Department of Electrical Engineering
Assignment
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Course Details

Course Title:	ELECTROMAGNETIC FIELD THEORY	**Module:**	
Instructor:	ENGINEER RAFIQ MANSOOR	**Total Marks:**	30

Student Details

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Answer to question (1)

(a) $\rho = \sqrt{(x^2+y^2)} = \sqrt{40} = 6.325$

$\phi = \tan^{-1}(y/x) = \tan^{-1}(-6/2) = -71.57$

$z = 3$

(b) $r = \sqrt{(x^2+y^2+z^2)} = \sqrt{50} = 7.07$

$\theta = \cos^{-1}(z/r) = \cos^{-1}(5/5\sqrt{2}) = 45^\circ$

$\phi = \tan^{-1}(y/x) = \tan^{-1}(4/3) = 53^\circ$

(c) $r = \sqrt{x^2+y^2+z^2} = \sqrt{14} = 3.74$

$\theta = \cos^{-1}(z/r) = \cos^{-1}(-1/3.74) = 105.5^\circ$

$\phi = \tan^{-1}(y/x) = \tan^{-1}(3/2) = 56.31.$

(d) $x = r\sin\theta\cos\phi = 4\sin 25^\circ \cos 120^\circ = 0.845$

$y = r\sin\theta\sin\phi = 4\sin 25^\circ \sin 120^\circ = 1.462$

$z = r\cos\theta = 4\cos 25^\circ = 3.625.$

(e) Before the charges are brought into contact, $F = 11.234 \mu N$.
After the charges are brought into contact and then seperated, charge on each sphere is, $(q_1 + q_2)/2 = 0.5nC$
On calculating the force with $q_1 = q_2 = 0.5nC$, $F = 1.404 \mu N$.

(f) $F = q_1 q_2 / (4\pi\epsilon_0 r^2) = -2 \times 9(10^{-9} \times 12) = -18 \times 10^{9}$

$E = F/q = 18 \times 10^{9}/2 = 9 \times 10^{9}$

(g) $E = Q/(4\pi\epsilon_0 r^2)$
$Q = (4000 \times 0.3^2)/(9 \times 10^9) = 4 \times 10^{-8} C$.

(h) $F = q_1 q_2 (4\pi\epsilon_0 r^2)$, substituting q_1, q_2 and F, $r^2 = q_1 q_2 / (4\pi\epsilon_0 F) =$
we get $r = 0.09m$.

Answer to question 2(a)

$A = (\sqrt{3}, 1)$, $B = (2, 0)$

$A \cdot B = (\sqrt{3}, 1)\ (2, 0)$
$A \cdot B = (2\sqrt{3})$

$|A| = \sqrt{(3)^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

$|B| = \sqrt{(2)^2 + (0)^2} = \sqrt{4+0} = \sqrt{4} = 2$

$\cos\theta = \frac{A \cdot B}{|A|\,|B|}$

$\theta = \cos^{-1} \frac{2\sqrt{3}}{2 \cdot 2}$

$= \cos^{-1} \frac{\sqrt{3}}{2}$

$\theta = 30.0029^\circ$.

Answer to question 2 (b)

(i)

$f = ax^2 + by^3z$

$\nabla = \hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}$

$= \hat{i}\frac{\delta}{\delta x}(ax^2 + by^3z) + \hat{j}\frac{\delta}{\delta y}(ax^2 + by^3z) +$
$\hat{k}\frac{\delta}{\delta z}(ax^2 + by^3z)$

$= \hat{i}(2ax + 0) + \hat{j}(0 + 3by^2z) +$
$\hat{k}(0 + by^3)$

$= \hat{i}(2ax) + \hat{j}(3by^2z) + \hat{k}(by^3)$

$= 2ax\hat{i} + 3by^2z\hat{j} + by^3\hat{k}$

$$\nabla f(x,y,z) = \begin{bmatrix} 2ax \\ 3by^2 \\ by^3 \end{bmatrix}$$

(ii) $f = ar^2\sin\theta + brz\cos2\phi$

$$\nabla = \frac{\delta f}{\delta r} + \frac{1}{r}\frac{\delta f}{\delta\theta} + \frac{1}{r\sin\theta}\frac{\delta f}{\delta\phi}$$

$$= \frac{\delta}{\delta r}(ar^2\sin\theta + brz\cos2\theta) +$$

$$\frac{1}{r}\frac{\delta}{\delta\theta}(ar^2\sin\theta + brz\cos2\phi) +$$

$$\frac{1}{r\sin\theta}\frac{\delta}{\delta\phi}(ar^2\sin\theta + brz\cos2\phi)$$

$$\nabla = 2ar\sin\theta + bz\cos2\phi +$$

$$\frac{1}{r}(ar^2\cos\theta + 0) +$$

$$\frac{1}{r\sin\theta}(0 + brz(-2\sin2\phi))$$

$$= 2ar\sin\theta + bz\cos2\theta + ar\cos\phi$$

$$+\frac{1}{r\sin\theta}(-brz2\sin2\phi)$$

$$= 2ar\sin\theta + bz\cos2\theta + ar\cos\theta$$

$$-\frac{1}{r\sin\theta}brz2\sin2\phi$$

Answer to question 3:-

$$E = \frac{KQ}{R^2}$$

Due to 2Q charge at the top

$$E_{1x} = \frac{K(2Q)}{a^2+b^2}\cos\theta$$

Due to -Q charge:-

$$E_{2x} = -\frac{KQ}{a^2}$$

Due to 2Q charge at bottom

$$E_{3x} = \frac{2KQ}{a^2+b^2}\cos\theta$$

Add $E_{1x} + E_{2x} + E_{3x}$

$$E_x = \frac{4KQ}{a^2+b^2}\cos\theta - \frac{KQ}{a^2}$$

$$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$$

$$= KQ\left[\frac{4\cos\theta}{a^2+b^2} - \frac{1}{a^2}\right]$$