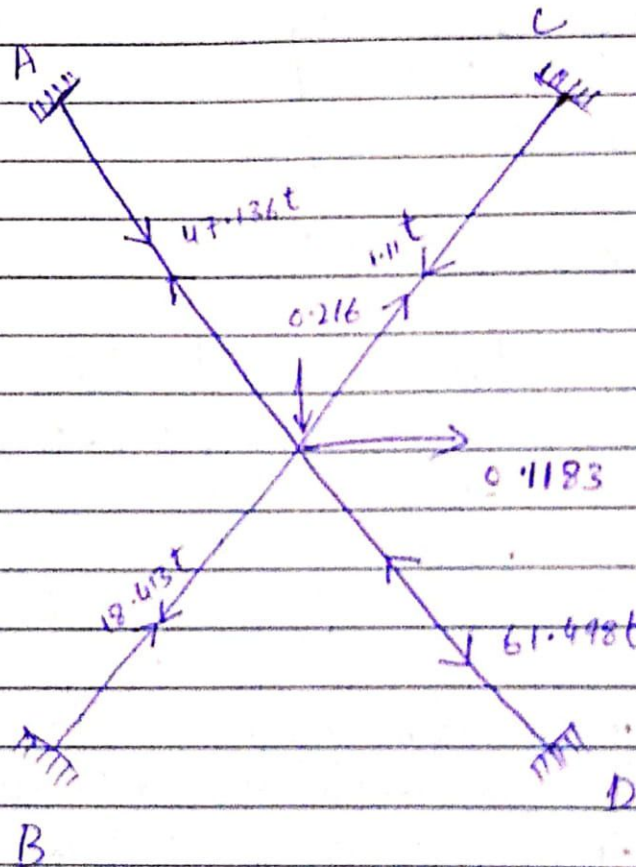
Step # 06 :- [AM]

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

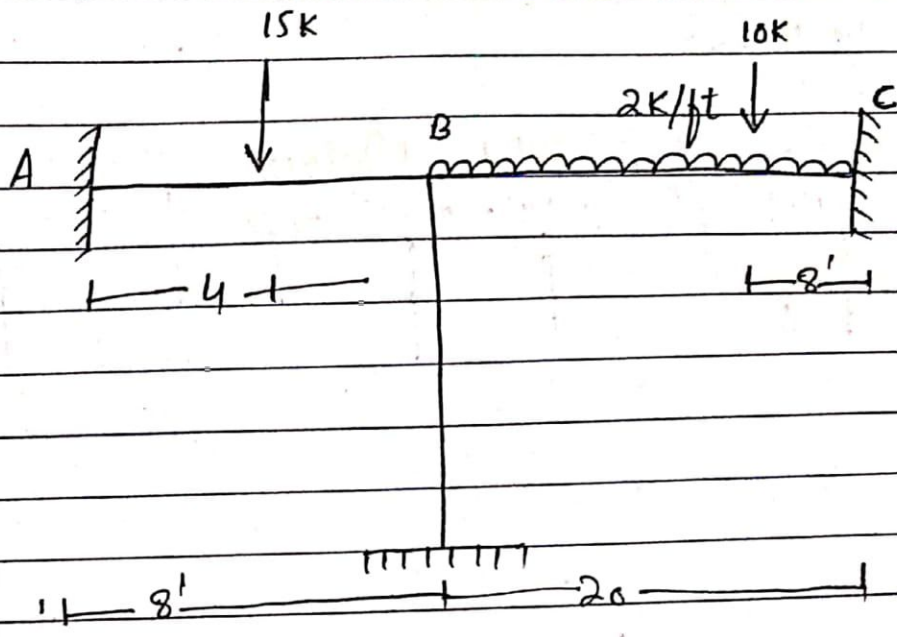
$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (-0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



Q No 3



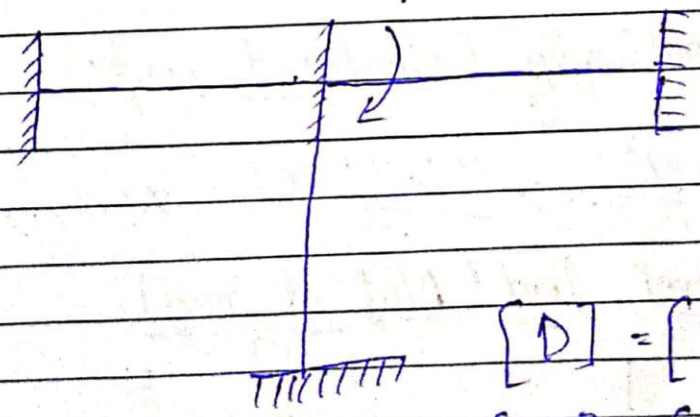
Sol

Step # 1
~~X~~ ~~X~~ ~~X~~

Determine Kinematic Indeterminacy

$$K.I = 1^\circ$$

Step 2:- Determine unknown joint Displacement:

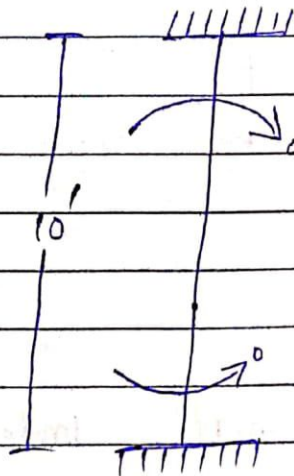
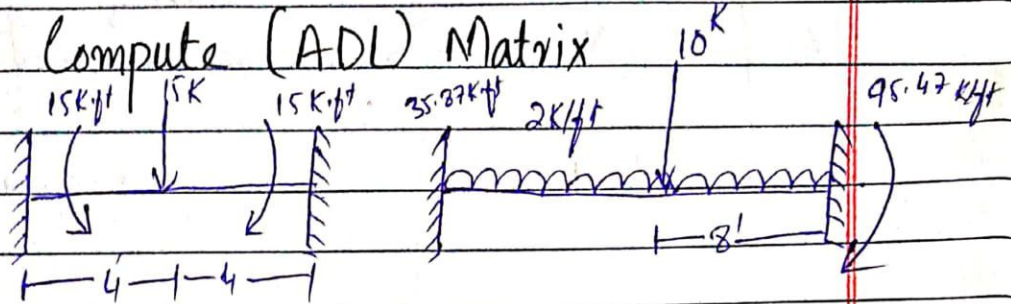


$$[D] = [?]$$

$$[AD] = [0]$$

Step # 3

Compute (ADL) Matrix



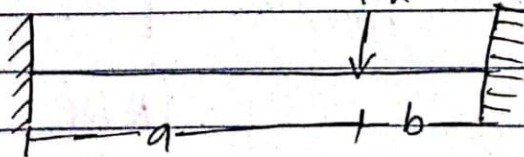
⇒ Point load at Center:

$$\frac{PL}{8} = \frac{(15)(8)}{8} = 15 \text{ Kip.ft}$$

⇒ Uniformly Distributed load:-

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ K.ft}$$

⇒ point load (Not at mid)



For Left End:-

$$\frac{Pab^2}{L^2} = \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ K.ft}$$

For Right End:-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ K.ft}$$

So total moment at left end:

$$19.2 + 66.67 = 85.87 \text{ K.ft}$$

Similarly at right end:-

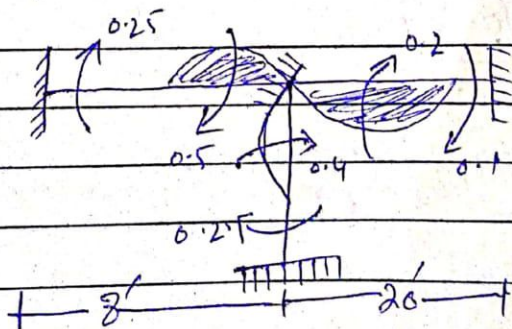
$$28.8 + 66.67 = 95.47 \text{ K.ft}$$

$$\text{So } [ADU] = -85.87 + 15 = -70.87 \text{ K.ft}$$

Step # 4
Determine $[S]$ Matrix

$$S = [S_{11}]$$

$$\text{Now } D = 1 \text{ K}$$



$$\Rightarrow \frac{4EI}{8} = 0.5 \quad \frac{2EI}{8} = 0.25$$

$$\Rightarrow \frac{4EI}{20} = 0.2 \quad \frac{2EI}{20} = 0.1$$

$$\Rightarrow \frac{4EI}{10} = 0.4 \quad \frac{2EI}{10} = 0.2$$

$$[S] = (0.5 + 0.4 + 0.2) EI$$

$$= 1.1 EI$$

$$[S] = 1.1 EI$$

Step # 5

Compute $[D]$ Matrix

$$[D] = [S]^{-1} \times [AD] - [AD]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87]$$

$$= \frac{70.87}{1.1}$$

$$[D] = [64.42] \frac{1}{EI}$$