

NAME

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ID

7957

SECTION

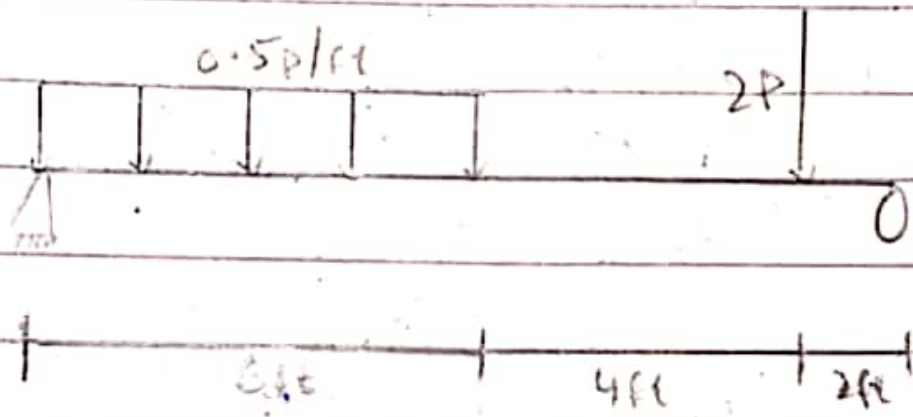
"B"

Deptt:

BE Civil

Question : 2.

Construct the Mohr's Circle diagram and --- equations?



2mch

2mch

## REACTION:-

$$\sum F_y = 0 \quad \uparrow + \text{upward is positive.}$$

$$R_A + R_B - 28.5 \times 6 - 114 = 0$$

$$R_A + R_B = 288 \rightarrow \textcircled{A}$$

$$\sum M_A = 0 \quad \curvearrowright + \text{Anticlockwise is +ve}$$

$$R_B (12) - 114 (10) - 28.5 (6) \times 3 = 0$$

$$R_B (12) - 1653$$

$$R_B = \frac{1653}{12}$$

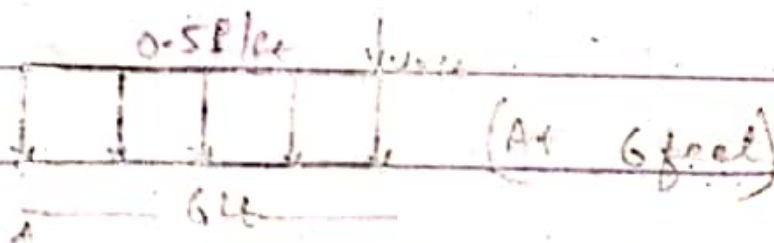
$$R_B = 137.75 \text{ lb}$$

Now

$$R_A = 288 - 137.75$$

$$R_A = 147.25$$

## SHEAR FORCE:-



147.25



$$\sum f_y = 0 \uparrow + (\text{At } 6\text{ft from left support})$$

$$- V_{6\text{ft}} + 147.25 - 28.5 \times 6 = 0$$

$$V_{6\text{ft}} = -23.75 \text{ lb}$$

$$\sum f_y = 0 \quad (\text{At } 10\text{ft from left support})$$

$$147.25 - 28.5 \times 6 - 114 - V_{10\text{ft}} = 0$$

$$- 137.75 - V_{10\text{ft}} = 0$$

$$= V_{10\text{ft}} = \cancel{137.75} = 137.75$$

$$V_{10\text{ft}} = -137.75$$

## BENDING MOMENT:-

$$\sum M_{6\text{ft}} = 0$$

$$\sum M_{6\text{ft}} = -147.25(6) + 28.5(6)(6/3)$$

$$\sum M_{6\text{ft}} = 541.5$$

Now we have to find moment  
at 3ft

$$\sum M_{3\text{ft}} = 0$$

$$\sum M_{3\text{ft}} = -147.25(3) + 28.5(6)(3)$$

$$= 71.25$$

4 ft

Now we have to find the moment at change point

$$\frac{147.25}{x} = \frac{23.75}{(6-x)}$$

$$147.25(6-x) = 23.75(x)$$

$$853.5 - 147.25x = 23.75x$$

$$853.5 = 23.75x + 147.25x$$

$$853.5 = 171x$$

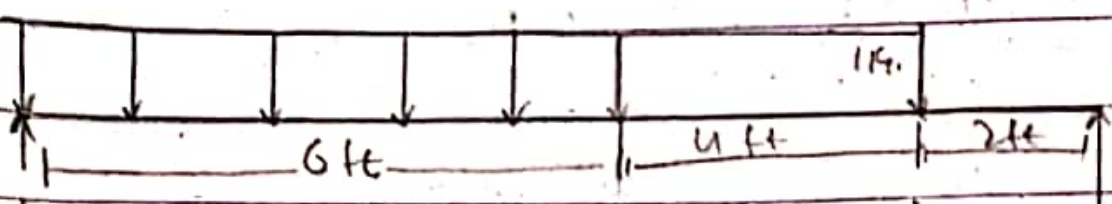
$$x = \frac{853.5}{171} = 4.991 \text{ ft}$$

$$\sum M_{4.991} = 0 +$$

$$\sum M_{4.991} = -147.25(4.991) + 23.75(6)(4.991)$$

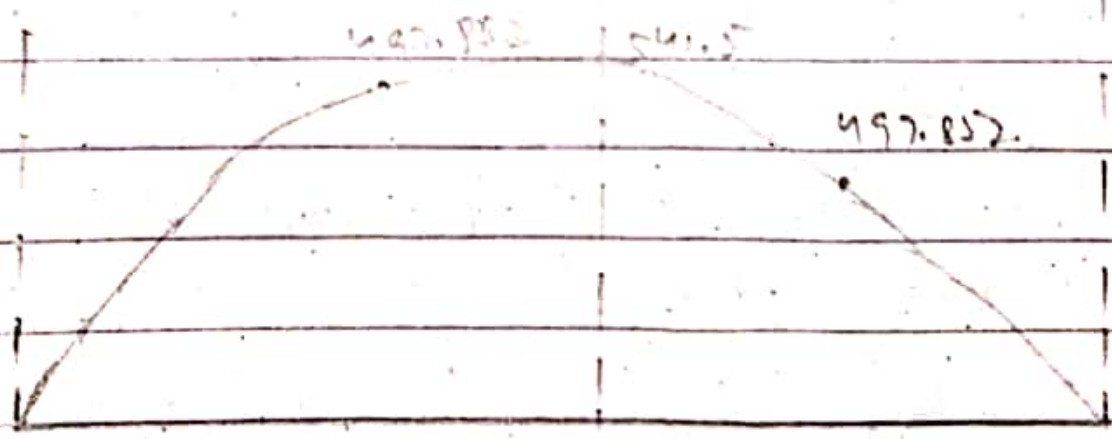
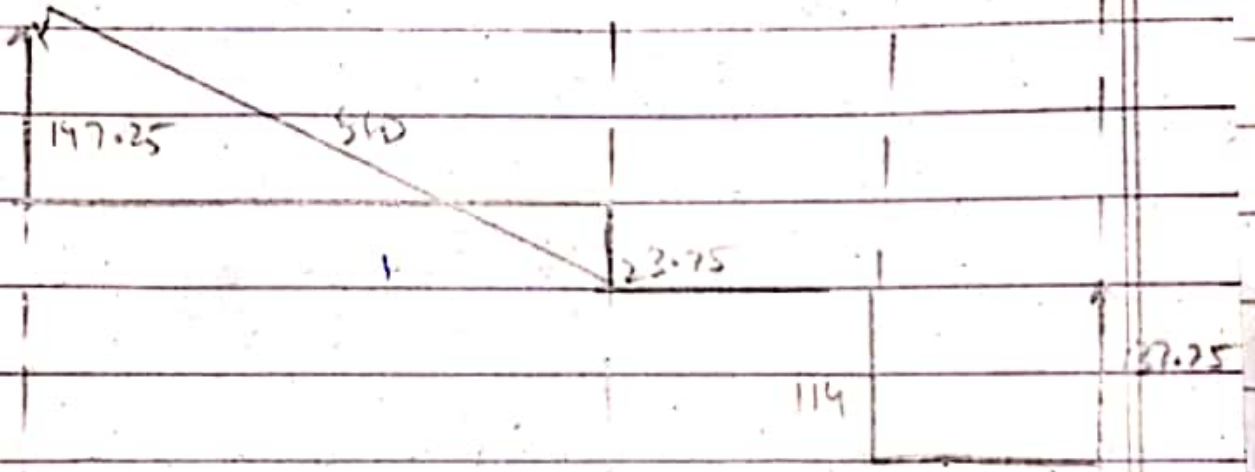
$$\sum M_{4.991} = 497.852 \text{ lb-ft}$$

4(B)



$R_A = 147.25$

$R_B = 137.25$



# MOMENT OF INERTIA-

$$y_1 = 5.5$$

$$y_2 = 3$$

$$y_3 = 0.5$$

$$A_1 = 4 \text{ inch}^2$$

$$A_2 = 4 \text{ inch}^2$$

$$A_3 = 4 \text{ inch}^2$$

$$\text{Now, } \bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{4 \times 5.5 + 4 \times 3 + 4 \times 0.5}{4 + 4 + 4}$$

$$\boxed{\bar{y} = 3}$$

$$I_1 = \frac{bh^3}{12}$$

$$I_3 = \frac{b \times h^3}{12}$$

$$I_1 = \frac{4 \times 1^3}{12}$$

$$I_3 = \frac{4 \times 1^3}{12}$$

$$I_1 = 0.33 \text{ inch}^4$$

$$I_3 = 0.33 \text{ inch}^4$$

$$I_2 = \frac{b^3 \times h}{12}$$

$$I_2 = \frac{4^3 \times 1}{12}$$

$$I_2 = 5.33 \text{ inch}^4$$

d
---

$$d_1 = \bar{y}' - y_1$$

$$d_1 = 3 - 5.5$$

$$d_1 = -2.5$$

$$d_2 = \bar{y}' - y_2$$

$$d_2 = 3 - 3$$

$$d_2 = 0$$

$$d_3 = \bar{y}' - y_3$$

$$d_3 = 3 - 0.5$$

$$d_3 = 2.5$$

$A d^2$
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$$\textcircled{1} A_1 d_1^2$$

$$= 4 \times (-2.5)^2$$

$$= 25 \text{ inch}^4$$

$$\textcircled{2} A_2 d_2^2$$

$$= 4 \times 0$$

$$= 0$$

$$\textcircled{3} A_3 d_3^2$$

$$= 4 \times (2.5)^2$$

$$= 25 \text{ inch}^4$$

$$\frac{I}{I_x} = I_1 + A_1 d_1^2$$

$$\frac{I}{I_x} = 0.33 + 25$$

$$\frac{I}{I_x} = 25.33 \text{ inch}^4$$



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$$I_{3x} = I_3 + A_3 d_3^2$$

$$I_{3x} = 0.33 + 25$$

$$I_{3x} = 25.33 \text{ inch}^4$$

Now

$$I_{xx} = I_{1x} + I_{2x} + I_{3x}$$

$$I_{xx} = 25.33 + 5.33 + 25.33$$

$$I_{xx} = 56 \text{ inch}^4$$

## SHEAR STRESS:-

$$\text{For } V = 137.75$$

Case No 1:-

$$\begin{aligned} \tau_{\text{top fiber}} &= \frac{VQ}{Ib} \\ &= \frac{137.75 \times 0}{56} \\ &= 0 \text{ psi} \end{aligned}$$

Case No 2:- (for 1 inch below top fiber)

$$\begin{aligned} \tau_{2A} &= \frac{137.75 \times 10}{56(4)} \quad (\because b=4) \\ &= 6.79 \text{ psi} \end{aligned}$$

Case No 3:-

$$\tau_{2B} = \frac{137.75 \times (10)}{56(1)}$$

$$= 29.59 \text{ psi} \quad \because Q = \bar{y} \times A$$

$$\bar{y} = 2 \times \frac{1}{2}$$

$$\bar{y} = 2.5$$

$$A = 4 \times 4$$

$$Q = 4 \times 2.5$$

$$Q = 10$$

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Case No 3: (Stress At. Centroidal Axis)

$$\tau_{max} = \frac{VQ}{Ib}$$

$$Q = Q_1 + Q_2$$

$$Q = 10 + 1(2)$$

$$= 12$$

Sol:-

$$\tau_{max} = \frac{137.75 \times 12}{56}$$

$$= 29.517 \text{ psi}$$

Case No 4A:- (1 inch above bottom)

$$\tau_{2A} = \frac{VQ}{It} \quad \because b=4$$

$$= \frac{137.75 \times 10}{56 \times 4}$$

$$= 6.149 \text{ psi}$$

Case No 4B:-

$$\tau_{2B} = \frac{VQ}{It}$$

$$= \frac{137.75 \times 10}{56 \times 1} \quad \because b=1$$

$$= 24.598 \text{ psi}$$

$$= 24.598 \text{ psi}$$

Case No: 5 (At Bottom fiber)

$$\tau = \text{Bottom} = VQ$$

It

$$\therefore \tau_{\text{Bottom}} = \frac{137.75 \times 0}{56(1)}$$

$$\therefore = 0 \text{ psi}$$

Case 6:

shear force at a distance of 3ft from left support of beam along its length.

$$V_{3ft} = -23.75$$

$$Q = 12$$

$$\tau_{\text{max}} = \frac{23.75 \times 12}{56(1)}$$

$$= 5.089 \text{ psi}$$

Case 7: (At a distance 6 inches below)

$$\text{For } b = 4 \quad \tau_A = \frac{23.75 \times 10}{56(1)} = \boxed{1.058 \text{ psi}}$$

For  $b = 1$

$$\tau_B = \frac{23.75 \times 10}{56(1)} = 4.241 \text{ psi}$$

①

## FLEXURAL STRESS:

$$\sigma = \frac{My}{I}$$

$$\text{Moment} = \dots \dots \dots 541.5$$

$$\text{Moment of Inertia } I = 56$$

Case No 1:- (stress at top fiber)

$$\begin{aligned} \sigma_{\text{TOP}} &= \frac{My}{I} \\ &= \frac{541.5 \times 3}{56} \end{aligned}$$

$$= 29.008 \text{ psi}$$

Case No: 2 (1 inch below top fiber)

$$\begin{aligned} \sigma_1 &= \frac{My}{I} \\ &= \frac{541.5 \times 2}{56} \end{aligned}$$

$$= \boxed{19.339} \text{ psi}$$

(12A)

Case No: 3: (At Geometrical Centroid)

$$\sigma_{\text{center}} = \frac{My}{I} \therefore y(0)$$
$$= 0 \text{ psi}$$

Case No: 4: (1 inch above the bottom)

$$\sigma = \frac{My}{I}$$
$$= \frac{541.5 \times 2}{56}$$
$$= 19.339 \text{ psi}$$

Case No: 5: (At bottom Fiber)

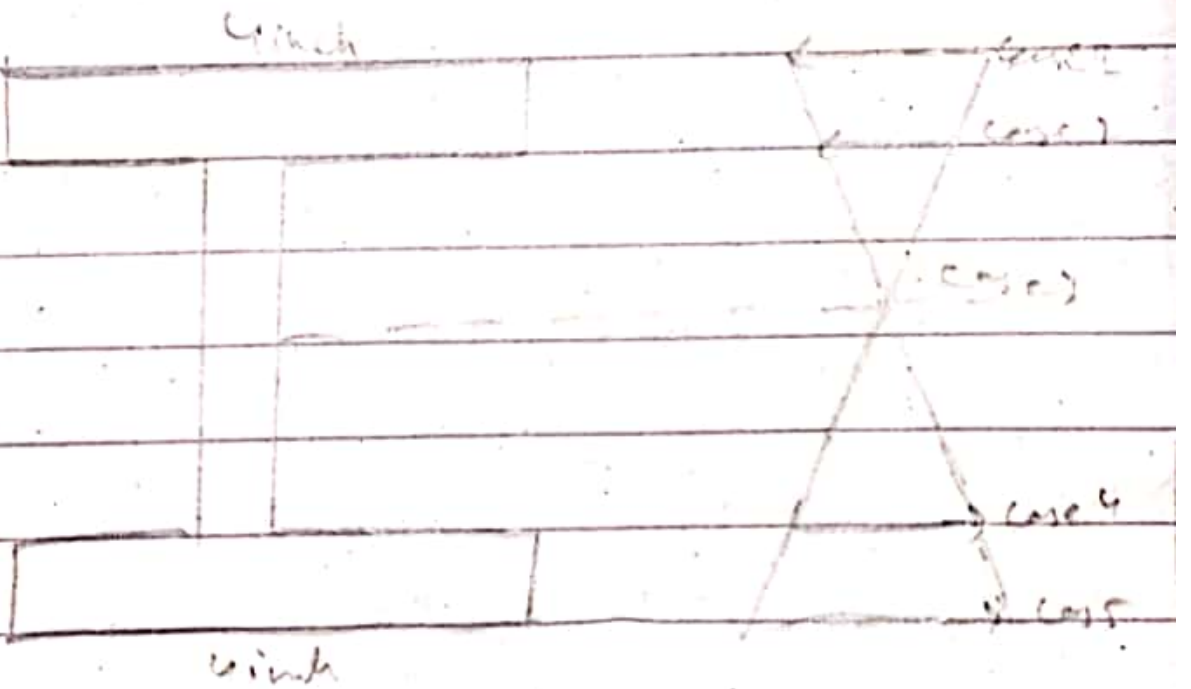
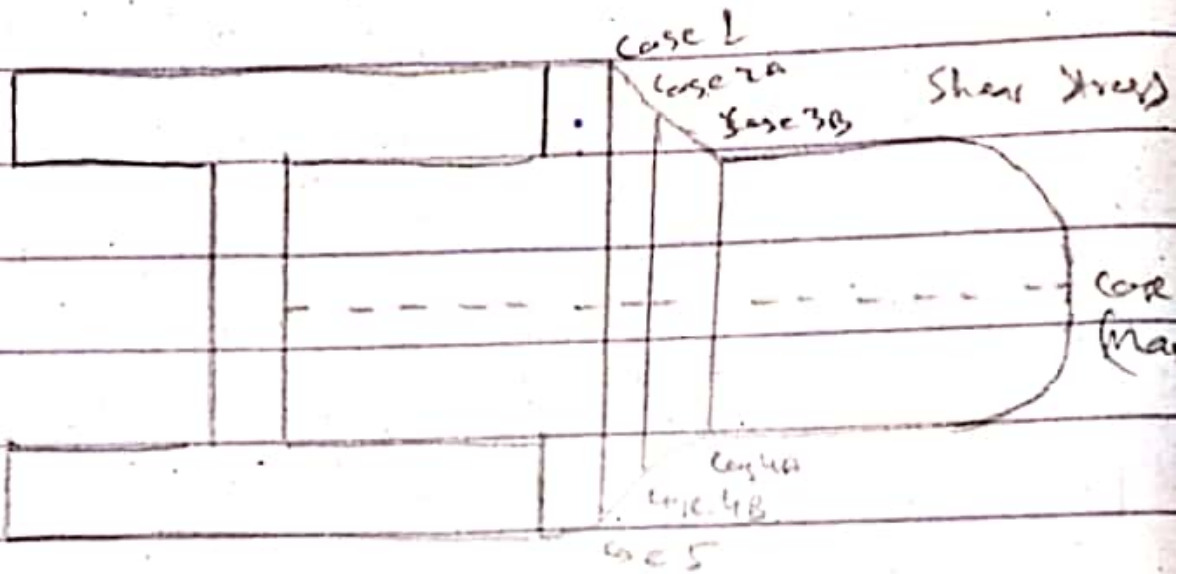
$$\sigma_{\text{Bottom}} = \frac{My}{I}$$
$$\sigma_{\text{Bottom}} = \frac{541.5 \times 3}{56}$$
$$= 29.008 \text{ psi}$$

(12B)

# SHEAR FORCE AND BENDING STRESS

## Diagram

### VARIATION:-



\* Stress state of a point element:

We find all stresses, which is acting on the I section beam. Given stress state condition is at point c which is the center of Uell at 3ft

Thus Flexural stress at point "c"

$\sigma_u = 19.339 \text{ psi}$   $\therefore$  from Case 2, point c lies in this case At 1 inch below top fiber.

Now shear stress at point "c"

$\tau_{xy} = 1.2090 \text{ psi} =$   $\therefore$  from Case No 7.

consider this point "c" is a planar element.

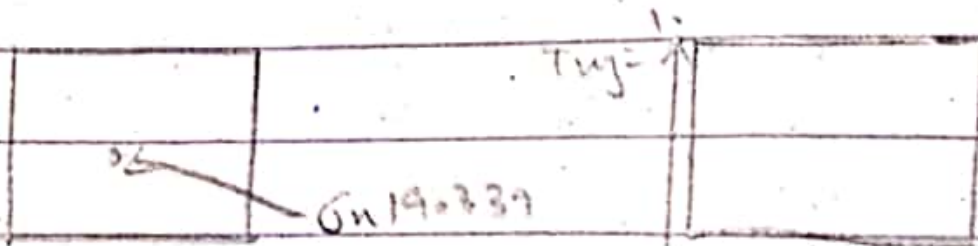


(9)

$\sigma_x = -19.339$  Compressive

$-19.339$  is compressive because its point lies in the center and in the compression zone of beam cross section.

• If point C lies below the centroid then stress would be tensile.

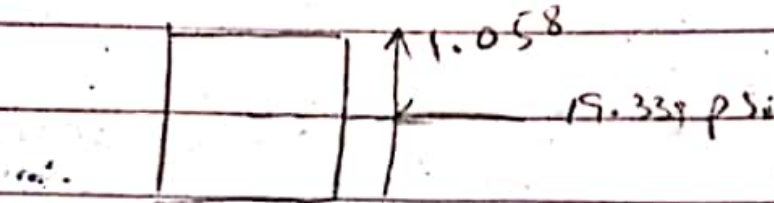


Combine stress in 2D element.



(15)

## Stress Transformation:



let assume  $\theta = -15^\circ$   
for new orientation.

$\theta = -15$   
(clockwise)

Now

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma'_x = \frac{-19.339 + 0}{2} + \frac{-19.339 - 0}{2} \cos 2(-15)$$

$$+ 1.058 \{ \sin 2(-15) \}$$

$$= -9.699 + -9.699(0.866) + 1.058(-0.5)$$

$$= -18.614 \text{ psi}$$

$$\sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

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$$\begin{aligned}\sigma_y &= \left( \frac{-19.399 + 0}{2} \right) - \left( \frac{-19.399 - 0}{2} \right) \cos 2(-15) \\ &\quad - 1.058 \{ \sin 2(-15) \} \\ &= 9.699 + 9.699(0.866) - 1.058(-0.5) \\ &= -0.753 \text{ psi}\end{aligned}$$

$$\begin{aligned}\tau_{x'y'} &= -\left( \frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\left( \frac{19.399 - 0}{2} \right) \sin 2(-15) + 1.058 \{ \cos 2(-15) \} \\ &= +9.699(0.5) + 1.058 \{ 0.866 \}\end{aligned}$$

$$\tau_{x'y'} = 5.750 \text{ psi} - 4.147 \text{ psi}$$

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## PRINCIPLE STRESS:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_x = -19.399, \quad \sigma_y = 0, \quad \tau_{xy} = 1.058$$

$$\tan 2\theta_p = \frac{1.058}{(19.399 - 0)/2}$$

$$= \frac{1.058}{-9.699}$$

$$= -0.109$$

$$\tan \theta_p = \frac{-0.109}{2}$$

$$= -0.05$$

$$\theta_p = \tan^{-1}(0.05)$$

$$\theta_p = -2.86^\circ$$

Now

$$\sigma_n' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_n' = \left( \frac{19.399 + 0}{2} \right) + \left( \frac{-19.399 - 0}{2} \right) \cos 2(-2.86^\circ)$$

$$+ 1.058 \left\{ \sin 2(-2.86^\circ) \right\}$$

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$$= -9.699 - 9.699(0.99) + 1.058(-0.09)$$

$$= -19.366 \text{ psi}$$

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \left( \frac{-19.399 + 0}{2} \right) - \left( \frac{-19.399 - 0}{2} \right) \cos 2(-2.86)$$

$$- (1.058) \{ \sin 2(-2.86) \}$$

$$= -9.699 + 9.699(0.99) - 1.058(-0.09)$$

$$= -0.001 \text{ psi}$$

(9)

SHEAR STRESS:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_s = \frac{-((-19.399 - 0)/2)}{1.058}$$

$$= \frac{-(-19.399 - 0)/2}{1.058}$$

$$\tan 2\theta_s = \frac{9.699}{1.058} = \frac{18.335}{2}$$

$$\tan \theta_s = 9.167$$

$$\theta_s = \tan^{-1}(9.167)$$
$$= 74.6^\circ$$

Now  $\tau_{xy}' = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$

$$\tau_{xy}' = \left( \frac{-19.399 - 0}{2} \right) \sin 2(74.6^\circ) + 1.058$$

$$\left\{ \cos 2(74.6^\circ) \right\}$$

$$= 9.699(0.419) + 1.058(-0.907)$$

(20)

$$\tau_{xy} = 3.09 \text{ psi}$$

## MOHR'S CIRCLE:-

Mohr's Circle. Center Coordinates.

$$(h, k) = \left[ \frac{\sigma_x + \sigma_y}{2}, 0 \right]$$

$$= \left[ \frac{-19.399 + 0}{2}, 0 \right]$$

$$= \left[ -9.699, 0 \right]$$

$$\text{Radius, } r = \sqrt{\left( \frac{-19.399 - 0}{2} \right)^2 + (1.058)^2}$$

$$= \sqrt{(-9.699)^2 + (1.058)^2}$$

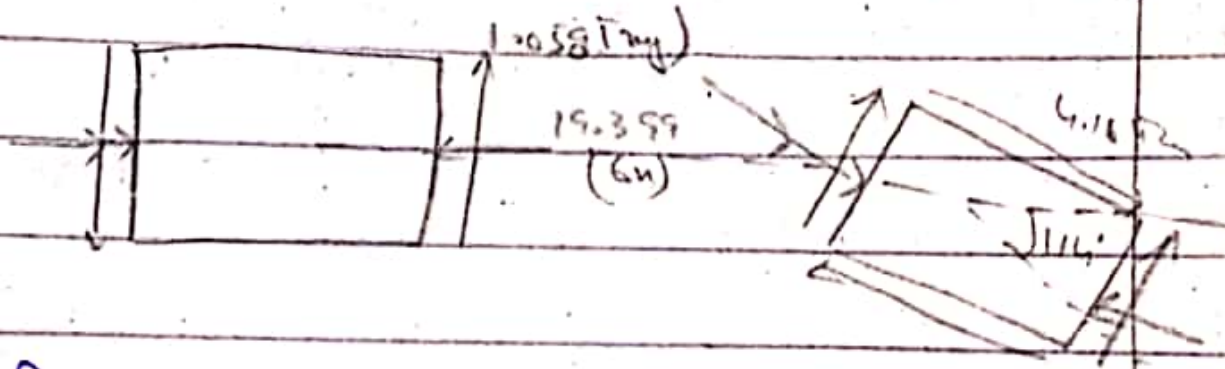
$$\text{Radius} = 8.699 \text{ psi}$$

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### Result Comprasion:-

$$\theta = 0^\circ$$

$$\theta = 15^\circ \text{ (clockwise)}$$



### PRINCIPLE AND SHEAR STRESS:-

$$\theta = 1^\circ \text{ (Anti clockwise)}$$

$$\theta = 44.68^\circ$$

