Muhammad Asif 16950

Q 12:-

$$x^2y'' - 4xy' + 6y = 0$$

$y(1) = 0.4$

$y'(1) = 0$

Sol: $x^2y'' - 4xy' + 6y = 0 \longrightarrow (1)$

$y(1) = 0.4$

$y'(1) = 0$

eq (1) is a Cauchy-Euler's equation:

so putting $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

eq (1) =>

$x^2 \cdot m(m-1)x^{m-2} - 4x\,m x^{m-1} + 6x^m = 0$

$m(m-1)x^m - 4m x^m + 6x^m = 0$

$[m(m-1) - 4m + 6]x^m = 0$

The auxiliary equation is

2

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$m(m-1) - 4m + 6 = 0$

$m^2 - m - 4m + 6 = 0$

$m^2 - 5m + 6 = 0$

$m^2 - 3m - 2m + 6 = 0$

$m(m-3) - 2(m-3) = 0$

$(m-2)(m-3) = 0$

$m - 2 = 0, \quad m - 3 = 0$

$m = 2, \quad m = 3.$

Since roots are real and distinct so it Solution is.

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$y(x) = c_1 x^{m_1} + c_2 x^{m_2}$

$y(x) = c_1 x^2 + c_2 x^3. \longrightarrow (2)$

$y(1) = 0.4 = c_1(1) + c_2(1)$

$0.4 = c_1 + c_2 \longrightarrow (3)$

$y'(x) = 2 c_1 x + 3 c_2 x^2.$

$y'(1) = 0 = 2 c_1 + 3 c_2.$

$\cancel{2 c_1} + 3 c_2 = 0 \longrightarrow (4)$

solving (3) and (4)

$$
\begin{aligned}
2\cancel{c_1} + 2 c_2 &= 0.8 \\
\pm 2\cancel{c_1} \pm 3 c_2 &= 0 \\
\hline
- c_2 &= 0.8
\end{aligned}
$$

$\boxed{c_2 = -0.8}$

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eq ③ ⇒ $c_1 - 0.8 = 0.4$

$c_1 = 0.4 + 0.8$

$c_1 = 1.2$.

eq ② ⇒

$y(x) = 1.2\,x^2 + (-0.8)\,x^3$

$= 1.2\,x^2 - 0.8\,x^3$.

13) $x^2y'' + 3xy' + 0.75y = 0$,

$y(1) = 1$

$y'(1) = -1.5$

Sol:- $x^2y'' + 3xy' + 0.75y = 0 \longrightarrow (1)$

$y(1) = 1$

$y'(1) = -1.5$

Eq (1) is an Euler-Cauchy equation so putting $y = x^m$

$y' = m\,x^{m-1}$

$y'' = m(m-1)\,x^{m-2}$.

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$x^2 m(m-1) x^{m-2} + 3x\, m\, x^{m-1} +$
$0.75 x^m = 0$

$m(m-1) x^m + 3m x^m +$
$0.75 x^m = 0$

$[m(m-1) + 3m + 0.75] x^m = 0$
$m(m-1) + 3m + 0.75 = 0$
$m^2 - m + 3m + 0.75 = 0$
$m^2 + 2m + 0.75 = 0$

here $a = 1$, $b = 2$, $c = 0.75$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4(1)(0.75)}}{2}$$

$$= \frac{-2 \pm \sqrt{4-3}}{2}$$

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$-\frac{3}{2}-1$

$-5/_2$

$= \frac{-2 \pm 1}{2}$

$= -\frac{2}{2} \pm \frac{1}{2}$

$m = -1 \pm \frac{1}{2}$

$m_1 = -1 + \frac{1}{2}, \quad m_2 = -1 - \frac{1}{2}$

$m_1 = \frac{1}{2}, \quad m_2 = -\frac{3}{2}$

The solation is.

$y(x) = c_1 x^{m_1} + c_2 x^{m_2}$

$= c_1 x^{1/2} + c_2 x^{-3/2} \longrightarrow A$

$y(1) = 1$

$c_1 + c_2 = 1 \longrightarrow 2$

$y'(x) = \frac{1}{2} c_1 x^{-1/2} - \frac{3}{2} c_2 x^{-5/2}$

$y'(1) = -1.5 = \frac{1}{2} c_1 - \frac{3}{2} c_2$

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$\frac{1}{2} C_1 - \frac{3}{2} C_2 = -1.5$

(x) ing by (2)

$2 \times \frac{1}{2} C_1 - \frac{2 \times 3}{2} C_2 = \frac{-3}{\cancel{2}} \times \cancel{2}$

$C_1 - 3 C_2 = -3 \longrightarrow (3)$

Solving (2) & (3)

$\cancel{C_1} + C_2 = 1$

$\underline{\pm \cancel{C_1} \mp 3 C_2 = \mp 3}$

$4 C_2 = 4$

$\Rightarrow C_2 = 1.$

eq (3) $\Rightarrow$

$C_1 - 3(1) = -3.$

$C_1 - 3 = -3.$

$C_1 = -3 + 3$

$C_1 = 0$

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So eq (1) $\Rightarrow$

$$y(x) = 0 + x^{-3/2}$$

$$\boxed{y(x) = x^{-3/2}}$$

14:- $x^2y'' + xy' + 9y = 0, \longrightarrow (1)$

$y(1) = 0$

$y'(1) = 2.5$

putting $y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1)x^{m-2}$

eq (1) $\Rightarrow$ $x^2 m(m-1)x^{m-2} + x\, m x^{m-1} + 9x^m = 0$

$(m^2 - m)x^m + m x^m + 9x^m = 0$

$[m^2 \cancel{- m} \cancel{+ m} + 9]x^m = 0$

$m^2 + 9 = 0$

$m^2 = -9$

$\Rightarrow m = \pm i3$

$\Rightarrow m = 0 \pm i3$, $\alpha = 0, \beta = 3$

Since the roots are complex So its

Solution is.

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$$y(x) = x^0 \left\{ C_1 \cos 3\ln x + C_2 \sin 3\ln x \right\}$$

$$y(x) = C_1 \cos 3\ln x + C_2 \sin 3\ln x. \rightarrow (2)$$

$$y'(x) = -3C_1 \cdot \frac{1}{x} \sin 3\ln x +$$

$$3C_2 \frac{1}{x} \cos 3\ln x$$

$$= -\frac{3}{x} \left\{ C_1 \sin 3\ln x - C_2 \cos 3\ln x \right\} \rightarrow (3)$$

$$y(1) = 0$$

$$C_1 \cos 3\ln 1 + C_2 \sin 3\ln 1 = 0$$

$$C_1 \cos 3(0) + C_2 \sin 3(0) = 0$$

$$C_1 \cos 0 + C_2 \sin 0 = 0$$

$$\boxed{C_1 = 0}$$

$$y'(1) = 2.5$$

$$-3\left(C_1 \sin 0 - C_2 \cos 0\right) = 2.5$$

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$$3C_2 = 2.5$$

$$C_2 = \frac{1}{3} \times \frac{25}{10}$$

$$= \frac{\cancel{25}\,5}{\cancel{30}\,6}$$

$$C_2 = 5/6 = 0.83$$

eq (2) $\Rightarrow$ $y(x) = 0 + 0.83 \sin 3 \ln x$

$\Rightarrow y(x) = (0.83) \sin 3 \ln x$.

15) $x^2 y'' + 3x y' + y = 0 \longrightarrow (1)$

$y(1) = 3.6$

$y'(1) = 0.4$

Sol:- Putting $y = x^m$

$\Rightarrow y' = m x^{m-1}$

$\Rightarrow y'' = m(m-1) x^{m-2}$

in eq (1) we get:

$m(m-1) x^2 x^{m-2} + 3x\, m x^{m-1} + x^m = 0$

$\{m^2 - m + 3m + 1\} x^m = 0$

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$m^2 + 2m + 1 = 0$

$\Rightarrow (m+1)^2 = 0$

$\Rightarrow (m+1)(m+1) = 0$

$m = -1, -1$

Since roots are double so its solution is.

$y(x) = \{c_1 + c_2 \ln x\} x^{-1}$

$y'(x) = (c_1 + c_2 \ln x)(-1 x^{-2})$

$+ \frac{c_2}{x} x^{-1}$

$= -x^{-2}\{c_1 + c_2 \ln x\} +$

$c_2 x^{-2}.$

$y(1) = 3.6$

$(c_1 + c_2 \ln 1)(1)^{-1} = 3.6$

$c_1 + 0 = 3.6$

$\boxed{c_1 = 3.6}$

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$y'(1) = 0.4$

$-1\left(C_1 + C_2 \ln 1\right) + C_2\, x(1) = 0.4$

$-C_1 + 0 + C_2 = 0.4.$

$C_2 = 0.4 + C_1$

$C_2 = 0.4 + 3.6$

$C_2 = 4.$

$y(x) = (3.6 + 4 \ln x)\, x^{-1}$

17) $(x^2D^2 - 3xD + 4I)y = 0$

$y(1) = -\pi,\ y'(1) = 2\pi$

$x^2y'' - 3xy' + 4y = 0 \longrightarrow (1)$

putting $y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1)x^{m-2}$

eq (1) $\Rightarrow x^2\, m(m-1)x^{m-2} - 3x\, m x^{m-1} + 4x^m = 0$

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$m(m-1)x^m - 3mx^m + 4x^m = 0$

$\{m^2 - m - 3m + 4\}x^m = 0$

The auxiliary eqn is.

$m^2 - 4m + 4 = 0$

$m^2 + 2(-2m) + (-2)^2 = 0$

$(m-2)^2 = 0$

$m - 2 = 0 \quad , \quad m - 2 = 0$

$m = 2 \quad , \quad m = 2.$

Since roots are double so its solution is.

$y(x) = x^2\{c_1 + c_2 \ln x\}$

$y(1) = -\pi$

$1(c_1 + 0) = -\pi$

$\Rightarrow c_1 = -\pi \quad ; \quad \ln 1 = 0$

$y'(x) = 2x(c_1 + c_2 \ln x)$

$+ x^2\left(\cancel{c_2} + \frac{c_2}{x}\right)$

$= 2x(c_1 + c_2 \ln x) + xc_2$

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$y'(1) = 2\pi$

$\Rightarrow 2(c_1 + 0) + c_2 = 2\pi$

$\Rightarrow 2(-\pi) + c_2 = 2\pi$

$\Rightarrow c_2 = 2\pi + 2\pi$

$\Rightarrow c_2 = 4\pi$

$y(x) = x^2(-\pi + 2\pi \ln x)$.

$= \pi[\ln x^2 - 1] x^2$.

17):- $(x^2 D^2 + xD + I)y = 0 \; ; \longrightarrow (1)$

$y(1) = 1 \; , \; y'(1) = 1$

Sol:- $x^2 y'' + xy' + y = 0$

$y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1) x^{m-2}$

$x^2 m(m-1) x^{m-2} + x m x^{m-1} + x^m = 0$

$[m^2 - m + m + 1] x^m = 0$

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$(m^2+1) = 0$

$m^2 = -1$

$m = \pm i$

Since roots are Complex so its solution

$\alpha = 0$

$\beta = 1.$

$y(x) = x^0 \{ C_1 \cos \ln x + C_2 \sin \ln x \}.$

$y(x) = C_1 \cos \ln x + C_2 \sin \ln x$

$y(1) = 1$

$C_1 \cos 0 + C_2 \sin 0 = 1.$

$\boxed{C_1 = 1}$

$y'(x) = \frac{-C_1}{x} \sin \ln x + \frac{C_2}{x} \cos \ln x$

$y'(1) = 0$

$-C_1 \sin 0 + C_2 \cos 0 = 0$

$\Rightarrow C_2 = 0$

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Hence $\boxed{y(x) = \cos \ln x}$

18):- $(9x^2 D^2 + 3xD + I)y = 0$; $\longrightarrow$ (1)

$y(1) = 1, \quad y'(1) = 0$

Sol:-

$$9x^2 y'' + 3x y' + y = 0$$

$$y = x^m$$

$$\Rightarrow y' = m x^{m-1}$$

$$y'' = m(m-1) x^{m-2}$$

$$9x^2 m(m-1) x^{m-2} + 3x\, m x^{m-1} + x^m = 0$$

$$\Rightarrow (9m^2 - 9m + 3m + 1) x^m = 0$$

$$\Rightarrow 9m^2 - 6m + 1 = 0$$

$$9m^2 - 3m - 3m + 1 = 0$$

$$\Rightarrow 3m(3m-1) - 1(3m-1) = 0$$

$$\Rightarrow (3m-1)(3m-1) = 0$$

$$\Rightarrow 3m - 1 = 0, \quad 3m - 1 = 0$$

$$\Rightarrow m = 1/3, \quad m = 1/3$$

$\frac{1}{3} - 1 = -\frac{2}{3}$

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Since roots are double so its solution is:

$$y(x) = x^{1/3}\left(c_1 + c_2 \ln x\right).$$

$$y(1) = 1$$

$$\Rightarrow (1)^{1/3}\left(c_1 + 0\right) = 1$$

$$\Rightarrow \boxed{c_1 = 1}$$

$$y'(x) = \frac{1}{3}x^{-2/3}\left(c_1 + c_2 \ln x\right) + x^{1/3} \cdot \frac{c_2}{x}$$

$$= \frac{1}{3}x^{-2/3}\left(c_1 + c_2 \ln x\right) + x^{-2/3} c_2$$

$$y'(1) = 0$$

$$\Rightarrow \frac{1}{3}c_1 + c_2 = 0$$

$$\frac{1}{3}(1) + c_2 = 0$$

$$\Rightarrow c_2 = -\frac{1}{3}.$$

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$y(x) = x^{1/3}\left(1 + \left(-\frac{1}{3}\right)\ln x\right)$

$= \sqrt[3]{x}\left[1 + \ln x^{-1/3}\right]$

$= \sqrt[3]{x}\left[1 + \ln \frac{1}{\sqrt[3]{x}}\right]$.

19) $\left((xD)^2 - xD - 15I\right)y = 0$;

$y(1) = 0.1$

$y'(1) = -4.5$

$x^2y'' - xy' - 15y = 0$

$y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} - x\,m\,x^{m-1} - 15x^m = 0$

$(m^2 - m)x^m - mx^m - 15x^m = 0$

$\{m^2 - m - m - 15\}x^m = 0$

The auxiliary eq: is :

$\Rightarrow m^2 - 2m - 15 = 0$

$\Rightarrow m^2 - 5m + 3m - 15 = 0$

$\Rightarrow m(m-5) + 3(m-5) = 0$

$\Rightarrow (m+3)(m-5) = 0$

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$m+3=0 \quad , \quad m-5=0$

$\Rightarrow m=-3, \quad m=5$

Since roots are real and distince Hence its solution is:

$$y(x) = c_1 x^{-3} + c_2 x^5$$

$$y(1) = 0.1$$

$$c_1 + c_2 = 0.1 \longrightarrow (i).$$

$$y'(x) = -3c_1 x^{-4} + 5c_2 x^4.$$

$$y'(1) = -4.5$$

$$-3c_1 + 5c_2 = -4.5 \longrightarrow (ii)$$

solving (i) and (ii) we have:

$$\cancel{3c_1} + 3c_2 = 0.3.$$

$$\cancel{-3c_1} + 5c_2 = -4.5$$

$$8c_2 = -4.2$$

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$C_2 = -0.53$

$C_1 + (-0.53) = 0.1$

$\Rightarrow C_1 = 0.1 + 0.53$

$C_1 = 0.63.$

$y(x) = 0.63x^{-3} - 0.53x^{5}$

for $x = 0$

$y(0) = 0$

for $x = 1$

$y(1) = 0.63 - 0.53$

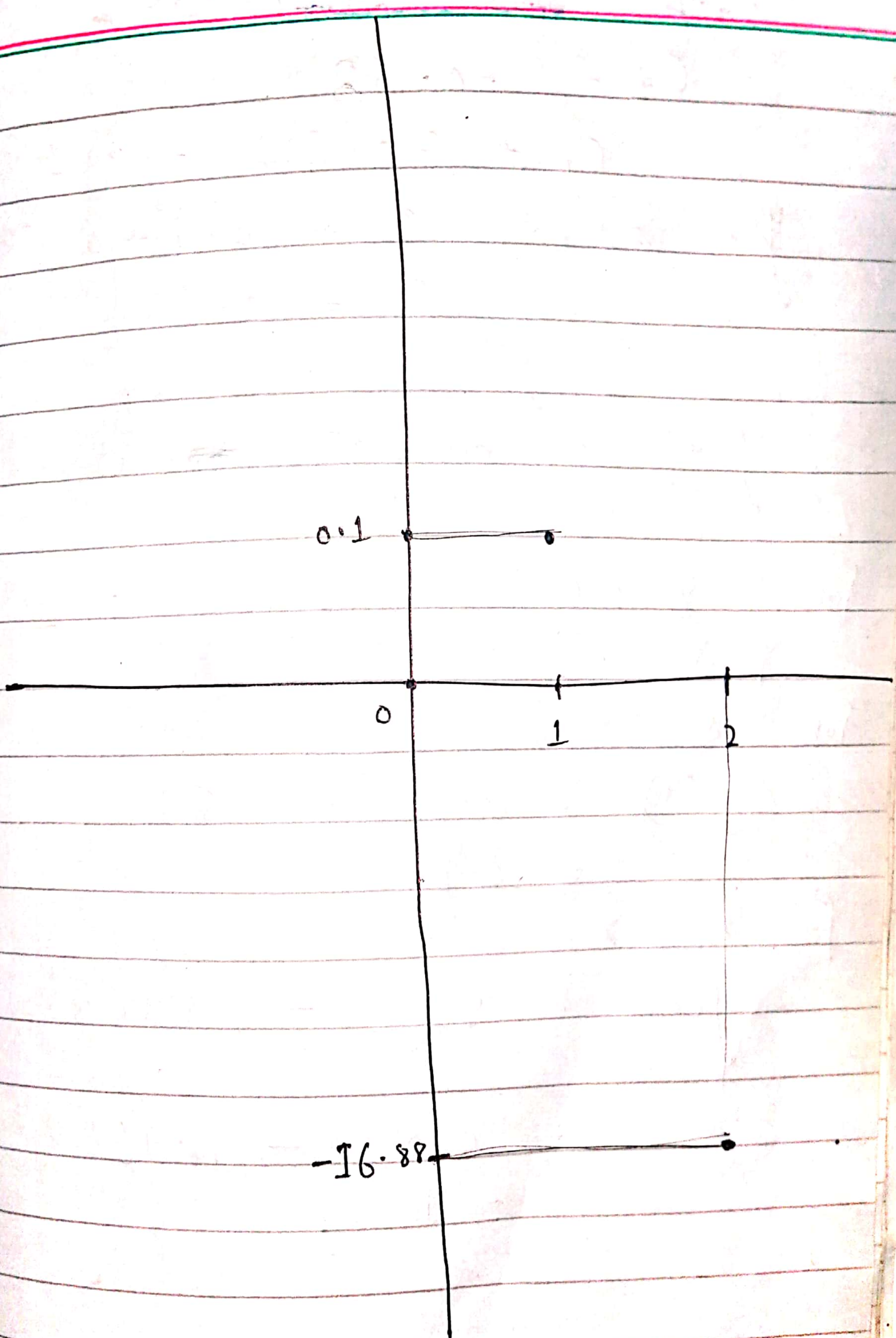
$= 0.1.$

for $x = 2.$

$y(2) = \frac{0.63}{8} - 0.53(32)$

$= 0.08 - 16.96$

$= -16.88$


0·1
0
1
2
-16·88

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Q4: Find the general Solution

a) $x' = \dfrac{2x}{t+1}$

Sol:- $x' = \dfrac{2x}{t+1}$

$$\frac{dx}{dt} = \frac{2x}{t+1}$$

$$\frac{1}{2x}\,dx = \frac{dt}{t+1}$$

$$\frac{1}{2}\int \frac{1}{x}\,dx = \int \frac{dt}{t+1}$$

$$\frac{1}{2}\ln x + \ln c = \ln(t+1)$$

$$\ln x^{1/2} + \ln c = \ln(t+1)$$

$$\ln c x^{1/2} = \ln(t+1)$$

$$c x^{1/2} = t+1$$

$$c^2 x = (t+1)^2$$

2

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$$c^2 = c_1$$

$$c_1 x = (t+1)^2 \quad \text{Ans.}$$

b) $\theta' = t\sqrt{t^2+1}\ \sec\theta$

Sol:-

$$\frac{d\theta}{dt} = t\sqrt{t^2+1}\ \sec\theta$$

$$\frac{1}{\sec\theta}\, d\theta = t\sqrt{t^2+1}\, dt$$

$$\int \frac{1}{\sec\theta}\, d\theta = \int t\sqrt{t^2+1}\, dt$$

$$\int \cos\theta\, d\theta = \int t\sqrt{t^2+1}\, dt$$

$$\sin\theta + c = \frac{1}{2}\int \sqrt{t^2+1}\ 2t\, dt$$

$$= \frac{1}{2}\int (t^2+1)^{\frac{1}{2}}\, 2t\, dt$$

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Muhammad Asif ID $\frac{1}{2}+1$

$$\sin\theta + C = \frac{1}{2}\frac{(t^2+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$= \frac{1}{2}\frac{(t^2+1)^{3/2}}{3/2}$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}(t^2+1)^{3/2}.$$

$$\sin\theta + C = \frac{1}{3}(t^2+1)^{3/2}$$

c) $(2u+1)u' - (t+1) = 0$

Sol:-

$$(2u+1)\frac{du}{dt} = (t+1)$$

$$(2u+1)du = (t+1)dt$$

$$\int 2u\,du + \int du = \int t\,dt + \int dt.$$

$$\cancel{2}\frac{u^2}{\cancel{2}} + u = \frac{t^2}{2} + t + C.$$

$$u^2 + u = \frac{t^2}{2} + t + C$$

Ans

d) $R' = (t+1)(R^2+1)$

Sol: $$\frac{dR}{dt} = (t+1)(R^2+1)$$

$$\frac{1}{R^2+1}\, dR = (t+1)\, dt$$

$$\int \frac{1}{R^2+1}\, dR = \int (t+1)\, dt$$

$$\tan^{-1} R + C = \frac{t^2}{2} + t$$

Ans.

e) $y' + y + \frac{1}{y} = 0$

Sol:

$$\frac{dy}{dt} = -y - \frac{1}{y}$$

$$\frac{dy}{dt} = -\left(y + \frac{1}{y}\right)$$

$$\frac{dy}{dt} = -\left(\frac{1+y^2}{y}\right)$$

$$\Rightarrow \left(\frac{y}{1+y^2}\right) dy = -dt$$

$$\int \frac{y}{1+y^2}\,dy = -\int dt.$$

$$\frac{1}{2}\int \frac{2y}{1+y^2}\,dy = -t + C$$

$$\frac{1}{2}\ln(1+y^2) = -t + C.$$

$$\ln(1+y^2)^{\frac{1}{2}} = -t + C$$

Ans

f) $(t+1)x' + x^2 = 0$

$$(t+1)\frac{dx}{dt} = -x^2.$$

$$\frac{1}{x^2}\,dx = -\frac{1}{(t+1)}\,dt$$

$$\int x^{-2}\,dx = -\int \frac{dt}{t+1}$$

$$\frac{x^{-2+1}}{-2+1} = -\ln(t+1) + C_1$$

$$-x^{-1} = -\ln(t+1) + C_1$$

$$\frac{1}{x} = \ln(t+1) + C$$

Ans

$\therefore -C_1 = C.$

$Q_2)$:- $y' = r(a-y)$

Sol:- $\dfrac{dy}{dt} = r(a-y)$

$\dfrac{dy}{a-y} = r\,dt$

$\displaystyle\int \frac{dy}{a-y} = r\int dt.$

$-\ln(a-y) = rt + C.$

$\ln\left(\dfrac{1}{a-y}\right) = rt + C$

Ans.

$Q_1)$ a) $x' = \sqrt{x}.$

$\dfrac{dx}{dt} = \sqrt{x}$

$\dfrac{dx}{\sqrt{x}} = dt$

$\displaystyle\int x^{-1/2}\,dx = \int dt$

$$\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = t + c$$

$$\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = t + c.$$

$2\sqrt{x} = t + c$ Ans.

When $t = 0$, $x = 1 \longrightarrow$ (1)

eq (1) $\Rightarrow$ $2(1) = 0 + c$

$c = 2.$

(1) $\Rightarrow$ $2\sqrt{x} = t + 2$

B) $x' = au + b$, $a, b > 0$

Sol:- $\frac{dx}{dt} = au + b.$

$dx = (au + b)\, dt$

$\int dx = \int (au + b)\, dt$

$x = (au + b)t + c$

Using $x(0) = 1$ we get

$1 = (au + b)(0) + c$

$\Rightarrow \quad c = 1.$

$x = (au + b)t + 1.$

b) $x' = e^{-2x}.$

Sol: $\frac{dx}{dt} = e^{-2x}.$

$\Rightarrow \quad e^{2x}\,dx = dt.$

$\int e^{2x}\,dx = \int dt.$

$\frac{e^{2x}}{2} = t + c_1.$

$\Rightarrow \quad e^{2x} = 2t + 2c_1$

$\Rightarrow \quad e^{2x} = 2t + c$

$\therefore 2c_1 = c$

using $x(0) = 1$

$e^2 = 0 + c.$

$c = e^2.$

$e^{2x} = 2t + e^2.$

c) $y' = 1 + y^2$

Sol: $\frac{dy}{dt} = 1 + y^2$

$$\Rightarrow \frac{dy}{1+y^2} = dt$$

$$\int \frac{dy}{1+y^2} = \int dt$$

$$\tan^{-1} y + c = t.$$

d) $$u' = \frac{1}{5-2u}.$$

$$\frac{du}{dt} = \frac{1}{5-2u}.$$

$$\int (5-2u)\, du = \int dt$$

$$5\int du - 2\int u\, du = \int dt$$

$$5u - \cancel{2}\frac{u^2}{\cancel{2}} = t + c$$

$$5u - u^2 = t + c$$

As.

f) $Q' = \frac{Q}{4+Q^2}$.

$\Rightarrow \frac{dQ}{dt} = \frac{Q}{4+Q^2}$.

$\Rightarrow \left(\frac{4+Q^2}{Q}\right) dQ = dt$.

$\Rightarrow 4\int \frac{1}{Q} dQ + \int \frac{Q^2}{Q} dQ$

$= \int dt$.

$\Rightarrow 4\int \frac{1}{Q} dQ + \int Q\, dQ =$

$\int dt$

$\Rightarrow 4 \ln Q + \frac{Q^2}{2} = t + c$

$\Rightarrow \ln Q^4 + \frac{Q^2}{2} = t + c$

g) $x' = e^{x^2}$.

Sol:- $\dfrac{dx}{dt} = e^{x^2}$.

$\dfrac{dx}{e^{x^2}} = dt$

$\int e^{-x^2} dx = \int dt$.

$\int e^{-t} \dfrac{dt}{2\sqrt{t}} = t + c$

Ans.