

Question No 1 a

Sol:→

$$y_h(n) = C_1(-1)^n + C_2(4)^n$$

an exponential sequence of the same form as $x(n)$, Normally we could assume a solution of the form

$$y_p(n) = k(4)^n u(n)$$

we observe that $y_p(n)$ is already contained in the homogenous solution.

Thus we assume that

$$y_p(n) = kn(4)^n u(n).$$

we obtain.

$$kn(4)^n u(n) - 3k(n-1)(4)^{n-1} u(n-1) - 4k(n-2)(4)^{n-2} u(n-2) = (4)^n u(n) + 2(4)^{n-1} u(n-1).$$

To determine k we evaluate this equation for any $n \geq 2$ - To simplify the arithmetic we select $n=2$ from

which we obtain $k = \frac{6}{5}$

Therefore,

$$y_p(n) = \frac{6}{5} n (4)^n u(n)$$

Total solution to the difference equation is obtained

$$y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{5} n (4)^n \quad n \geq 0$$

↳

question 2b:

first we eliminate the negative powers of 'z' by multiplying both numerator and denominator by z^2

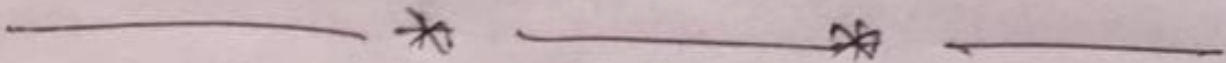
$$\text{Thus we obtain } X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

The poles of $X(z)$ are $p_1 = 1$ and $p_2 = 0.5$
consequently, the expansion will be

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

(obtained by applying partial fraction)

$$\Rightarrow X(z) = \frac{2z}{z-1} - \frac{z}{z-0.5}$$



Question (1b)

Sol:is

$$y(n] = 0.6y[n-1] - 0.08y[n-2] + x[n]$$

$$Y(z) = \frac{X(z)}{1 - 0.6z^{-1} + 0.08z^{-2}}$$

Impulse response .

$$x[n] = \delta[n]$$

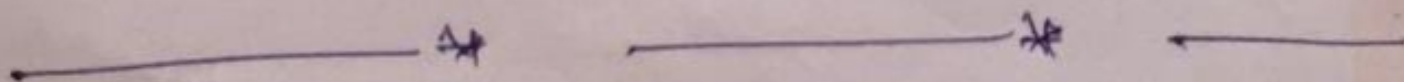
$$X(z) = 1$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.6z^{-1} + 0.08z^{-2}}$$

$$= \frac{1}{\left(1 - \frac{1}{5}z^{-1}\right)\left(1 - \frac{2}{5}z^{-1}\right)}$$

$$H(z) = \frac{-1}{1 - \frac{1}{5}z^{-1}} + \frac{2}{1 - \frac{2}{5}z^{-1}}$$

$$h[n] = \left[-1 \left(\frac{1}{5}\right)^n + 2 \left(\frac{2}{5}\right)^n \right] u[n]$$



Question no 3.

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

$$H(0) = 1 \quad ; \quad |H(\pi/4)|^2 = 1/2.$$

$$H(z) = \frac{b_0}{(1-pz^{-1})^2} \quad \cdot \quad H(0) = \frac{b_0}{(1-pe^{-j0})^2}$$

$$|H(0)| = 1$$

$$1 = \frac{b_0}{(1-p)^2} \Rightarrow \boxed{b_0 = (1-p)^2}$$

$$H(0) = \frac{b_0}{(1-pe^{-j0})} \cdot \frac{1}{1-pe^{-j0}}$$

$$|H(\omega)|^2 = b_0^2 \frac{1}{1+p^2-2p\cos\omega} \cdot \frac{1}{1+p^2+2p\cos\omega}$$

$$|H(\pi/4)|^2 = 1/2 \quad \frac{b_0^2}{(1+p^2-2p\frac{1}{\sqrt{2}})^2}$$

$$= (1+p^2-\sqrt{2}p)^2 = 2b^2$$

$$1+p^2-\sqrt{2}p = \sqrt{2}(1-p)^2$$

$$\boxed{p = 0.32}$$

$$\text{and } b_0 = (1-0.32)^2 = \boxed{0.46}$$

where the constants c_1 and c_2 are determined that the initial conditions

$$y(0) = 3y(-1) + 4y(-2) + 1$$

$$y(1) = 3y(0) + 4y(-1) + 6$$

$$= 13y(-1) + 12y(-2) + 9$$

on other hand evaluated at $n=0$ and $n=1$

$$y(0) = c_1 + c_2$$

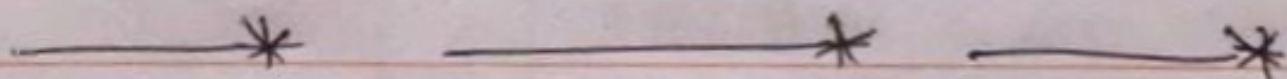
$$y(1) = -c_1 + 4c_2 + \frac{24}{25}$$

$$\text{Hence } c_1 = -\frac{1}{25} \text{ and } c_2 = \frac{26}{25}$$

finally we have the zero state response to the forcing function.

$x(n) = (4)^n u(n)$ in the form.

$$y_{zs}(n) = -\frac{1}{25} (-1)^n + \frac{26}{25} (4)^n + \frac{6}{5} n (4)^n$$



Question No 4 (A)

A filter duration sequence of length 1 is given as.

$$x(n) = \begin{cases} 1 & 0 \leq n \leq L-1 \\ 0 & \text{otherwise} \end{cases}$$

Solution :->

The Fourier transform of this sequence

$$\begin{aligned} X(\omega) &= \sum_{n=0}^{L-1} x(n) e^{-j\omega n} \\ &= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} \\ &= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2} \end{aligned}$$

The magnitude and phase of $x(\omega)$ are illustrated $L=10$. The N -point DFT of $x(n)$ is simply $x(\omega)$ evaluated at the set of N equally spaced frequencies $k=0, 1, \dots, N-1$. Hence,

$$X(k) = \frac{1 - e^{-j2\pi k L/N}}{1 - e^{-j2\pi k/N}} \quad k=0, 1, \dots, N-1$$

as equivalently

$$2(1-p)^2 = 1 + p^2 - \sqrt{2p}$$

The value of $p = 0.32$ Satisfied this equation - consequently the system function for the desired filter is

$$H(z) = \frac{0.46}{(1 - 0.32z^{-1})^2}$$

The Samp principle can be applied for the design of bandpass filter.

Question No 3(b).

Sol:.

Clearly the filter must have pass at.

$$P_b = re^{jkz}$$

and zeros at $z=1$ and $z=-1$

Consequently the System function

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2 - 1}{z^2 + r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ at the filter at

$$\omega = \pi/2 \text{ thus.}$$

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/q$

$x_2((1-n))_4$ rotated by one unit
in time as illustrated,

for $m=2$

we have

$$x_3(2) = \sum_{n=0}^3 x_1(n) x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded
sequence.