Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th

Instructor: Sir Pir Mehar **Total Marks:** 50

Student Details

Name: Rimsha Khan **Student ID: 13672**

Q1.	(a)	Determine the response y(n), n ≥ 0, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks 7** **CLO 2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks 7** **CLO 2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks 6** **CLO 2**
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z \rvert > \lvert a \rvert$$	**Marks 6** **CLO 2**

Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1 - pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$.	**Marks 6** **CLO 3**
	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	**Marks 6** **CLO 3**
Q 4	(a)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \ge L$	**Marks 6** **CLO 2**
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$ $$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$	**Marks 6** **CLO 2**

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SUBJECT :- DSP SUBMITTED To :- PIR MEHER

QUESTION 1 :-

PART A :-

a) Determine the response $y(n), n \geq 0$ of the system described by the second order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

To the input $x(n) = (-1)^n u(n)$. And the initial condition are $y(-1)$ and $y(-2) = 0$

Solution :-

First we find $y(n)$.

The characteristic equation is.

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2$$

Hence,

$$y(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is.

$$y_p(n) = K(-1)^n u(n)$$

Substituting this solution into the difference equation, we obtain.

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$,

$$K(1+4+4) = 2 \Rightarrow K = \frac{2}{9}$$

$$\Downarrow$$

$$K(9)$$

$$K = \frac{2}{9}$$

The total solution is.

$$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n \right] u(n)$$

$\downarrow$

equation of $y(n)$.

But values of C_1 and C_2 is not found.

So,

For finding C_1 and C_2.

We take initial conditions from the question.

From initial conditions we obtain $y(0) = 1$, $y(1) = 2$, then

$$C_1 + \frac{2}{9} = 1$$

$$\Rightarrow \boxed{C_1 = \frac{7}{9}}$$

$$2C_1 + 2C_2 - \frac{2}{9} = 2$$

$$\Rightarrow \boxed{C_2 = \frac{1}{3}}$$

putting values of C_1 and C_2 in equation.

So,

$$\boxed{y(n) = \left[\left(\frac{7}{9}\right) 2^n + \left(\frac{1}{3}\right) n 2^n + \frac{2}{9}(-1)^n \right] u(n)}$$

PART B :-

b, Determine the impulse response and unit step response of the system described by the difference equation.

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Solution :-

The characteristic equation is.

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{5}$$

Hence

$$y(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{5}^n$$

With $x(n)$ we have.

$$y(0) = 2$$

$$y(1) = -0.7y(0)$$

$$= 0 \Rightarrow y(1) = 1.4$$

Hence $C_1 + C_2 = 2$

and

$$\frac{1}{2}C_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$\Rightarrow C_1 + \frac{2}{5}C_2 = \frac{14}{5}$$

These equations yield

$$C_1 = \frac{10}{3}, \quad C_2 = -\frac{4}{3}$$

So,

$$\boxed{h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)}$$

Impulse response

Now,

For step response.

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$\boxed{= \frac{10}{3} \left(\frac{1^n}{2} \left(2^{n+1} - 1\right) u(n) - \frac{1}{3} \left[\frac{1}{5} \left(5^{n+1} - 1\right) u(n)\right.\right.}$$

Unit step response.

Question 2:-
PART A:-
a, Determine the causal signal $x(n)$ having the Z-transform.

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol:-

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

By partialy factorized factors are

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$x(z) = A \frac{(1-2z^{-1})}{(1-z^{-1})} B + \frac{(1-2z^{-1})}{(1-z^{-1})^2}$$

when evaluated at $z = -1$.
As apply this $z = -1$ limit we get:-

$A = 4$

Similarly $B = -3$

and $C = -1$.

So putting values of A, B and C.
we get.

$$\boxed{x(n) = [4(2)^n - 3 - n]\, u(n)}$$

PART B :-

b, Evaluate the inverse Z-transform using the complex inversion integral?

$$X(z) = \frac{1}{1-az^{-1}} \qquad |z| > |a|$$

Solution :-

We have

$$x(n) = \frac{1}{2\pi j} \oint \frac{z^{n-1}}{C_1 - az^{-1}} dz = \frac{1}{2\pi j} \oint \frac{z^n dz}{cz-a}$$

Where c is the circle at radius greater than $|a|$. We shall evaluate this integral with $f(z) = z^n$. We distinguish two cases.

1, If $n \geq 0$, $f(z)$ has only zeros and hence no poles inside C. The only pole inside C is $z = a$. Hence

$$x(n) = f(z_0) = a^n \qquad n \geq 0$$

2, If $n < 0$, $f(z) = z^n$ has an n^{th} order pole at $z = 0$, which is also inside C. Thus there are contributions from both places or poles. For $n = 1$ we have.

$$x(-1) = \frac{1}{2\pi j} \oint \frac{1}{C_z(z-a)} dz$$

$$= \frac{1}{z-a}\bigg|_{z=0} + \frac{1}{z}\bigg|_{z=a} = 0$$

If $n = -2$ we have.

$$x(-2) = \frac{1}{2\pi j} \oint \frac{1}{cz^2(z-a)} dz$$

$$= \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By continuing the same way we show that $x(n) = 0$ for $n < 0$. Thus

$$\boxed{x(n) = a^n u(n)}$$

QUESTION 3:-

PART A:-

A two pole low-pass filter has the system response.

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

Determine the values of b_0 and p such that the frequency response $H(\omega)$ satisfies that condition $H(0) = 1$ and $\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$.

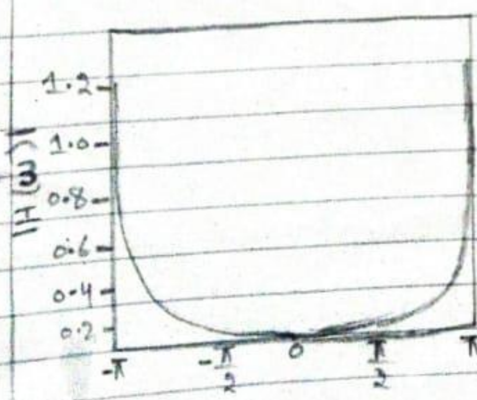
Solution:-

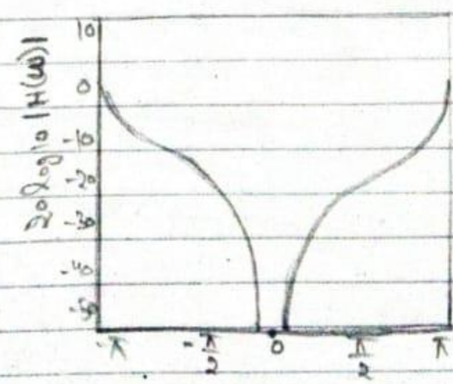
At $\omega = 0$ we have

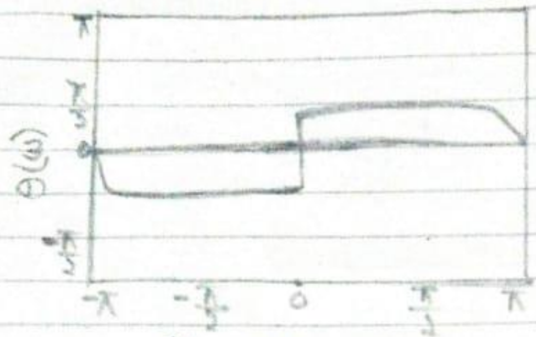
$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$







At $\omega = \frac{\pi}{4}$

$$H = \left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4)+jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2}+jp/\sqrt{2})^2}$$

Hence

$$= \frac{(1-p)^4}{[(1-p/\sqrt{2})^2+p^2/2]^2} = \frac{1}{2}$$

or equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p = 0.32$, satisfies the equation. So the system function for desired filter is.

$$\boxed{H(z) = \frac{0.46}{(1-0.32z^{-1})^2}}$$

PART B :-

b, Design a bandpass filter that has centre of its pass band at $\omega = \pi/2$ zero in its frequency response at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.

Solution :-

The filter must have poles at.

$$P_{1,2} = re^{\pm j\pi/2}$$

and zeros at $z = 1$ and $z = -1$.

The system function is.

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{(z^2-1)}{(z^2-r^2)}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \frac{\pi}{2}$. Thus we have

$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1-r^2} = 1$$

$$\Rightarrow G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have.

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= \frac{1}{2}$$

or equivalently

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ satisfies the equation so the system filter for the desired function is.

$$\boxed{H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}}$$

PART A:- QUESTION 4.

a, A finite duration sequence of length L is given as.

$$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Determine the N-pole or N-point DFT of this sequence for $N \geq L$.

Solution :-

The fourier transform of this sequence is.

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

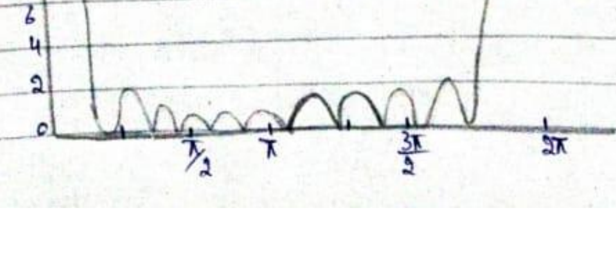
$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1/2)}$$

The magnitude and phase of $X(\omega)$ are illustrated. for $L = 10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally speed frequencies $\omega_K = 2\pi K/N$. $K = 0, 1, \ldots, N-1$.

Hence

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}} \quad (K = 0, 1, \ldots N-1)$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$



If N is selected such that $N = L$ then the DFT becomes.

$$X(K) = \begin{cases} L, & K = 0 \\ 0, & K = 1, 2 \cdots L-1 \end{cases}$$

QUESTION 4:-

PART B:-

b, Perform the circular convolution of the following two sequencies. Solve the problem step by step.

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Solution:-

Each sequence consist of four non-zero points. Now $X_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified by. Begining with $m=0$ we have.

$$X_3(0) = \sum_{n=0}^{3} x_1(n)x_2((-n))_N$$

$x_2((-n))_4$ is simply the sequence $x_2(n)$ folded and graphed on circle.

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$ point by point. We sum the values in product sequence to obtain.

$$\boxed{X_3(0) = 14}$$

For $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)x_2((1-n))_4$$

Now again sum the product values we get.

$$\boxed{X_3(1) = 16}$$

Now for $n=2$ we have.

$$x_3(2) = \sum_{n=0}^{3} x_1(n)x_2((2-n))_4$$

Now $x_2((2-n))_4$ is folded sequence.
So Summing the four terms in product Sequences.

$$\boxed{x_3(2) = 14}$$

For $m = 3$

$$x_3(3) = \sum_{n=0}^{3} x_1(n)x_2((3-n))_4$$

The folded Sequence is now $x_2((-n))_4$. Three units yield in time $x_2((3-n))_4$. So the product sum of this sequence is

$$\boxed{x_3(3) = 16}$$

We observe that if the computation above is continued beyond $m = 3$, we simply repeat the sequence of four values we obtained. Therefore the circular convolution of two Sequences $x_1(n)$ and $x_2(n)$.

$$\boxed{x_3(n) = \left\{ \underset{\uparrow}{14}, 16, 14, 16 \right\}}$$

THE END