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paper :- Calculus & Analytical Geometry.

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Q No (1) (a)

Identify: $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

Sol:-

$$\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

$$= \frac{\sqrt{2+0} - \sqrt{2}}{0} = \frac{\sqrt{2} - \sqrt{2}}{0}$$

$$= \frac{0}{0} \Rightarrow 0/0 \text{ Form}$$

So then

$$\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

xing and ÷ing by $\sqrt{2+h} + \sqrt{2}$

$$= \lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$= \lim_{h \to 0} \frac{(\sqrt{2+h} - \sqrt{2})(\sqrt{2+h} + \sqrt{2})}{(h)(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \to 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \to 0} \frac{\cancel{2} + h - \cancel{2}}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{2+h} + \sqrt{2})}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2+0}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

(b) Q No (4):

$$y = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$$

Sol:

$$y = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$$

$$= (x+x^{-1})\frac{d}{dx}(x-x^{-1}+1) + (x-x^{-1}+1)\frac{d}{dx}(x+x^{-1})$$

$$= (x+x^{-1})(1+x^{-2}) + (x-x^{-1}+1)(1-x^{-2})$$

$$= \left(x+\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right) + \left(x-\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)$$

$$= x + x\frac{1}{\cancel{x^2}} - \frac{1}{\cancel{x}} + \frac{1}{x^3} + x - x\frac{1}{x^2} - \frac{1}{\cancel{x}} + \frac{1}{x^3} + 1 - \frac{1}{x^2}$$

$$= 2x + 1 - \frac{1}{x^2} + \frac{1}{x^3} \text{ Ans}$$

Q2 (a)

$$S = 160t - 16t^2 \text{ ft}$$

Given:

a) $$S = 160t - 16t^2 \text{ ft}$$

Velocity is,

$$V = \frac{ds}{dt} = \frac{d}{dt}\left(160t - 16t^2\right)$$

$$= \frac{d}{dt} 160t - \frac{d}{dt} 16t^2$$

$$V = 160 - 32t$$

Maximum hight

$$V = 0$$

So

$$160 - 32t = 0$$

$$\frac{160}{32} = \frac{32t}{32}$$

$$t = 5 \text{ sec}$$

$$S_{max} = S(5) = 160(5) - 16(5)^2$$

$$S_{max} = 400 \text{ ft}$$

Q No (2) (b)

Given that

$$S = 256 \text{ ft}$$

then

$$160t - 16t^2 = 256$$

$$16t^2 - 160t + 256 = 0$$

$$\frac{16}{16}\left(t^2 - 10t + 16\right) = \frac{0}{16}$$

$$t^2 - 10t + 16 = 0$$

$$t^2 - 8t - 2t + 16 = 0$$

$$t(t-8) - 2(t-8) = 0$$

$$(t-8)(t-2) = 0$$

$t - 8 = 0$ $t - 2 = 0$

$t = 8$ $t = 2$

$t_1 = 8\,\text{sec}$ $t_2 = 2\,\text{sec}$

Since

$$V = 160 - 32t$$

$t_1 = 2s$

$$V_{(2)} = 160 - 32(2)$$
$$= 160 - 64$$

$$V_{(2)} = 96\,\text{m/s} \Rightarrow \text{Velocity}$$

(5)

$t_2 = 8s$

$$V_{(8)} = 160 - 32(8)$$

$$= 160 - 256$$

$$= -96\ m/s \Rightarrow \text{velocity downword}$$

(c) Since,

$$V = 160 - 32t$$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(160 - 32t)$

$$a = 0 - 32\ m/s^2$$

$$a = -32\ m/s^2$$

Q3(a)

$$y = x^4 - 2x^2 + 2$$

Sol:.

$$y = x^4 - 2x^2 + 2$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 2)$$

$$= \frac{d}{dx}x^4 - \frac{d}{dx}2x^2 + \frac{d}{dx}2$$

$$= 4x^3 \cdot \cancel{\frac{dx}{dx}} - 2 \times 2x \cancel{\frac{dx}{dx}} + 0$$

$$= 4x^3 - 4x + 0$$

$$\frac{dy}{dx} = 4x^3 - 4x$$

if the tangent is horizontal then $\frac{dy}{dx} = 0$

So,

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$4x = 0$, $x^2 - 4 = 0$

$\frac{4x}{4} = \frac{0}{4}$ $\quad$ $x^2 - 1 = 0$

$x = 0$ $\quad$ $x^2 = 1$

$\sqrt{x^2} = \sqrt{1}$

$x = 0$ $\quad$ $x = \pm 1$

So

$x = 0, 1, -1$

their corresponding point in

$y = x^4 - 2x^2 + 2$

For, $x = 0$

$y = x^4 - 2x^2 + 2$

$= (0)^4 - 2(0)^2 + 2$

$= 0 - 0 + 2$

$y = 2$

$x = 1$

$y = x^4 - 2x^2 + 2$

$= (1)^4 - 2(1)^2 + 2$

$= 1 - 2 + 2$

$y = 1$

$x = -1$

$y = x^4 - 2x^2 + 2$

$= (-1)^4 - 2(-1)^2 + 2$

$= 1 - \cancel{2} + \cancel{2}$

$= 1$

Hence, $(0, 2), (1, 1), (-1, 1)$