Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Total Marks:** 20

Student Details

Name: **Rimsha khan** **Student ID:** **13672**

Q1.	(a)	Determine the response $y(n)$, $n \geq 0$, of the system described by the second order difference equation $y(n)-3y(n-1)-4y(n-2) = x(n) + 2x(n-1)$ To the input $x(n) = 4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $y(n) = 0.6y(n-1)-0.8y(n-2) + x(n)$	
Q2.	(a)	Determine the causal signal $x(n)$ having the z-transform $x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$	
Q. 3	(a)	A two- pole low pass filter has the system response $H(z) = \frac{b_0}{(1-pz^{-1})^2}$ Determine the values of b_0 and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\|H(\frac{\pi}{4})\right\|^2 = \frac{1}{2}$.	Marks 4

<table>
<tr><td></td><td>(b)</td><td>Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.</td><td></td></tr>
<tr><td rowspan="2">Q 4</td><td>(c)</td><td>A finite duration sequence of Length L is given as
$$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$$
Determine the N- point DFT of this sequence for $N \ge L$</td><td rowspan="2">Marks 4</td></tr>
<tr><td>(d)</td><td>Compute the DFT of the four-point sequence
$$x(n) = (0 \;\; 1 \;\; 2 \;\; 3)$$</td></tr>
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"SESSIONAL ASSIGNEMENT"

NAME:- RIMSHA KHAN I.D:- 13672
SUBJECT:- DSP SUBMITTED TO:- PIR MEHER

QUESTION 1:-

PART A:-

Determine the response $y(n), n \geq 0$ of the System described by the second order difference equation.

$$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$$

To the input $x(n) = 4^n u(n)$.

Solution:-

We have already determined the solution to the homogenous difference equation. So we have

$$y(n) = C_1(-1)^n + C_2(4)^n$$

The particular solution is assumed to an exponential Sequence.

$$y_p(n) = K(4)^n u(n)$$

Now we do substitution.

$$Kn(4)^n u(n) - 3K(n-1)(4)^{n-1}u(n-1) - 4K(n-2)(4)^{n-2}u(n-2)$$
$$= (4)^n u(n) + 2(4)^{n-1}u(n-1)$$

To determine K we evaluate the equation for any $n \geq 2$. We select $n=2$ from which we obtain $K = \frac{6}{5}$. Therefore

$$y_p = \frac{6}{5}n(4)^n u(n)$$

The total solution to the difference equation obtained by adding. Thus

$$y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{5}n(4)^n \quad n \geq 0$$

Where C_1 and C_2 determined initial conditions.

$$y(0) = 3y(-1)+4y(-2)+1$$
$$y(1) = 3y(0)+4y(-1)+6$$
$$= 13y(-1)+12y(-2)+9$$

on the other hand evaluated at $n=0$ and $n=1$ yields.

$$y(0) = C_1+C_2$$
$$y(1) = -C_1+4C_2+\frac{24}{5}$$

We have already solved for the zero input response. We can simplify the computations above by setting $y(-1) = y(-2) = 0$. So,

$$C_1+C_2 = 1$$
$$-C_1+4C_2+\frac{24}{5} = 9$$

Hence $C_1 = -\frac{1}{25}$ and $C_2 = \frac{26}{25}$. So $y(n)$ is.

$$\boxed{y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5}n(4)^n \qquad n \geq 0}$$

PART B :-

b, Determine the impulse response and unit step response of the system described by difference equation.

$$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$

Solution:-

The characteristic equation is

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$\lambda = 0.2,\ 0.4$ Hence

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$$y(h) = C_1 \frac{1^n}{5} + C_2 \frac{2^n}{5}$$

With $x(n) = \delta(n)$ the initial conditions are.

$$y(0) = 1$$

$$y(1) - 0.6\,y(0) = 0 \Rightarrow y(1) = 0.6$$

Hence, $C_1 + C_2 = 1$ and

$$\frac{1}{5}C_1 + \frac{2}{5} = 0.6 \Rightarrow C_1 = -1,\ C_2 = 3$$

Therefore $h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$

The step response is.

$$s(n) = \sum_{k=0}^{n} h(n-k),\ n \geq 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$\boxed{= \left\{\frac{1}{0.12}\left[\left(\frac{2^{n+1}}{5} - 1\right)\right] - \frac{1}{0.16}\left[\left(\frac{1^{n+1}}{5} - 1\right)\right]\right\} u(n)}$$

Question 2:-

PART A:-

a, Determine the causal signal $x(n)$ having the Z-transform.

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol:-

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

By partialy factorized factors are

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$x(z) = A \frac{(1-2z^{-1})}{(1-z^{-1})} B + \frac{(1-2z^{-1})}{(1-z^{-1})^2}$$

when evaluated at $z=-1$.

As apply this $z=-1$ limit we get:-

$A = 4$

Similarly $B = -3$

and $C = -1$.

So

putting values of A, B and C.

we get.

$$\boxed{x(n) = [4(2)^n - 3 - n]\, u(n)}$$

PART B :-

b, Determine the partial fraction expression of the following function.

$$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$

Solution :-

First we eliminate the negtive powers by multiplying both numerator and denominator by z^2.

$$X(z) = \frac{z^2}{z^2-1.5z+0.5}$$

The poles of $X(z)$ are $p_1 = 1$ and $p_2 = 0.5$.

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

Multiply equation with denominator.

$$z = (z-0.5)A_1 + (z-1)A_2$$

Now set $z = p_1 = 1$.

$$1 = (1-0.5)A_1 \text{ and } 0.5 = (0.5-1)A_2$$

$A_2 = 1$ result of expression is..

$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

So we have pole positions.

$$\frac{(z-P_k)X(z)}{z} = \frac{(z-P_k)A_1}{z-P_1} + \text{-----} A_k + \text{----}$$

$$\frac{(z-P_k)A_N}{z-P_N}.$$

Consquently, with $z = P_K$ yields the Kh cofficient as.

$$\boxed{A_K = \frac{(z - P_K)\,X(z)}{z}\Bigg|_{z = P_K} \qquad K = 1, 2, \ldots N}$$

QUESTION 3:-

PART A:-

A two pole low-pass filter has the system response.

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

Determine the values of b_0 and p such that the frequency response $H(\omega)$ satisfies that condition $H(0) = 1$ and $\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$.

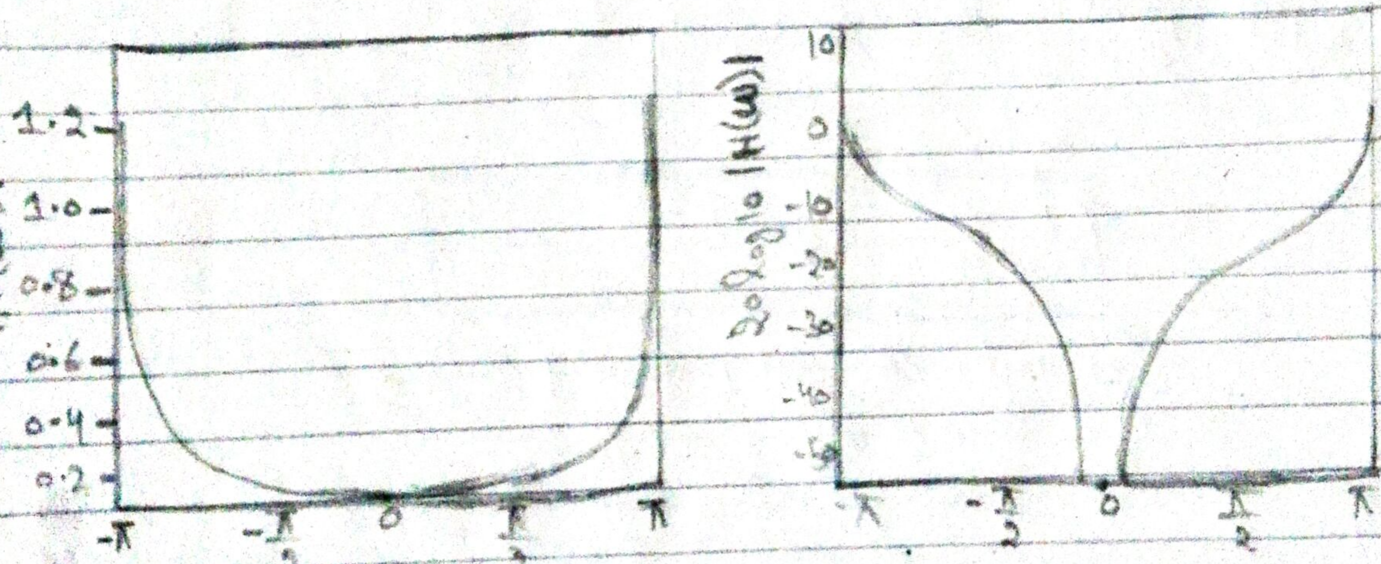
Solution:-

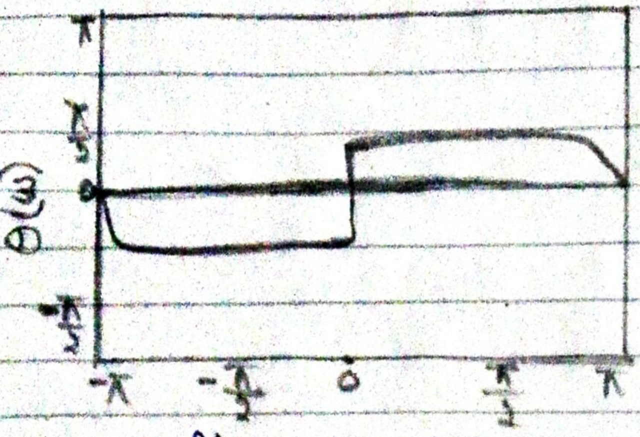
At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$





At $\omega = \frac{\pi}{4}$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4)+jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2}+jp/\sqrt{2})^2}$$

Hence

$$= \frac{(1-p)^4}{[(1-p/\sqrt{2})^2+p^2/2]^2} = \frac{1}{2}$$

or equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p = 0.32$, satisfies the equation. So the system function for desired filter is.

$$\boxed{H(z) = \frac{0.46}{(1-0.32z^{-1})^2}}$$

PART B :-

b) Design a bandpass filter that has centre of its pass band at $\omega = \pi/2$ zero in its frequency response at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.

Solution :-

The filter must have poles at.

$$P_{1,2} = re^{\pm j\pi/2}$$

and zeros at $z = 1$ and $z = -1$.

The System function is.

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{(z^2-1)}{(z^2-r^2)}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \frac{\pi}{2}$. Thus we have

$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1-r^2} = 1$$

$$\Rightarrow G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have.

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= \frac{1}{2}$$

or equivalently

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ satisfies the equation so the system filter for the desired function is.

$$\boxed{H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}}$$

PART A:- QUESTION 4.

a) A finite duration sequence of Length L is given as.

$$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Determine the N-pole or N-point DFT of this sequence for $N \geq L$.

Solution :-

The fourier transform of this sequence is.

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

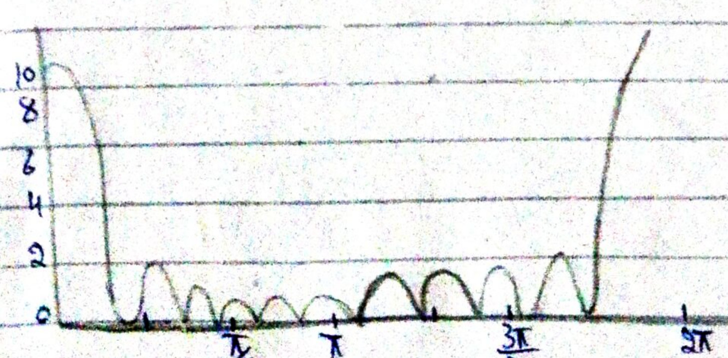
$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1/2)}$$

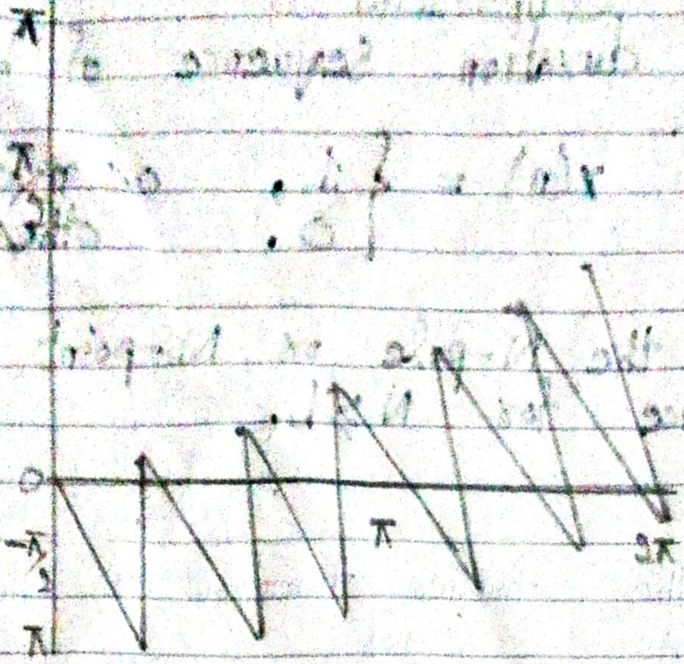
The magnitude and phase of $X(\omega)$ are illustrated for $L = 10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally speed frequencies $\omega_K = 2\pi K/N$. $K = 0, 1 \ldots N-1$.

Hence

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}} \qquad (K = 0, 1 \ldots N-1)$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





If N is selected such that N = L then the DFT becomes.

$$X(K) = \begin{cases} L, & K = 0 \\ 0, & K = 1, 2 \cdots L-1. \end{cases}$$

PART B :-

b, Compute the DFT of four point sequence.

$$X(n) = (0, 1, 2, 3)$$

Solution :-

The first step is to determine the matrix W_4. By exploiting the periodically property of W_4.

$$W_N^{K+N/2} = W_N^K$$

The matrix W_4 may be expressed as.

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then

$$X_4 = W_4 x_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The IDFT of X_4 may be determined by conjugating the elements in W_4 to obtain W_4.