INTRODUCTION TO STRUCTURAL DYNAMICS & EARTHQUAKE ENGINEERING

Submitted by:

Osama Habib

ID: 7703

Section : C

Submitted to:

Engr. Yaseen Mahmood

IQRA NATIONAL UNIVERSITY PESHAWAR

Question No. 1

Given data:-

- A beam is pulled in a downward direction = 1/2 inch.
- Ignoure the self weight of the beam as well as the damping effect.
- $E = 24000$ ksi
- $I = 150 \text{ in}^4$
- δ_{st} = Defleition due to 7703 lb static load

Required data:-

- Natural time period of system = ?
- Develop and slove equation of motion for vibration.

Sol :-

As we know that

The general EOM for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

In our case system is undamped ($c=0$) undergoing free vibration ($P(t)=0$)

Hence

The general EOM becomes;

$$ku = m\ddot{u} = 0 \rightarrow (2)$$

Here

$$K = 3EI$$

$$K = \frac{3 \times (29000) \times (150)}{(10 \times 12)^5}$$

$$K = 7.55 \text{ k/in}$$

Also,

$$K = 7.55 \text{ k/in} \times 1000 \times 12$$

$$K = 90625 \text{ lb/ft}$$

Similarly;

$$m = \frac{7703}{32.2} = 239.223 \text{ slug}$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{239.223}}$$

$$\omega_n = 19.463 \text{ rad/sec}$$

For natural time period of systems

$$T_n = \frac{2\pi}{\omega_n} = ~~2\pi/54.9 = 0.114 \text{ sec}~~$$

$$= \frac{2\pi}{19.463} = \boxed{T_n = 0.323 \text{ sec}}$$

Substituting the corrosponds value in equ (1)

we get

$$90625\,u + 239.19\,\ddot{u} = 0$$

where "K" is in lb/ft and m is in lb sec/ft²

General solution to the EOM for undamped free vibration is

$u(t) = u(0)\cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t)$

$u(t) = 1/2'' = 1/24$ ft and $\dot{u}(0) = 0$

$u(t) = 1/24 \times \cos(19.464t) + 0$

$= 1/24 \cos(19.464t)$

Equivalent static forces at any time "t" is

$f_s(t) = k \cdot u(t) = \dfrac{90625 \times \cos(19.464t)}{24}$

$f_s(t) = 3776.041 \cos(19.464t)$

Amplitude of dynamic displacement, u_0 for undamped free vibration is

$u_0 = \sqrt{\left[(u(0))^2 + \left(\dfrac{\dot{u}(0)}{\omega_n}\right)^2\right]}$

$$v_0 = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$v_0 = 1/24$ ft

Amplitude of equilent static force,

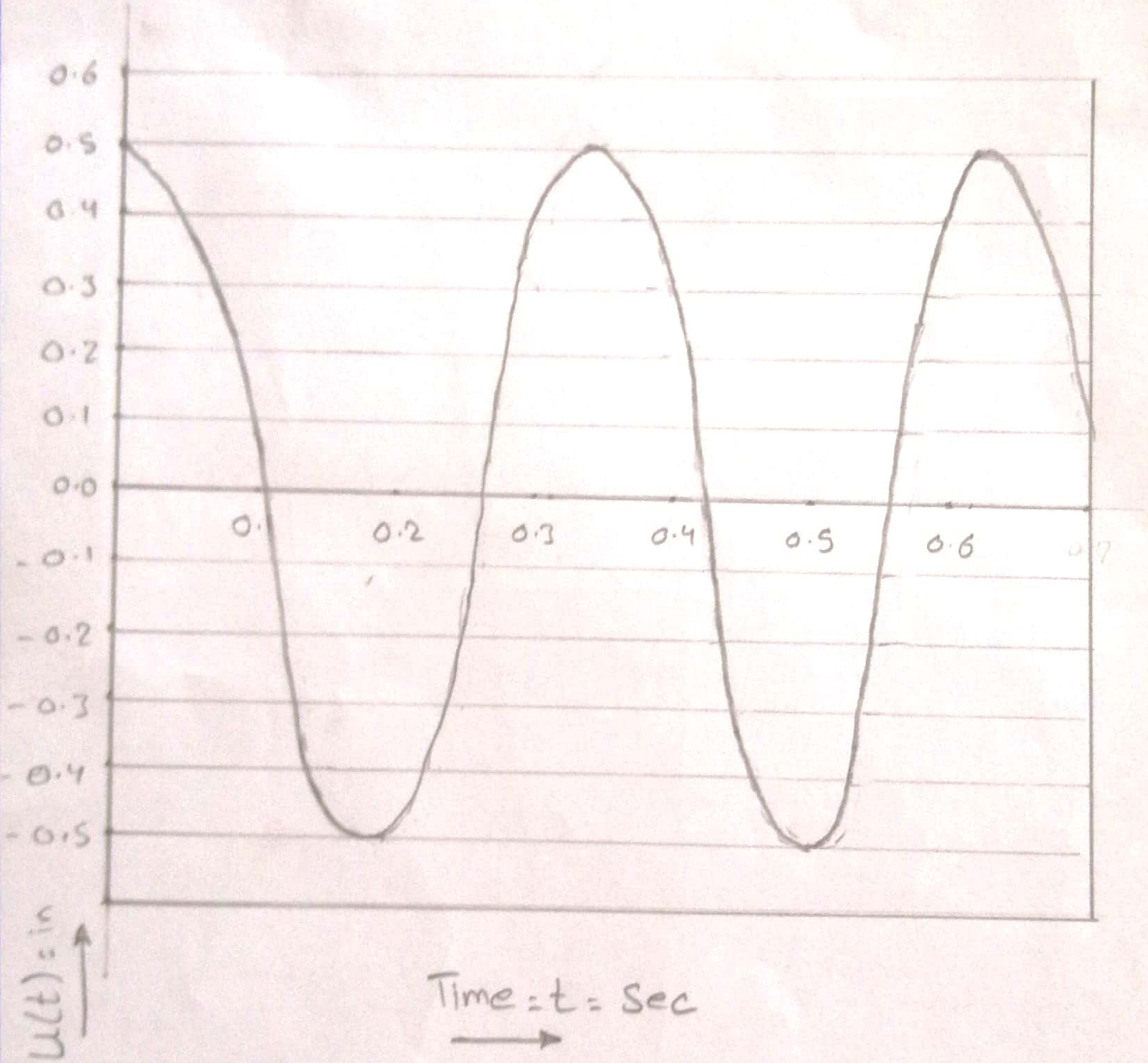
f_{so}

$k v_0 = 90625 \times 1/24 = 3776.04$ lb

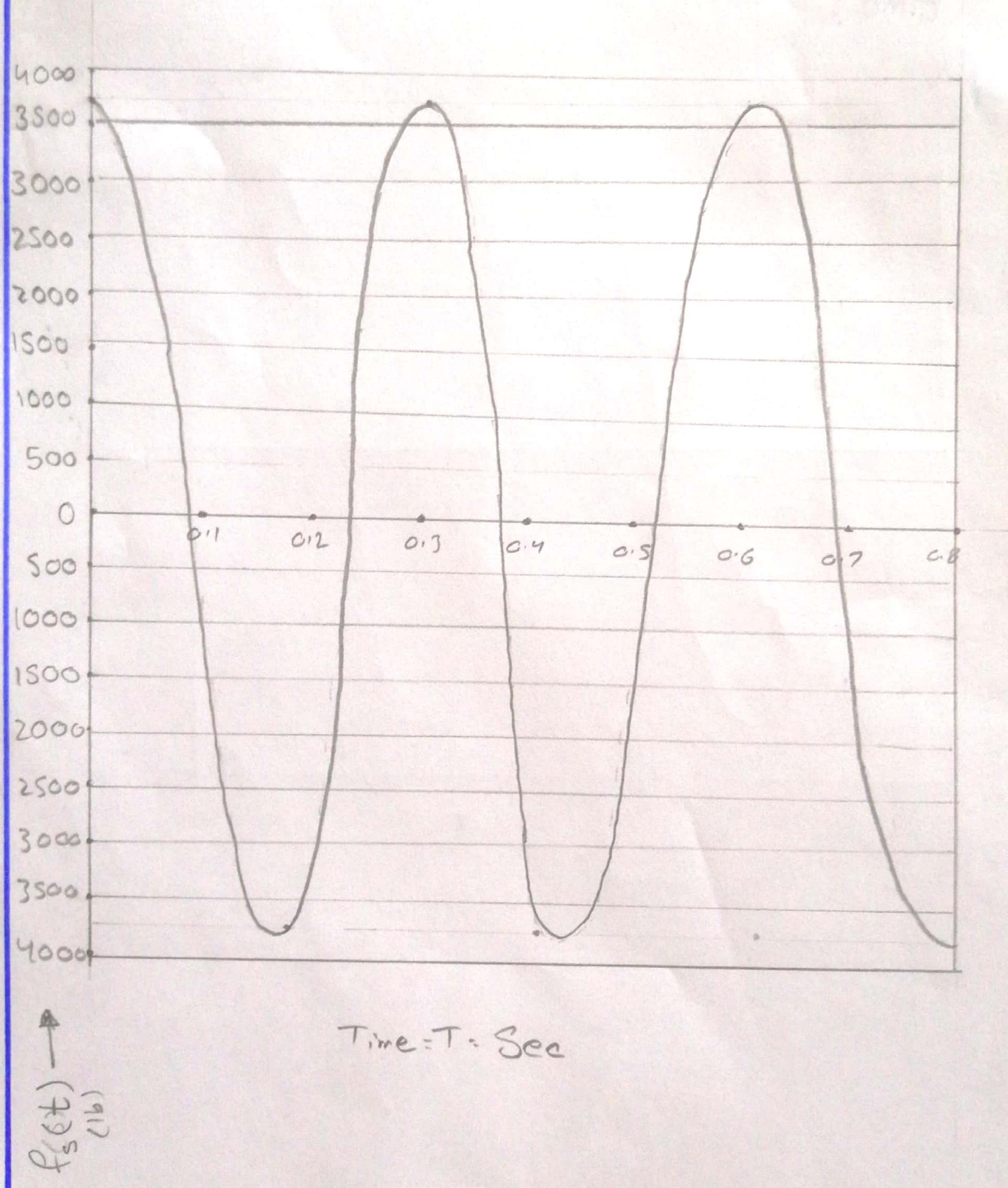
$$k v_0 = 3776.04 \text{ lb}$$

Graphs:

a- Showing Variation of displacement with time;



b. Showing Variation of Equivalent Static force with time :



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Question # 2:-

Given data:-

Damping ratio of renforce concrete with considerable cracking - 3-5 %

→ So we take $\xi = 3\%$

→ other data are taken from question no 1.

Required data

Develop and Solve the equation of motion for vibration at free end = ?

Develop an equation showing variation in equivalent state Force with time = ?

Sol:-

As we know that

EOM (equation of motion) for damped free vibration is;

$$ku + c\dot{u} + m\ddot{u} = 0 \rightarrow (1)$$

As we know that from question No 1 data

i.e

$k = 90625 \ lb/ft$

$m = 239.23 \ \frac{lb \ sec^2}{ft}$

$\omega_n = 19.463 \ rad/sec$

As we know that

$c = \xi * 2 m \omega_n$

$= 0.03 \times 2 \times 239.22 \times 19.463$

$c = 279.335 \ lb \ sec/ft$

By putting values in eq (1) we get

$90625u + 279.61\dot{u} + 239\ddot{u} = 0$

Solution to the EOM for damped free vibration is

$$u(t) = e^{-\xi \omega_n t} \left[u(0) \cos(\omega_D t) + \frac{1}{\omega_D} \left(\dot{u}(0) + u(0) \xi \omega_n \right) \sin(\omega_D t) \right]$$

Here;

$\omega_D = 19.464 \ rad/sec$

$$U(t) = e^{-(0.03 \times 19.464 \times t)} \left[\frac{1}{24} \times \cos(19.464t) + \frac{1}{19.464} \times \left[0 + \frac{1}{24} \times 0.03 \times 19.464 \times \sin(19.464) \right] \right]$$

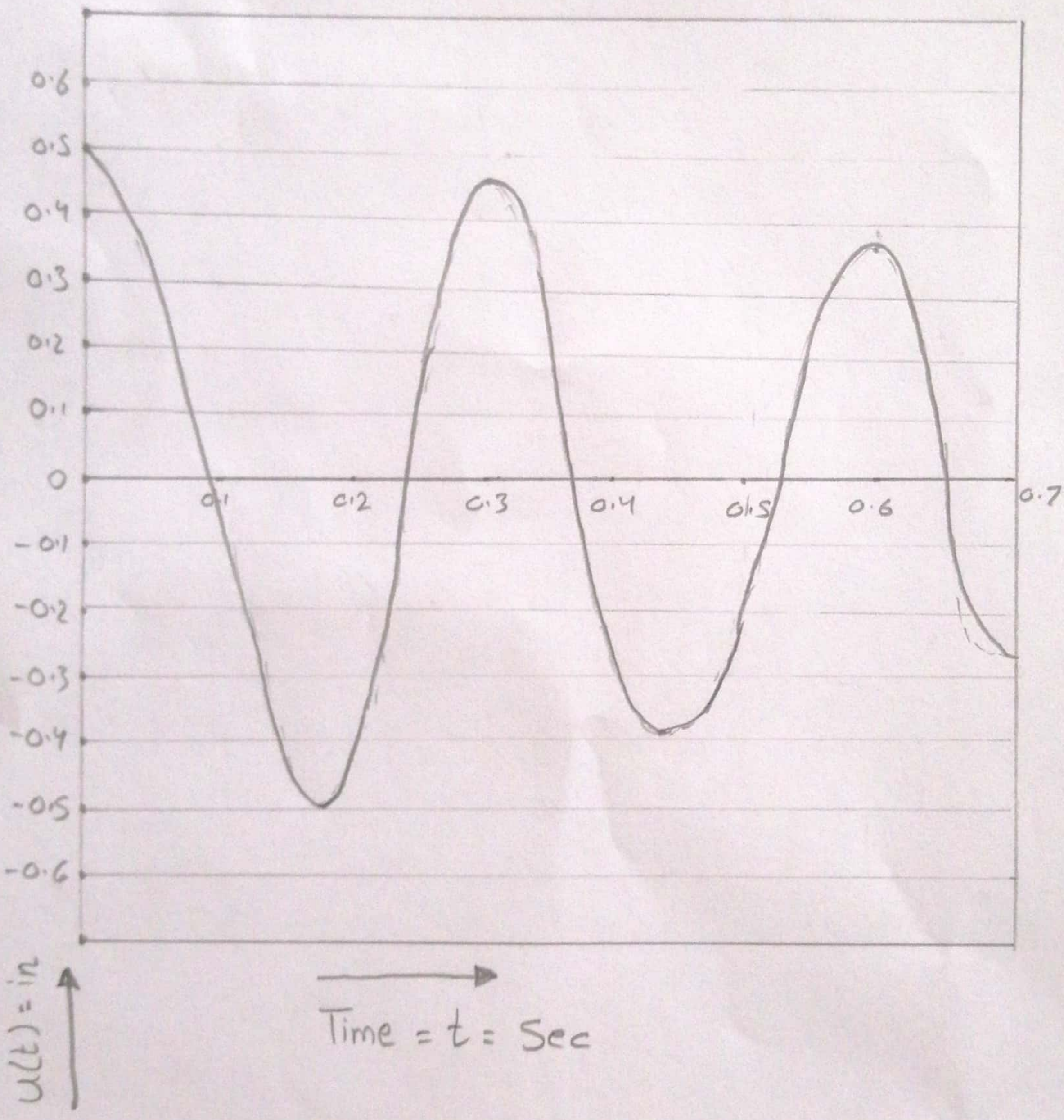
$$U(t) = e^{-(0.583t)} \left[0.0416 \times \cos(19.464t) + 0.024 \sin(19.464t) \right]$$

$$fs(t) = 90625 \times u(t)$$

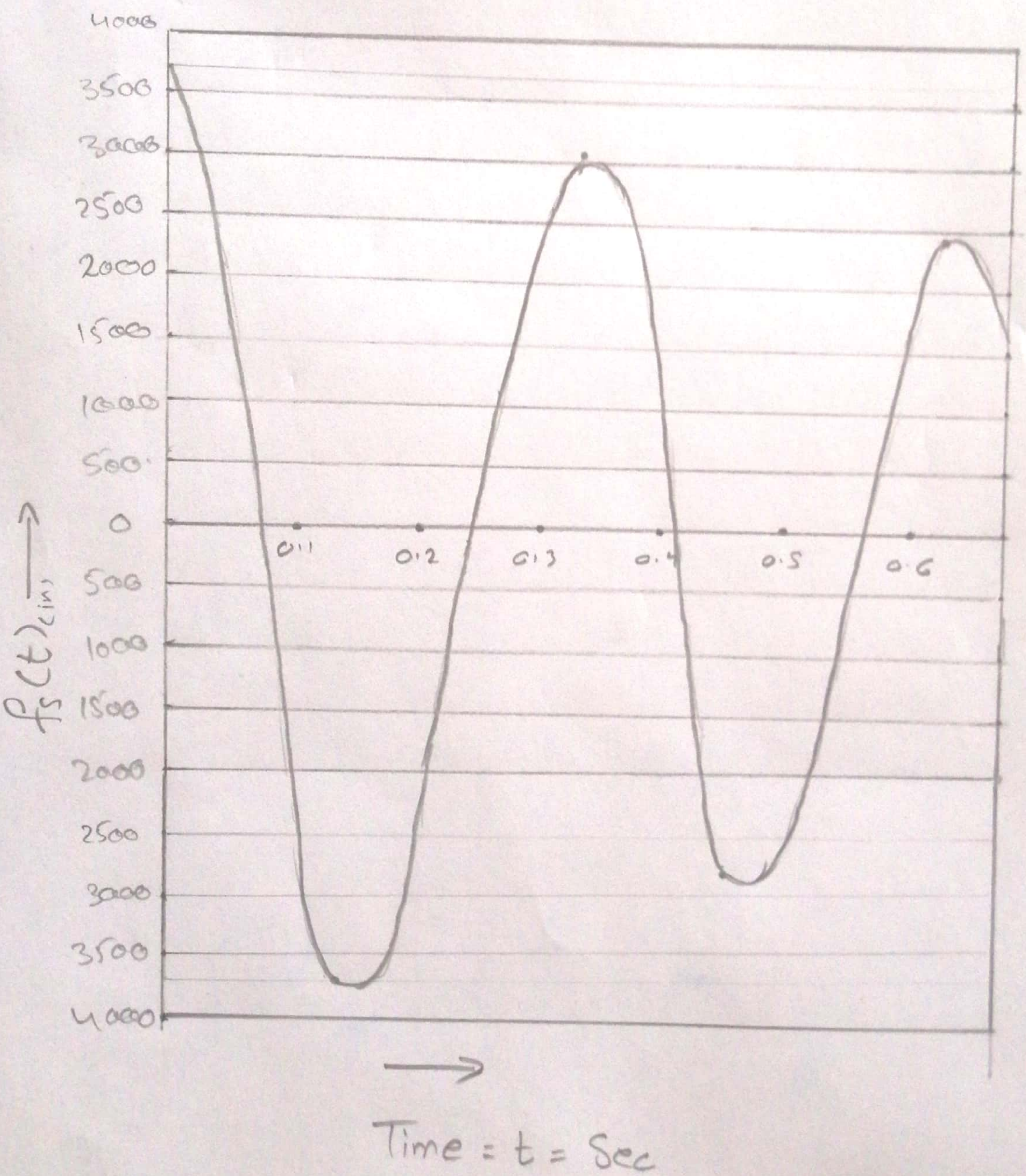
$$fs(t) = e^{-0.584t} \left[3806.25 \cos(19.464t) + 2175 \sin(19.464t) \right]$$

Graph:

a. Shows variation of displacement with time;



b. Showing Variation of Equivalent Static force with time:



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Question No. 03

Given data :-

- → Amplitude cable force = 60 kips
- → Horizental displacement of tank = $\frac{7703}{1000}$ = 7.703 in
- → Cycles = 7
- → cycle completion time = 3.57 sec
- → Amplitude of displacement = 2.286 cm = 0.9 in

Required data:-

Complete the following:

- → Damping ratios
- → Natural period of ~~under~~ undamped vibration
- → Stiffness of structures
- → Weight of tank
- → Damping co-efficient
- → Number of cycles to reduce the displacement amplitude to 0.5"

Sol:-

As given in question

$u_1 = 7.703$ in

After $J = 7$

$U_{J+1} = U_8 = 20.286$ cm $= 0.9$ in

a. ζ - Damping ratio = ?

$$J = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{J+1}}\right)$$

By putting values we get

$$7 = \frac{1}{2\pi\zeta} \ln\left(\frac{7.703}{0.9}\right)$$

$$\zeta = \frac{1}{2\pi 7} \ln\left(\frac{7.702}{0.9}\right)$$

$\zeta = 0.0488 = 4.88\%$ Ans

$\zeta = 4.88\%$

b. natural period of undamped vibration

According to question 7 cycles are completed in 3.57 sec

Time period to complete one cycle $\frac{3.57}{7}$

$T_0 = 0.51$ sec

As we know that

$$\omega_0 = \omega_n \sqrt{(1-\zeta)^2}$$

$$\frac{2\pi}{\omega_0} = 2\pi / \omega_n \sqrt{(1) - (\zeta)^2}$$

$$T_0 = T_n \sqrt{1-(\zeta)^2}$$

$$T_n = T_0 \sqrt{(1) - (\zeta)^2}$$

By putting values we get

$$T_n = 0.5 \sqrt{(1) - (0.0488)^2}$$

$$T_n = 0.509$$

$$T_n = 0.509 \text{ sec}$$

c = stiffness of structure = k

As we know that

$$k = \frac{60 \cos(60)}{7.703}$$

k = 3.89 k/in

k = 46741.10

d = weight of tank = w =

As we know that

$$w_n = \sqrt{k/m} = \sqrt{k/w_g}$$

$$= \sqrt{\frac{k \cdot g}{w}}$$

$$w_n^2 = \frac{k \cdot g}{w}$$

$$w = \frac{k \cdot g}{w_n^2} \rightarrow (a)$$

Also $w_n = \frac{2\pi}{T_n}$

equ

$$w = \frac{k \cdot g \times T_n^2}{4\pi^2}$$

By putting values we get

$$w = \frac{46741.10 \times 32.2 \times (0.51)^2}{4 \times \pi^2}$$

$w = 9530.925\ lb$

$w = 9.530\ k$

e. Damping co-efficient = c =

As we know that

$$\xi = \frac{c}{2m\omega_n}$$

$c = \xi \times 2m\omega_n = \xi \times 2m \times (2\pi / T_n)$

$$c = \frac{\xi \times 4 \times \pi \times m}{T_n}$$

By putting values we get

$$c = \frac{0.0488 \times 4 \times \pi \times \left(\frac{9530.925}{32.2}\right)}{0.51}$$

$c = 355.908\ lb \cdot sec / ft$

7. Number of cycles to reduce the displacement amplitude to 0.5" .5"

As we know that

$$j = \frac{1}{2\pi\xi} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$j = \frac{2}{2\times\pi\times0.0488} \ln\left(\frac{7.703}{0.5}\right)$$

$j = 8.918$ say 9 cycles

$j = 8.9 = 9$ cycles