Question No 1 a

Sol:-

$$y_h(n) = C_1(-1)^n + C_2(4)^n.$$

an exponential sequence of the same from as x(n), Normally, we could assume a solution of the form

$$y_p(n) = K(4)^n u(n)$$

we absorve that yp(n) is already contained in the homogenous solution.

Thus we assume that

$$y_p(n) = kn(4)^n u(n).$$

we obtain.

$$kn(4)^n u(n) - 3k(n-1)(4)^{n-1}u(n-1) - 4k(n-2)(4)^{n-2}u(n-2) = (4)^n u(n) + 2(4)^{n-1}u(n-1).$$

To determine k we evaluate this equation for any $n \geq 2$ - To simplify the arthimetic we select $n=2$ from which we obtain $\boxed{k = 6/5}$

There fore,

$$y_p(n) = \frac{6}{5} n (4)^n u(n)$$

Total solution to the difference equation is abtained

$$y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{5} n(4)^n \quad n \geq 0$$

Question 2b :>

first we eliminate the negative powers of 'z' by multiplying both numerator and denominator by z^2

Thus we obtain $X(z) = \dfrac{z^2}{z^2 - 1.5z + 0.5}$

The poles of $X(z)$ are $p_1 = 1$ and $p_2 = 0.5$ consequently, the expansion will be

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{2}{(z-1)} - \frac{1}{(z-0.5)}$$

(obtained by applying partial fraction)

$$\Rightarrow X(z) = \frac{2z}{(z-1)} - \frac{z}{(z-0.5)}.$$

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Question (1 b)

Soln:

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

$$y(z) = \frac{X(z)}{1-0.6z^{-1}+0.08z^{-2}}.$$

Impulse response.

$$x(n) = \delta(n)$$

$$x(z) = 1$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0.6z^{-1}+0.08z^{-2}}$$

$$= \frac{1}{(1-\frac{1}{5}z^{-1})(1-\frac{2}{5}z^{-1})}$$

$$H(z) = \frac{-1}{1-\frac{1}{5}z^{-1}} + \frac{2}{1-\frac{2}{5}z^{-1}}$$

$$h(n) = \left[-1\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]u(n).$$

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Question No 3.

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

$H(0) = 1$; $|H(\pi/4)|^2 = 1/2$.

$$H(z) = \frac{b_0}{(1-pz^{-1})^2} \quad . \quad H(\theta) = \frac{b_0}{(1-pe^{-j\theta})^2}$$

$|H(0)| = 1$

$$1 = \frac{b_0}{(1-p)^2} \Rightarrow \boxed{b_0 = (1-p)^2}$$

$$H(\theta) = \frac{b_0}{(1-pe^{-j\theta})} \cdot \frac{1}{1-pe^{-j\theta}}$$

$$|H(\theta)|^2 = b_0^2 \frac{1}{1+p^2-2p\cos\theta} \cdot \frac{1}{1+p^2-2p\cos\theta}$$

$$|H(\pi/4)|^2 = 1/2 \quad \frac{b_0^2}{(1+p^2-2p\frac{1}{\sqrt{2}})^2}$$

$$= (1+p^2-\sqrt{2}p)^2 = 2b^2$$

$$1+p^2-\sqrt{2}p = \sqrt{2}(1-p)^2$$

$\boxed{p = 0.32}$

and $b_0 = (1-0.32)^2 = \boxed{0.46}$

where the constants c_1 and c_2 are determined that the initial conditions

$$y(0) = 3y(-1) + 4y(-2) + 1$$

$$y(1) = 3y(0) + 4y(-1) + 6$$

$$= 13y(-1) + 12y(-2) + 9$$

on other hand evaluated at n=0 and n=1

$$y(0) = c_1 + c_2$$

$$y(1) = -c_1 + 4c_2 + \cancel{24}\tfrac{24}{5}$$

Hence $c_1 = -\frac{1}{25}$ and $c_2 = \frac{26}{25}$

finally we have the zero state response to the forcing function.

$x(n) = (4)^n u(n)$ in the form.

$$y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5}n(4)^n$$

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Question No 4 (A).

A filter duration sequence of length 1 is given as.

$$x(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Solution:→

The fourier transform of this sequence

$$X(\omega) = \sum_{n=0}^{1-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{1-1} e j^{-\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin n(\omega L/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated $L=10$ The N-1 point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally $K=0,1,\ldots N-1$ Hence.

$$X(K) = \frac{1 - e^{-j2\pi K L/N}}{1 - e^{-j2\pi K/N}} \qquad K = 0, 1, \ldots N-1$$

as equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p = 0.32$ satisfied this equation - consequently the system function for the desired filter is

$$H(z) = 0.46/(1-0.32z^{-1})^2$$

The same principle can be applied for the design of bandpass filter.

Question No 3(b).

Sol:.

Clearly the filter must have pass at.

$$P_{b} = re^{\pm jK_2}$$

and zeros at $z=1$ and $z=-1$

Consequently the system function

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency response $H(w)$ at the filter at

$w = \pi/2$ thus.

$$H(\pi/2) = G\,2/1-r^2 = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(w)$ at $w = 4\pi/9$

M T W T F S Date:

$x_2((1-n))_4$ rotated by one unit in time as illustrated

for $m=2$

we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence.