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Subject # Calculus

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Question # 1

Find PQ where P is the point in ....?

Sol:-

Coordinate of $P = (4, 1, 3)$

$OP = 4i + 1j + 3k$

or $PQ = \vec{OQ} - \vec{OP}$

$= (i + 2j + 4k) - (4i + 1j + 3k)$

$= -3i + 1j + 1k \rightarrow (1)$

Now distance between P & Q $= |PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9+1+1}$

$= \sqrt{11} \rightarrow (2)$ $d = 3.3167$

Let M be the point which divided PQ in ratio 1:3 then by ratio theorm.

(2)

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Position vector of M = $\vec{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i+2j+4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \longrightarrow (3)$$

Hence eq (1), (2) & (3) are the required sol.

Question # 2

Evaluate

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}\, dx$$

Sol:-

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x} = \int \frac{2(2x^3 + 5x + 2)}{x(2x+1)}\, dx$$

Apply linearity

$$= 2\int \frac{2x^3 + 5x + 2}{x(2x+1)}\, dx$$

Now solving

$$\int \frac{2x^3 + 5x + 2}{x(2x+1)}\, dx$$

Perform polynomial long division

$$= \int \left(\frac{11x + 4}{2x(2x+1)} + \frac{2x - 1}{2} \right) dx$$

Apply linearity

$$= \frac{1}{2}\int \frac{11x+4}{x(2x+1)}\,dx + \int x\,dx - \frac{1}{2}\int 1\,dx$$

Now solving

$$\int \frac{11x+4}{x(2x+1)}\,dx$$

Perform partial fraction decomposition

$$= \int \left(\frac{3}{2x+1} + \frac{4}{x}\right) dx$$

$$= 3\int \frac{1}{2x+1}\,dx + 4\int \frac{1}{x}\,dx$$

Now solving

$$\int \frac{1}{2x+1}\,dx$$

Substitute $u = 2x+1$ $\quad \frac{du}{dx} = 2$

$$dx = \frac{1}{2}\,du \qquad = \frac{1}{2}\int \frac{1}{u}\,du$$

Now Solving

$$\int \frac{1}{u} du$$

This is standard Integral

$$= In(u)$$

$$= \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{ln(u)}{2}$$

Put again $u = 2x+1$

$$= \frac{In(2x+1)}{2}$$

Now - Solving

$$\int \frac{1}{x} dx$$

$$3 \int \frac{1}{2x+1} dx + 4 \int \frac{1}{x} dx$$

$$= \frac{3\ln(2x+1)}{2} + 4\ln(x)$$

Now Solving

$$\int x \, dx$$

Apply power Rule

$\int x^n dx = \frac{x^{n+1}}{n+1}$ $n=1$

$$= \frac{x^2}{2}$$

Now

$$\int 1 \, dx$$

Apply Constant rule

$\cancel{\frac{1}{2}\int 1\,dx}$

$$\frac{1}{2}\int \frac{11x+4}{x(2x+1)}\,dx + \int x\,dx - \frac{1}{2}\int 1\,dx$$

$$= \frac{3\ln(2x+1)}{4} + 2\left(\ln(x)\right) + \frac{x^2}{2} - \frac{x}{2}$$

$$= \frac{3\ln(2x+1)}{2} + 4\ln(x) + x^2 - x$$

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$= 3\ln(|2x+1|) + 4\ln(|x|) + x^2 - x + c$

$= 3\ln\dfrac{(|2x+1|)}{2} + 4\ln(|x|)$

$+ (x-1)\ x + c$

$= x^2 - x + \dfrac{3}{2}\ln|2x+1|$

$+ 4\ln|x| - \dfrac{3}{4} + c$ Ans

Question #3

Part (a) $\int_0^2 x^2 e^x \, dx$

Sol:

$$\int_0^2 x^2 e^x \, dx$$

Now first find Integration

$$= \int x^2 e^x \, dx$$

$$= x^2 \int e^x \, dx - \int \left(\left(\int e^x dx \right) \frac{d}{dx} x^2 \right) dx$$

$$= x^2 e^x - \int e^x (2x) \, dx$$

$$= x^2 e^x - 2 \int x e^x \, dx$$

$$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\left(\int e^x dx \right) \frac{d}{dx} x \right) dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2x e^x + 2e^x$$

Now Put limits

$$= \left| x^2 e^x - 2xe^x + 2e^x \right|_0^2$$

$$= \left(2^2 e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0\right)$$

$$= \cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2$$

$$\boxed{= 2e^2 - 2}$$ Ans

Part (b)

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

Sol:

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

First find Integration

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = ? \rightarrow (1)$$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\boxed{2dy = \frac{1}{\sqrt{x}}dx}$$ Point (1)

$$\int \sin(y)(2dy) = 2\int \sin(y)dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

Put $y = \sqrt{x}$

$$= -2\cos\sqrt{x}$$

Put limits

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{2} - \cos 1)$$

$$= -2\cos\sqrt{2} + 2\cos(1)$$ Ans

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Question # 4

The leplace eq in 3.d is

Sol:-

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

So

$$u(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

$$u(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{2}(x^2 + y^2 + z^2)^{-3/2}(2x)$$

$$\frac{\partial u}{\partial x} = -x(x^2 + y^2 + z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(\frac{-3}{\cancel{2}}\right)(x^2 + y^2 + z^2)^{-5/2}(2x)\right.$$

$$\left. + (x^2 + y^2 + z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \rightarrow (1)$$

Now

$$\frac{\partial u}{\partial y} = \frac{-1}{\cancel{2}} (x^2+y^2+z^2)^{-3/2} (\cancel{2}y)$$

$$= -y (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y(-3/\cancel{2})(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) + (x^2+y^2+z^2)^{-3/2} \right]$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = 3y^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}} \rightarrow (2)$$

$$\frac{\partial u}{\partial z} = \frac{-1}{2} (x^2+y^2+z^2)^{-3/2} (2\cancel{z})$$

$$\frac{\partial u}{\partial z} = -z (x^2+y^2+z^2)^{-3/2}$$

$$\boxed{\frac{\partial^2 u}{\partial z^2} = 3z^2 (x^2+y^2+z^2)^{-5/3} - (x^2+y^2+z^2)^{-3/2}} \rightarrow (3)$$

Putting eq ①, ② & ③ in Ⓐ

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} +$$

$$3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$+3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$\Rightarrow (x^2+y^2+z^2)^{-5/2}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2 z^2\right]$$

$$\Rightarrow (x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2} - \cancel{x^2} - \cancel{y^2} - z^2 + 3y^2 - \cancel{x^2} - y^2 - z^2 + 3z^2 - \cancel{x^2} - \cancel{y^2} - \cancel{z^2}\right]$$

$$\Rightarrow (x^2+y^2+z^2)^{-5/2}(0) = (0)$$

So the given $u(x,y,z)$ is solution of laplace equation.