Name Fawad Gul

ID 7373

Subject Differential Equation.

Submmitted to Mam Shomaila Mazhar.

Date 30-06-2020.

Q1a.

$W = \sin(x+ct) + \cos(2x+2ct)$

Given $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2} \rightarrow ①$

Now $\partial w/\partial t = \frac{\partial}{\partial t}\left[\sin(x+ct)+\cos(2x+2ct)\right)$

$= \partial/\partial t\,(\sin(x+ct) + \partial/\partial t\,(\cos(2x+2ct)$

$\frac{\partial w}{\partial t} = c\cos(x+ct) - 2c\sin(2x+2ct)$

Now $\frac{\partial^2 w}{\partial t^2} = \partial/\partial t\left[c\cos(x+ct) - 2c\sin(2x+2ct)\right)$

$\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)$

Now $\partial w/\partial x = \partial/\partial x\left[\sin(x+ct)+\cos(2x+2ct)\right)$

$\partial w/\partial x = \cos(x+ct) - 2\sin(2x+2ct)$

$\partial^2 w/\partial w^2 = \partial/\partial x\left[\cos(x+ct) - 2\sin(2x+2ct)\right)$

$\partial^2 w/\partial x^2 = -\sin(x+ct) - 4\cos(2x+2ct)$.

① ⇒ ②

$-c^2\sin(x+ct)-4c^2\cos(2x+2ct)=c^2[\sin(x^2+ct)-4\cos(2x+2ct)]$

$-c^2\cancel{\sin}(x+ct)-4c^2\cos(\cancel{2}x+2ct)=-c\cancel{\sin}(x+ct)-4c^2\cancel{\cos}(x+2ct)$

$0=0$ $\boxed{\text{Satisfied}}$

Q2
B

$w=\tan(2x+ct)$

Now $\partial w/\partial t = c\sec^2(2x+ct)$

& $\partial^2 w/\partial t = \partial/\partial t\,(c\sec^2(2x+ct))$

$= c^2 2\cdot 2\sec^2(2x+ct)\tan(2x+ct)$

NOW $\partial w/\partial x = 2\sec^2(2x+ct)$

$\partial^2 w/\partial x^2 = 4\sec^2(2x+ct)\tan(2x+ct)$

① ⇒

$4c^2\sec^2(2x+ct)\tan(2x+ct)=4c^2\cancel{\sec^2}(2x+c)\tan(2x+ct)$

$0=0$ $\boxed{\text{Satisfied}}$

Scanned with CamScanner

Q No 2 Given Function is

$$F(x) = \begin{cases} x; & -\pi < x \le 0 \\ 2x; & 0 \le x \le \pi \end{cases}$$

we have to find the Fourier Co-effecient, a_0, a_n & b_n

Now,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\cancel{\pi}}\left[0 - \frac{\pi^{\cancel{2}}}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$\boxed{a_0 = \frac{-\pi}{2} + \pi = \pi/2} \rightarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

$$= \frac{1}{\pi}\left[x(\sin nx/n) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x(\sin nx/n) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$\boxed{a_n = \begin{cases} \frac{-2}{\pi n^2}; & \text{if } n \text{ is odd} \\ 0; & \text{if } n \text{ is even} \end{cases}} \longrightarrow (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$\boxed{b_n = \frac{1}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] = \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}} \quad -(3)$$

So The requried Fourier Since is : $f(x) \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)} + 3\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\sin nx}{n}$$

Q3 Given $y'' - 4y' + 13y = 8\sin 3x$

we have to find $y = y_c + y_p$

For y_c The Charactastic Cousidlary Eqn) Eqn is:

$$m^2 - 4m + 13 = 0$$

$$\Rightarrow m = \frac{4 \pm \sqrt{16-52}}{2} \Rightarrow m = \frac{4 \pm 6i}{2}$$

$$\Rightarrow m = 2 \pm 3i \quad ; \quad \alpha = 2 \ \& \ \beta = 3$$

So $$y_c = e^{2x}\left\{ c_1 \cos 3x + c_2 \sin 3x \right\}$$

For y_p let

$$y_p = \text{Imag.}\left(\frac{1}{m^2 - 4m + 13} 8e^{3ix} \right)$$

$$= 8 \text{ Imag } \frac{e^{3ix}}{(3i)^2 - 4(3i) + 13}$$

$$= 8 \text{ Imag } \frac{e^{3ix}}{-9 - 12i + 13}$$

$$= 8 \text{ Imag } \frac{e^{3ix}}{4 - 12i}$$

$$y_p = 2 \text{ Imag } \frac{e^{3ix}}{(1-3i)} \times \frac{(1+3i)}{(1+3i)}$$

$$y_p = 2 \text{ Imag } \frac{(1+3i)(e^{3ix})}{(1)^2 - (3i)^2}$$

$$y_p = 2 \text{ Imag } \frac{(1+3i)(e^{3ix})}{10}$$

$$y_p = 2/10 \left(\sin 3x + 3 \cos 3x \right)$$

So The general Solution is

$y = y_c + y_p$

$y = c_1 e^{2x} \cos 3x + c_2 e^{2x} \sin 3x + 2/10 \left(\sin 3x + 3\cos 3x \right)$

Now use The inteial Condition $y(0) = 1$

$y(0) = c_1 e^{0} \cos(0) + c_2 e^{(0)} \sin(0) + 2/10 \left(\sin(0) + 3\cos(0) \right)$

$1 = c_1 (1) + 0 + 0 + 2/10 \left(3(1) \right)$

$1 = c_1 + 6/10 \Rightarrow \boxed{c_1 = 1 - 6/10 = 4/10 = 2/5}$

Again use The another Inital Condition

$y'(0) = 2$

So $y' = c_1 2 e^{2x} \cos 3x + c_1 e^{2x} (-3 \sin 3x)$

$+ c_2 2 e^{2x} \sin 3x + c_2 e^{2x} \left(3 \cos 3x \right)$

$+ 2/10 \left(\cos 3x - 3 \sin 3x \right)$

$y'(0) = c_1 2 e^{(0)} \cos(0) + c_1 e^{(0)} \left(-3 \sin(0) \right)$

$+ c_2 2 e^{(0)} \sin(0) + c_2 e^{(0)} \left(3 \cos(0) \right)$

$+ 2/10 \left(\cos(0) - 3 (\sin(0)) \right)$

$2 = 2 c_1 + 0 + 0 + c_2 3(1) + 2/10 \left(1 - 3(0) \right)$

$2 = 2 c_1 + 3 c_2 + 2/10 \Rightarrow 2 = 2(2/5) + 3 c_2 + 2/10$

$1/3 (2 - 4/5 - 2/10) = 1/3 (2 - 4/5 - 2/10) = c_2$ $\boxed{\text{use } c_1 = 2/5}$

$\Rightarrow \boxed{c_2 = 1/3 \left(\frac{20 - 8 - 2}{10} \right) = 1/3}$

The General Solution is $y = 2/5 e^{2x} \cos 3x + 1/3 e^{2x} \sin 3x + 2/10 \left[\sin 3x + 3\cos 3x \right]$

b The required Solution

Q4

$(D^2 - DD')z = \cos x \cos 2y$

The auxiliary equation is

$m^2 - m = 0 \Rightarrow m = 0, \; m = 1$

Here the complementary function is given by

$z_c = f_1(y) + f_2(y + x)$

For the particular Integral, we have

$z_p = \frac{1}{D^2 - DD'} \cdot \cos x \cos 2y$

$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} \left[\cos(x - 2y) + \cos(x - 2y) \right]$

$= \frac{1}{2} \left[\frac{1}{D^2 - DD'} \cos(x + 2y) + \frac{1}{D^2 - DD'} \cos(x - 2y) \right]$

$= \frac{1}{2} \cos(x + 2y) - \frac{1}{6} \cos(x - 2y)$

Hence the complete Solution is given by

$z = f_1(y) + f_2(y + x) + \frac{1}{2} \cos(x + 2y) - \frac{1}{6} \cos(x - 2y)$

Ans.