Digital Signal Proccessing
Sessional Assingment

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Q1

(a)

Determine The response $y(n) \geq 0$ of The System described by secound order difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

to The Input $x(n) = 4^n u(n)$

Soll:

Consider The difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1) \quad (1)$$

the Homogenus equation of The system is

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

The charactrist equation of the system is

$$\lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

Determin The root of The Charactrastic equation

$$\cancel{\lambda^2 - 3\lambda} \quad \lambda^2 - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$$

p.t.o

$(\lambda-4)(\lambda+1)=0$

$\lambda=-1, 4$

The homogenous Sallution is

$y_h(n) = c_1(-1)^n u(n) + c_2(4)^n u(n)$

Since
4 is a charactrist
root and exitation is

$x(n) = 4^n u(n)$

We assume a particular Sallution
of The Form

$y_p(n) = Kn4^n u(n)$

Then $Kn4^n u(n) - 3K(n-1)4^{n-1}u(n-1) - 4K(n-2)$
$4^{n-2} u(n-2)$

$= 4^n u(n) + 2(4)^{n-1} u(n-1)$

For $n=2$

$K(32-1) = 4^2 + 8 = 24 \Rightarrow K = 6/5$

the Sollution is

$y(n) = y_p(n) + y_h(n)$

$= 6/5\, n4^n + c_1 4^n + c_2(-1)^n]\, u(n)$

p.t.o

(3)

To solve for c_1 and c_2 we have assume that

$y(-1) = y(-2) = 0$ Then

$y(0) = 1$ and

$y(1) = 3y(0) + 4 + 2 = 9$

Hence

$c_1 + c_2$ and

$\frac{24}{5} + 4c_1 - c_2 = 9$

$4c_1 - c_2 = \frac{21}{5}$

Therefore

$c_1 = \frac{26}{25}$ and $c_2 = \frac{-1}{25}$

The total Sallution is

$y(n) = \left[\frac{6}{5} n4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u(n)$

Q 1:

(b)

Determine The Impuls response and unit step response of the system describe by The difference equation

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

Soll:

Consider difference equation

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$
$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n)$$

To obtain The homogenous equation set Input.

$$x(n) = 0$$

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$$

Determine the sollution to homogenous equation

$$y_h(n) = \lambda^n$$

Substitude The sollution obtaind in the homogenous equation

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$
$$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$$

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$= (\lambda - 0.2)(\lambda - 0.4) = 0$

There fore The roots are

$\lambda_1 = 0.2, \quad \lambda_2 = 0.4$

Thus The general form of The Sollution to the homogenus is

$y_h(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$

$y(n) = C_1(0.2)^n + C_2(0.4)^n \quad \ldots (1)$

$\lambda = 0.2 \quad \lambda = 0.4$

Hence

$$y_h(n) = C_1 \frac{1^n}{5} + C_2 \frac{2^n}{5}$$

With $x(n) = \delta(n)$ The initial condition are

$y(0) = 1$

$y(1) - 0.6y(0) = 0$

$y(1) = 0.6 \quad = C_1 + C_2 = 1$ and

$= \frac{1}{5}C_1 + \frac{2}{5} = 0.6$

$\Rightarrow C_1 = -1, \quad C_2 = 3$

P.T.O

there fore $h(n) = \left[-\left(\frac{1}{3}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$

The Step response is

$$s(n) = \sum_{k=0}^{n} h(n-k), \; n > 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= \left\{\frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[\frac{1}{5}^{n+1} - 1\right]\right\} u(n)$$

$$\left\{\frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[\frac{1}{5}^{n+1} - 1\right]\right\} u(n)$$

Q2 (a)

Determine The Causal Signal $x(n)$ having Z Transform:

~~Hint~~ $$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Hint: take inverse Z-transform using Partial fraction method.

Soll

the Z- transform is

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

the expression is written as

$$Y(z) = \frac{1}{(1-2/z)(1-\frac{1}{z})^2}$$

$$= \frac{1}{\frac{(z-2)(z-1)^2}{z^3}}$$

$$= \frac{z^3}{(z-2)(z-1)^2} \longrightarrow (1)$$

P.T.O

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$X(z)$ has a simple pole at $P_1 = 2$
and a double $P_2 = P_3 = 1$
In such a case the appropiate
partial fraction expansion is

$$X(z) = \frac{z^3}{(z-2)(z-1)^2} = \frac{A}{z-2} + \frac{A_2}{z-1} + \frac{A_3}{(z-1)^2}$$

the problem is to determine the
coefficient A_1, A_2, A_3

we proced as in the case of
distinict pole to determin A_1
we multiply both side of
by $(z-2)$ and evalute the result

$z = -2$

$$(z-2)X(z) = A_1 + \frac{z-2}{z-1} A_2 + \frac{z-2}{(z-1)^2} A_3$$

which we evaluted at $z=2$

$$A_1 = \left.\frac{(z-2)\, x(z)}{2}\right|_{z=2}$$

$A_1 = 4$

$$A_2 = A_1 + \frac{z-2}{z-1}$$

$A_2 = -3$

$$A_3 = A_1 + \frac{z-2}{z-1} A_2 \Rightarrow A_3 = -1$$

p.to

Hence $x(n) = [4(2)^n - 3 - n] u(n)$

B Part "b"

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

First we eleminate the negative power by multiplying both nominator and denomenator by z^2 then

$$x(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

the pole $x(z)$ are $P_1 = 1$ and $P_2 = 0.5$ consequently the expension of the form

$$\frac{x(z)}{z} = \frac{z}{(z-1)(z-0.5)}$$

$$= \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

multiply the equation by $\frac{x(z)}{z}$ the denomentor term $(z-1)(z-0.5)$

p.t.o

$z = (z - 0.5)A_1 + (z-1)A_2$ — ①

$z = p = 1$ put in eq ①

$1 = (1-0.5)A_1 + (1-1)$

$\Rightarrow 1 = (1-0.5)A_1 + 0$

$\Rightarrow 1 = (0.5)A_1$

$\Rightarrow A_1 = \frac{1}{0.5}$

$\Rightarrow A_1 = 2$

$\Rightarrow$ put $P_2 = z = 0.5$ in eq ①

$0.5 = (0.5 - 0.5)A_1 + A_2(0.5 - 1)$

$0.5 = 0 + A_2(-0.5)$

$0.5 = A_2(-0.5)$

$A_2 = -\frac{\cancel{0.5}}{\cancel{0.5}}$

$A_2 = -1$

$\frac{x(z)}{z} = \frac{2}{z-1} - \frac{1}{z - 0.5}$ Ans.

Q3: (a)

Sol:

~~we have~~

At $w = 0$ we have

$$H(0) = \frac{b^o}{(1-P)^2} = 1$$

hence

$$b_0 = (1-P^2)$$

At $w = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{1-P\cos(\pi/4)+jP\sin(\pi/4)^2}$$

$$= \frac{(1-P)^2}{1-P/\sqrt{2}+jP(\sqrt{2})^2}$$

hene

$$\frac{(1-P)^4}{[1-P(\sqrt{2})^2+P^2/2]}$$

$\sqrt{2}\,(1-p^{2})^{2} = 1+p^{2}-\sqrt{2}p$

the value of $p = 0.32$
Satisfied equation Consequently
the System function for the
desired filter is

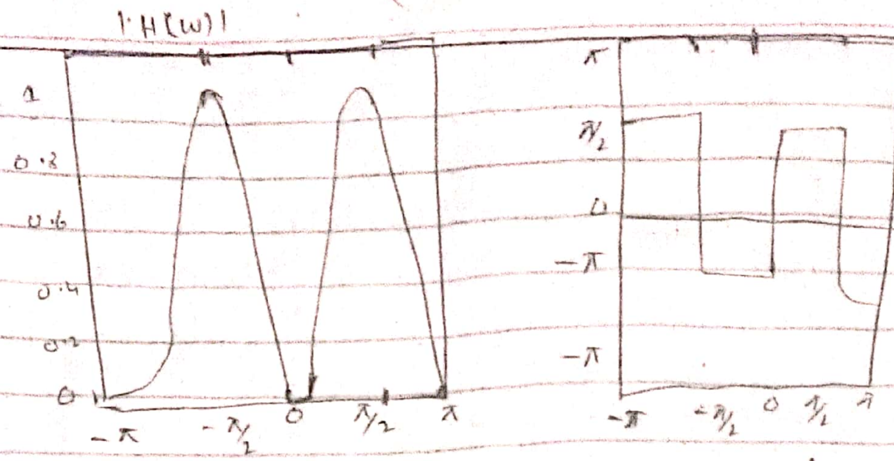
$$H(z) = \frac{0.46}{(1-0.32z^{-1})^{2}}$$

Q3. (b)

Soll:
Clearly filter must have
Pole at the $P_{1,2} = re^{\pm j\pi/2}$
and zeroes at $z=1$ and $z=-1$
Consequently the system function
is $H(z) = \frac{b(z-1)(z+1)}{(z-jr)(z+jr)}$

$= b_0 \frac{z^{2}-1}{z^{2}+r^{2}}$

P.T.O



the gain filter is determind by evaluting the frequency response $H(w)$ of the filter at

$w = \pi/2$ Thus we have

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of 'r' is determind by evaluating $H(w)$ at $w = 4\pi/9$

Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{\frac{(1-r^2)^2}{4}\,2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1.94(1-r^2)^2 = 1-1.88r^2+4$$

P.T.O

the value of $b^2 = 0.7$

Satisfied This equation
There fore The System function
for the desired filter

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

$$\boxed{H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}}$$

Q4:
(b)
Soll:

the first step is to determine the matrin W_4 ~~by~~ by expoliting the periodicity property of W_4 and the symmetry property

$$W_N^{k+N/2} = -W^{kn}$$

the Matrix W_4 may expressed

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^4 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

then

~~[illegible]~~

$$W_4 X_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

P.T.O.

(16)

The DFT of Y_4 may be determind by conjugate the element in W_4 to obtain and then applying the farmula;

Q4:

Part (a)

$$x(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Determind The N-point DFT of this sequence for $n \geq L$

Soll:

$$X(w) = \sum_{n=0}^{L-1} x(n) e^{-Jwn}$$

$$= \sum_{n=0}^{L-1} e^{-Jwn} = \frac{1 - e^{-JwL}}{1 - e^{-Jw}}$$

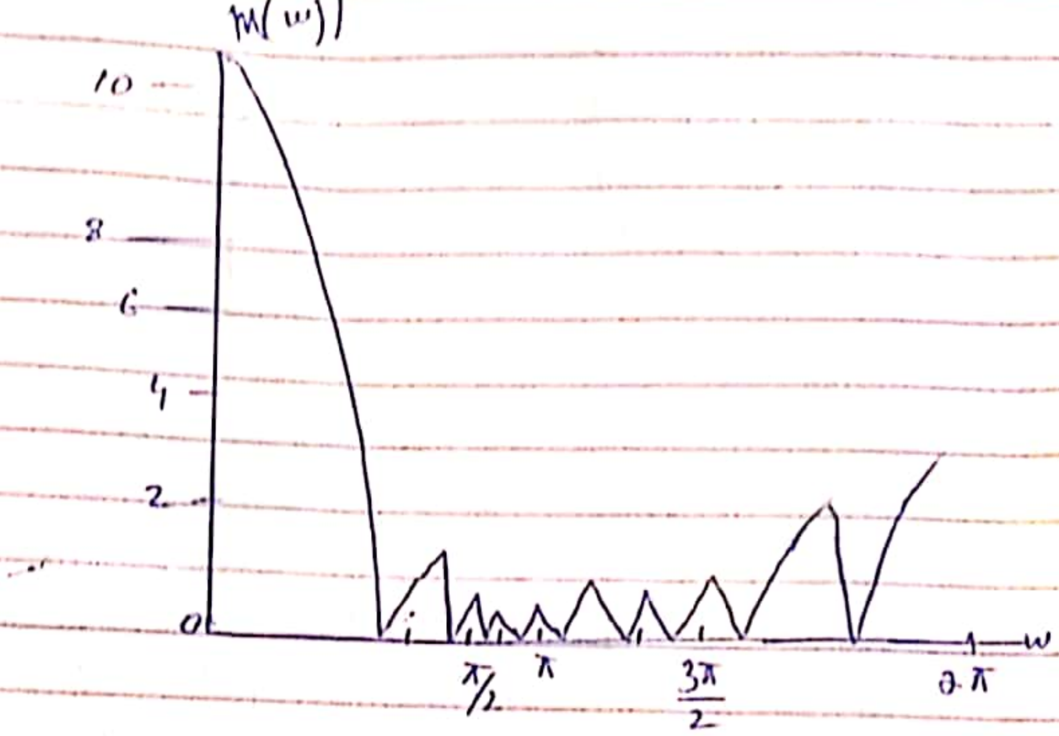
$$= \frac{\sin(wL/2)}{\sin(w/2)} e^{-Jw(L-1)/2}$$

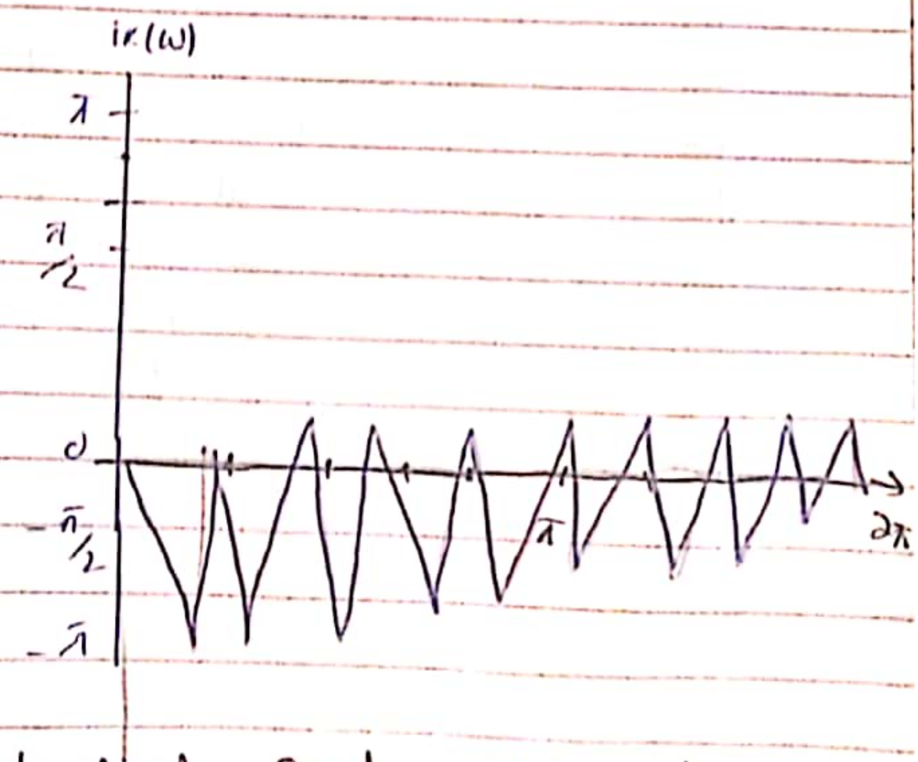
The Magnitude and phase of $x(w)$ are Illustrated for $L = 10$. The N Point DFT of $x(n)$ is simply $x(w)$ evaluted at the set of N equality spaced frequencies

$$W_L = \frac{2\pi k}{N}, \quad k = 0, 1, \ldots, N-1$$

$$x(k) = \frac{1 - e^{-2j\pi kL/N}}{1 - e^{-2j\pi k/N}}$$

$$\frac{\sin(\pi k L/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$





If N is selected such that $N = L$

Then DFT became

$$x(k) = \begin{cases} L & k = 0 \\ 0 & k = 1, 2, \cdots L-1 \end{cases}$$

Thus There is only one non zero value in the DFT This is apparent from observation of $X(\omega)$ sin $X(\omega) = 0$ at frequencies $\omega_k = 2\pi k/L$ $k \neq 0$ The reader should verify that $x(n)$ can be recovered from $X(k)$ by performing and L - point IDFT.

Although The L - point DFT is sufficient to uniquely represent the sequence $x(n)$ in the frequency domain it is apparent that it doesnot provide sufficient detail to yield a good picture. We must evaluate (Interpolate) $X(\omega)$ at more closely spaced frequency say $\omega_k = 2\pi k/N$ were $N > L$, in effect we can view this computation as expending the size of the sequence from 'L' points to 'N' by appending N-L zeroes to the sequence $x(n)$ that is zero size padding then the N - point DFT provides finer interpolation than the L - point

$L = 10$, $N = 50$, and $N = 100$

Now spectral characteristic sequence are more clearly as one will conclude by comparing these spectra with the continuse spectrum $X(\omega)$.