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Section = A

Subject = differential equation

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Assignment No = 1

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① $x^4 y''' + 2x^2 y' + 2y - 10x + \frac{10}{x}$.

Solution:-

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3 D^3 y + 2x^2 D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3 D^3 + 2x^2 D + 2)y = 10x + 10x^{-1} \quad \text{— ①}$$

Let $x = e^t \Rightarrow t = \ln x$.

$$xD = D$$

$$x^2 D^2 = D(\Delta - 1) = \Delta^2 - D$$

$$x^3 D^3 = \Delta(\Delta - 1)(\Delta - 2).$$

Subtituting into eq.(1).

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$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$

$(m^3 - m^2 + 2)y = 10e^{t} + \frac{10}{e^{t}}$

using Synthtic ~~dy~~division

	1	-1	0 2	
-1		-1	2 -2	
	1	-2	2	0

$D^2 - 2D + 2 = 0$

Now using Quadratic formula

$a = 1, \quad b = -2, \quad c = 2$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2a}$$

$$D = \frac{2 \pm \sqrt{4-8}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = 2 \pm \sqrt{\frac{-4}{2}}$$

$$D = 2 \pm \frac{\sqrt{-1 \times \sqrt{4}}}{2}$$

$$D = \frac{2 + 2i}{2}$$

$$D = \cancel{2}\left(\frac{1 \pm i}{\cancel{2}}\right)$$

$$D = 1 + i$$

Since roots are complex

$$y_c = e^{-x}(a \cos t + c_2 \sin t).$$

Now particular integration

$$y_p = \frac{1}{D^2 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot 10/et$$

$$= \frac{10et}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$= \frac{\overset{5}{\cancel{10}}et}{\cancel{2}} + \frac{\overset{5}{\cancel{10}}e^{-t}}{\cancel{2}}$$

$= 5e^{t} + 5e^{-t}$

$y_p = 5e^{t} + 5e^{-t}$

General Soloution

$y = y_c + y_p$

$y = e^{-x}(c_1 \cos t + (2 \sin t) + 5e^{-t} + 5e^{-t}$

put $e^{t} = x$ and $t = \ln x$

$y = e^{-x}(c_1 \ln x + c_2 \sin(\ln x)) + 5e^{x} + 5e^{-x}$ Ans

Question No 2 Page No. 5

2) $x^3\frac{d^3y}{dx^3} + 4x^2 \frac{d^2y}{dx^2} - 5x\frac{dy}{dx} - 15y = x^4$

Solution:

let $\frac{d}{dx} = D$

$x^3 D^3 y + 4x^2 D^2 y - 5xDy - 15y = x^4$

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

let: $x = e^t \Rightarrow t = \ln x$

$xD = D$

$x^2D^2 = D(D-1) = D^2 - D$

$x^3D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$

Now Subtituting

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

$(D^3 - 3D^2 + 2D + 4(D^2 - D) - 5(D) - 15)y = e^{4t}$

$(D^3 + D^2 - 7D - 15)y = e^{4t}$

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Synthetic division

5	1	+1	-7	-15
		3	12	15
	1	4	5	0

$$D^2+4D+5=0$$

Quadratic formula:~ $D=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$D=\frac{-b\pm\sqrt{4^2-4(1)(5)}}{2}$$

$$=\sqrt{\frac{-4\pm 16-20}{2}}$$

$$\Rightarrow -4\pm 2i$$

$$D=\frac{\cancel{2}(-2+i)}{\cancel{2}}$$

$$yc=e^{3x}(c_1\cos t+c_2\sin t)$$

For $yp=?$

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Page No 7:

$$yP = \frac{1}{D^3 + D^2 - 7D - 15} \cdot e^{4t}.$$

$$= \frac{1}{(4)^3 + (4)^2 - 7(4) - 5} e^{4t}.$$

$$= \frac{1}{64 + 16 - 28 - 15} e^{4t}.$$

$$= \frac{1}{80 - 43} e^{4t}.$$

$$yP = \frac{1}{37} e^{4t}.$$

Hence:-

$$y = yc + yP$$

$$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}.$$

again put $t = \ln x$ & $x = \ln x$.

$$y = e^{3x}(c_1 \cos \ln x + c_2 \sin \log x) + \frac{1}{37} e^{4x}$$ Ans

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Page NO. 8:-

Question # 03

$x^2 y''' + 2xy' - 6y = 10x^2.$

Solution:

$y(1) = 1$ and $y'(1) = -6$

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 6y = 10x^2.$$

$$\Rightarrow \left(x^2 d^2/dx^2 + 2x\, d/dx - 6\right) y = 10x^2.$$

Put $xD = D \Rightarrow x^2D^2 = D(D-1) = D^2 - D.$

$x = e^t$ & $\log x = t.$

$$(D^2 - D + 2D - 6)y = 10e^{2t}.$$

$$(D - D + 6)y = 10e^{2t}.$$

The characteristic equation.

$D^2 + D - 6 = 0$

$D^2 + 3D - 2D - 6 = 0$

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$\Rightarrow D(D+3)-2(D+3)=0$

$\Rightarrow (D+3)(D-2)=0$

$D+3=0, \; D-2=0$

$D=2, \; D=-3$

Since roots are real & distinct.

For $y_c = ?$

$$y_c = C_1 e^{-3t} + C_2 e^{2t}$$

For $y_P = ?$

$$y_P = \frac{1}{D^2-D-6} \cdot 10^{2t}.$$

$$= \frac{10}{D^2-D-6} e^{2t}.$$

$$= 10 \frac{1}{0} e^{2t} \text{ fails}$$

<u>Now</u>

$$10 \frac{1}{d/dD(D^2+D-6)} e^{2t}.$$

$$\Rightarrow 10 \frac{t}{2\lambda+1} e^{2t}.$$

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$= 10 \dfrac{1.t}{4+1} e^{2t}$

$y_P = 2te^{2t}$

General Solution:

$y = y_c + y_P$

$= c_1 e^{-3t} + c_2 e^{2t} + 2te^{2t}$

$y = c_1 x^{-3} + c_2 x^2 + 2(\log x)x^2 \to (B)$

Put $y(1) = 1$ i-e $x = 1, y = 1$ in (B)

$1 = c_1 (1)^{-3} + c_2 (1)^2 + 2\log(1)$

$1 = c_1 + c_2 \to (C)$

Now differentiate eq (B) w.r.t x.

$y' = -3c_1 x^{-4} + 2(2x + \frac{2}{x}(x^2) + 4x\log x$

Now put $y'(1) = -6$ i-e $y' = -6$ and $x = -6$

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$\Rightarrow -6 = -3c_1 + 2(2+2+0$

$\Rightarrow -6 = -3c_1 + 2(2+2.$

$\Rightarrow -6-2 = -3c_1 + 2c_2 + 2.$

$\Rightarrow -8 = -3c_1 + 2c_2 \rightarrow$ (D).

$\times$ing eq (C) with (2) Sub-ing from (D).

$$\begin{array}{l} 2c_1 + 2c_2 = 2 \\ \mp 3c_1 \pm \cancel{2c_2} = \mp 8 \\ \hline 5c_1 \qquad = 10 \end{array}$$

$c_1 = \frac{\cancel{10}^{2}}{\cancel{5}_{1}} \Rightarrow \boxed{c_1 = 2}$

$-8 = -3(2) + 2c_2$

$-8 = -6 + 2c_2$

$2c_2 = -8+6$

$2c_2 = -2.$

$$C_2 = \frac{-\cancel{2}}{\cancel{2}} = 1$$

$$\boxed{C_2 = -1}$$

Now put the value of c_1 & c_2 in eq (B).

$$y = 2x^{-3} - x^2 + 2\ln x \; x(x^2).$$

$$y = \frac{2}{x^3} - x^2 + 2x^2 \log x. \quad \text{Ans.}$$

Question # 04:

$$x^2y'' + 7xy' + 5y = x^5$$

$$y(0) = 2 \;\&\; y'(1) = 2.$$

Solution:

$$x^2 \frac{dy^2}{dx} + 7x\frac{dy}{dx} + 5y = x^5$$

$$\Rightarrow \left(x^2 \frac{d^2}{dx^2} + 7x\frac{d}{dx} + 5\right) y = x^5 \rightarrow (A).$$

Put $xD = D \Rightarrow x^2D^2 = D(D-1) = D^2 - D.$

$x = e^t \Rightarrow \log x = t$ in eq. (A).

$$\Rightarrow (D^2 - D + 7D + 5)y = e^{5t}.$$

$$\Rightarrow (D^2 + 6D + 5)y = e^{5t}.$$

By Quadratic formula

$$D = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}.$$

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$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}.$$

$$= \frac{-6 \pm \sqrt{36 - 20}}{2}.$$

$$= \frac{-6 \pm \sqrt{16}}{2}.$$

$$= \frac{-6 \pm \sqrt{4^2}}{2}.$$

$$= \frac{\cancel{2}(-3 \pm 2)}{\cancel{2}_1}$$

$D = -3 \pm 2$ Since roots are real & distinct.

$$y_c = c_1 e^{-5t} + c_2 e^{-t}.$$

For $y_p = ?$

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$$yP = \frac{1}{D^2+6D+5} e^{5t}.$$

$$= \frac{1}{(5)^2+6(5)+5} e^{5t}.$$

$$= \frac{1}{60} e^{5t}.$$

Now General Solution is

$$y = yc + yp.$$

$$y = c_1 e^{-5t} + c_2 e^{-t} + \frac{1}{60} e^{5t}.$$

$$y = c_1 x^{-5} + c_2 x^{-1} + \frac{1}{60} x^5 \rightarrow (B).$$

$x = 0$ put in this equation.

No in eq (B) $e^0 = 1$.

put $y(0) = 2$ i-e $y = 2$ & $x = 2$.

$$2 = c_1(2)^{-5} + c_2(2)^{-1} + \frac{1}{60}(2)^5.$$

$\Rightarrow 2 - \frac{4}{3} = 320c_1 + 4c_2$

$\Rightarrow 2/3 = 320c_1 + 4c_2 \rightarrow (D)$.

Ding eq (C) with 2 & then —ing eq (C) from (D).

$\frac{-44}{15} = 64c_1 + 4c_2$.

$\frac{-44}{15} = 64c_1 + 4c_2$.

$+2/3 = +320c_1 \pm 4c_2$

$+ \qquad -$

$\frac{34}{15} = -256c_1$

$c_1 = \frac{34}{15} \times 256$.

$\boxed{c_1 = 580}$

Put the value of c_1 in eq (C)

$\frac{22}{15} = -32(580) - 2c_2$

$$2 = -32c_1 - 2c_2 + \frac{1}{\cancel{60}_{15}}(\cancel{32}^{8}).$$

$$2 = -32c_1 - 2c_2 + \frac{8}{15}.$$

$$2 - \frac{8}{15} = -32c_1 - 2c_2.$$

$$\frac{22}{15} = -32c_1 - 2c_2 \rightarrow (C).$$

Now differentiate eq (B) w.r.t (x).

$$y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12} x^{4} \longrightarrow$$

Put $y'(1)=2$ i-e $y'=2$ & $x=2$ in above equation.

$$2 = -5c_1 c_2^{-6} - c_2(2)^{-2} + \frac{1}{12}(2)^4.$$

$$2 = -5c_1(-64) - c_2(4) + \frac{1}{12}(16).$$

$$2 = 320c_1 + 4c_2 + 4/3$$

$$\Rightarrow \frac{22}{15} = -18560 - 2C_2 .$$

$$\Rightarrow \frac{22}{15} + 18560 = -2C_2 .$$

$$\Rightarrow \frac{18561}{-2} = C_2 .$$

$$\boxed{-9280 = C_2}$$

Now put the value of C_1 & C_2 in eq (B).

$$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5 .$$

$$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5 .$$ **Ans.**

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Question #05

$$(x+1)^2 y'' - 3(x+1)' y' + 4y = x^2$$

Solution:-

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$$

$$\Rightarrow \left((x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\frac{d}{dx} + 4 \right] y = x^2$$

$$\Rightarrow \left\{ (x+1)^2 D^2 - 3(x+1)D + 4 \right] y = x^2 \rightarrow (A).$$

Put $(x+1)D = D \Rightarrow (x+1)^2 D^2 = D(D-1) = D^2 - D$

$x = e^t$ in eq (A)

$$\Rightarrow \left\{ D^2 - D - 3D + 4 \right] y = e^{2t}.$$

$$\Rightarrow \left\{ D^2 - 4D + 4 \right] y = e^{2t}.$$

$$\Rightarrow (D^2 - 4D + 4) 2 = e^{2t}.$$

For y_c we find the roots

$$D^2 - 4D + 4 = 0$$

$D^2 - 2D - 2D + 4 = 0$

$D(D-2) - 2(D-2) = 0$

$D - 2 = 0$

$D - 2 = 0$, $D = 2$

, $D = 2$

So the roots are real & reapd.

the General Solution are.

$y = (C_1 + C_2 x) e^{mx}$

$y = (C_1 + C_3 x)^{2x}$

For $y_P = ?$

$$y_P = \frac{1}{D^2 - 4D + 4}$$

$(2)^2 - 4(2) + 4$

$\Rightarrow 0$

$$y_P = \frac{2}{2D - 4} e^{2t}.$$

if we put 2

$2D - 4 \Rightarrow 2(2) - 4 = 0$

we take again derivation

$Y_P = \frac{\cancel{2}}{\cancel{2}} \cdot e^{2t}.$

$y = (C_1 + C_2 x)^{2t} + e^{2t}$ **Ans.**

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(1)

Application of Partial differential equation:-

Many engineering Problems are governed by differents types of Partial differential equations, and some of the more important types are given below.

Tricomi equation:-

$$y \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \begin{cases} y > 0 : \text{elliptic} \\ y < 0 : \text{hyperbolic} \end{cases}$$

Laplace equation (or variants): $\partial 2 \phi \partial x 2 + \partial 2 \phi \partial y 2 =$
$\nabla 2 \phi = 0$

Poisson's equation:-

$$\partial 2 \phi \partial x 2 + \partial 2 \phi \partial y 2 = f(x, y)$$

Helmholtz equation: $\partial 2 \phi \partial x 2 + \partial 2 \phi \partial y 2 + C 2 \phi = 0$

Plate bending:-

$$\nabla 2 \nabla 2 w = \nabla 4 W = qD$$

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Wave equation:-

$$\frac{\partial^2 u}{\partial t^2} - c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$$

Fourier equation:-

$$\partial T \, \partial t = a \, (\partial_2 T \, \partial x 2)$$

Separable differential equations:-

For equations which can be expressed in separable form as shown below, the solution can be obtained easily as,

$$\frac{dy}{dx} = F(x, y) \quad \frac{dy}{f(y)} = f(x)\, dx \quad \int \frac{dy}{f(y)} =$$

$$\int f(x) \, dx + C$$

$$M(x, y)\, dx + N(x, y)\, dy = 0 \quad M(x)\, dx = N(y)\, dy$$

then $\int M(x)\, dx = -\int N(y)\, dy + C.$

3) Separable differential equations:-

For equations which can be expressed in separable from as shown below, the Solution can be obtained easily as.

$$\frac{dy}{dx} = F(x,y) \quad \frac{dy}{\eta(y)} = f(x)\,dx \quad \int \frac{dy}{\eta(y)} = \int f(x)\,dx + C$$

$$M(x,y)\,dx + N(x,y)\,dy = 0 \quad M(x)\,dx = -N(y)\,dy$$

then $\int M(x)\,dx = -\int N(y)\,dy + C$

Example:-

$$\frac{dy}{dx} = 3x^3 + (y^2+1) \Rightarrow \frac{dy}{y^2+1} = x^3\,dx$$

$$\int \frac{dy}{y^2+1} = \int x^3\,dx + C \Rightarrow \tan^{-1} y = \frac{1}{4}x^4 + C$$

$$\Rightarrow y = \tan\left(\frac{1}{4}x^4 + c\right)$$

Example:-

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}$$ subject to $y(0) = -1$

Since is a separable function, the pr
Can be solved as.

$$2(y-1)\,dy = (3x^2+4x+2)\,dx$$
$$y^2-2y = x^3+2x^2+2x+c$$

Example:

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}$$ subject to

Since this is a separable function, the problem can be solved as

$$2(y-1)\,dy = (3x^2+4x+2)\,dx$$
$$y^2-2y = x^3+2x^2+2x+c$$

Based on the boundary condition, $C = 3$

hence $y^2-2y = x^3+2x^2+2x+3$

This quadratic equation in y^2 can be with two solutions by the quadratic as:

$y = 1 \quad \sqrt{x^3+2x^2+2x+4}$ and $y = 1 + \sqrt{x^3+2x^2+2x+4}$

Since the second solution does not satisfy the boundary Condition, it will not be accepted; hence, the Solution to this differential equation is obtained.

Variation of Parameters:-

for the following equation form, it is Possible to Solve it by variation of Parameters.

for $\dfrac{dy}{dx} = P(x)\,y + Q(x)$

put $y = C(x)\, e^{\int P(x)\,dx}$ by differentiating, it gives.

$$\frac{dy}{dx} = \frac{dc(x)}{dx} e^{\int p(x)\,dx} + \underbrace{c(x)\,p(x)\,e^{\int p(x)\,dx}}_{p(x)\,y}$$

Substitute it to the original ODE

$$\frac{dc(x)}{dx} = Q(x)e^{-\int P(x)dx}$$ Comparing the terms, it given

$$C(x) = \int Q(x) e^{-\int P(x)dx} dx + C.$$

Example:-

$$(x+1)\frac{dy}{dx} - ny = e^{x}(x+1)^{n+1}$$

This equation is now expressed as.

$$\frac{dy}{dx} = P(x)y + Q(x)$$

$$\frac{dy}{dx} = \frac{n}{x+1}y + \underbrace{e^{x}(x+1)^{n}}_{Q(x)}$$

for $x \neq -1$

Solving the homogeneous part of the ODE

$$\frac{dy}{dx} = \frac{n}{x+1}y \text{ then } \frac{dy}{y} = \frac{n}{x+1}dx$$

$$\ln|y| = \ln(n)|x+1| + c_1$$

$$y = c(x+1)^{n}$$

Look for solution $y = c(x)(x+1)^n$

where $c(x)$ is the variation of parameters

Substitute it to the ODE.

$$\frac{dc(x)}{dx}(x+1)^n + nc(x)(x+1)^{n-1} = nc(x)$$

$$(x+1)^{n-1} = nc(x)(x+1)^{n-1} + e^x(x+1)^n$$

$$\frac{dy}{dx} = \frac{n}{x+1}y + e^x(x+1)^n$$

Comparison givens $\frac{dc(x)}{dx} = e^x$

Integration of this equation gives.

$$C(x) = e^x + \bar{C}$$

General Solution is hence given by

$$y = (x+1)^n(e^x + \bar{C})$$

The Bernoulli equation is an important equation type which can be solved in a similar way by variation of parameters. Consider the following form of equation.

$$\frac{dy}{dx} = p(x)\,y + Q(x)\,y^n$$

step 1:- put $z = y^{1-n}$

step 2:- Then $\frac{dz}{dx} = (1-n)\,y^{-n}\,\frac{dx}{dy}$

$$\frac{dz}{dx} = (1-n)\,p(x)\,z + (1-n)\,Q(x)$$

The non linear ODE now becomes linear ODE. It can be solved by formula.

step 3:- $n = -1$, $z \cdot y^2$. Inverting z to get y

$$\frac{dy}{dx} = \frac{y}{2x} + \frac{x^2}{2y}$$

$$\frac{dz}{dx} = \frac{1}{x}\,z + x^2$$

$$z = c\int \frac{1}{x}\,dx\left(\int x^2 e^{-\int \frac{1}{x}dx}\,dx + c\right) = cx + \frac{1}{2}x^2$$

Back substitution of $z = y^2$ of $z = y^2$

$$y^2 = cx + \frac{1}{2}x^2$$

Homogeneous equations:.

for equation of the following type, where all the Coefficients are Constant, it Can be evaluated according to different.

Laplace equation.

Laplace equation forms an important governing Condition for many types of Problems. Some of the more Common forms are given by.

Three dimensional Laplace equation:-

$$u_{xx} + u_{yy} + u_{zz} = 0$$

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Two dimensional heat Conduction:-

$a^2 (u_{xx} + u_{yy})$ u_t

Two dimensional Seepage problem.

$(K_x u_{xx} + K_y u_{yy}) = 0$

There a two major types of boundary Conditions to this problem.

Dirichlet problem:-

Boundary Conditions prescribed as u.

THE END