Name:- Muhammad Adil Rehman

ID :- 7939

Section:- "B"

Q1:- $x^3y''' + 2x^2y' + 2y = 10x + \frac{10}{x}$

$x^3y''' + 2x^2y' + 2y = 0$

Let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$y''' = m(m-1)(m-2)x^{m-3}$

$x^3 m(m-1)(m-2)x^{m-3} + 2x^2 mx^{m-1} + 2x^m = 0$

$m(m-1)(m-3)x^m + 2mx^m + 2x^m = 0$

$x^m \left[m(m-1)(m-3) + 2mx + 2\right] = 0$

$x^m \left[(m^2-m)(m-3) + 2mx + 2\right] = 0$

$x^2 m(m-1)x^{m-2} + 17x^m = 6$

Q2:- $x^3y''' + 4x^2y'' - 5xy' - 15y = x^4$

$x^3y''' + 4x^2y'' - 5xy' - 15y = 0$

$y = x^m \Rightarrow y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$y''' = m(m-1)(m-2)x^{m-3}$

$x^3 m(m-1)(m-2)x^{m-3} + 4x^2 m(m-1)x^{m-2} - 5xmx^{m-1} - 15x^m = 0$

$\Rightarrow m(m-1)(m-2)x^m + 4m(m-1)x^m - 5mx^m - 15x^m = 0$

$\Rightarrow x^m\left[(m^2-m)(m-2) + 4(m^2-m) - 5m - 15\right] = 0$

$\Rightarrow (m^2-m)(m-2) + 4m^2 - 4m - 5m - 15 = 0$

$\Rightarrow m^3 - 2m^2 - m^2 + 2m + 4m^2 - 4m - 5m - 15 = 0$

$\Rightarrow m^3 + m^2 - 7m - 15 = 0$

$-1+1+7-15$ | $-8+4+14-15$ | $m = 3$

$m = 2$ | $-4-1$ | $m = -2+i$

$\Rightarrow 8+4-14-15$ | | $m = -2-i$

$12-14-15$

$27+9-21-15$

$36-36=0$

Q3 $y'' + 2/x\, y' - 6/x^2 y = 10$

$f(x) = 10$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} c_1 x^{-3} & c_2 x^2 \\ -3c_1 x^{-4} & 2c_2 x \end{vmatrix} = 2c_1c_2 x^{-2} + 3c_1c_2 x^{-2}$$

$$= 5c_1c_2 x^{-2}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & c_2 x^2 \\ 10 & 2c_2 x \end{vmatrix} = -10c_2 x^2$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} c_1 x^{-3} & 0 \\ -3c_1 x^{-4} & 10 \end{vmatrix} = 10c_1 x^{-3}$$

$$u_1' = \frac{W_1}{W} = \frac{-10c_2 x^2}{5c_1c_2 x^{-2}} = \frac{-2}{c_1} x^4$$

$$u_2' = \frac{W_2}{W} = \frac{10c_1 x^{-3}}{5c_1c_2 x^{-2}} = 2/c_2\, x^{-1}$$

so $y_p =$ $u_1' = -2/c_1\, x^4$

$$u_1 = \frac{-2}{c_1} \int x^4 dx = \frac{-2}{5c_1} x^5$$

$$u_2 = 2/c_2 \int \frac{1}{x} dx = \frac{2}{c_2} \ln x .$$

$$y_p = \frac{-2}{5c_1} x^5 c_1 x^{-3} + \frac{2}{c_2} \ln x\, c_2 x^2$$

$$\boxed{y_p = -2x^2 + 2\ln(x)\, x^2}$$

so $\boxed{y = c_1 x^{-3} + c_2 x^2 - 2x^2 + 2\ln x\, (x^2)}$ A

Q 4:-

$$x^2y'' + 7xy' + 5y = x^5$$

$y'(0) = 2$ and $y'(1) = 2$

A.H eq

$$x^2y'' + 7xy' + 5y \rightarrow (1)$$

$$y = x^m$$
$$y' = m x^{m-1}$$
$$y'' = m(m-1) x^{m-2}$$

$$x^2(m(m-1)x^{m-3}) + 7x(m x^{m-1}) + 5x^m = 0$$

$$(m^2 - m)x^m + 7mx^m + 5x^m = 0$$

$$(m^2 - m + 7m + 5)x^m = 0$$

let $x^m \neq 0$

$$m^2 + 6m + 5 = 0$$
$$m^2 + 5m + m = 0$$
$$m(m+5) + 1(m+5) = 0$$
$$(m+1)(m+5) = 0$$
$$(m+1)(m+5) = 0$$

$m = -1$ $\quad m = -5$

$$y_c = c_1 x^{-1} + c_2 x^{-5} \rightarrow (a)$$

(A) $\Rightarrow$ $x^2y'' + 7xy' + 5y = x^5$

$$y'' + \frac{7}{x}y' + \frac{5}{x^2}y = x^3 \quad -(2)$$

$$y_p = u_1 x^{-1} + u_2 x^{-5} \quad -(*)$$

$$W = \begin{vmatrix} x^{-1} & x^{-5} \\ -x^{-2} & 5x^{-6} \end{vmatrix} = -5x^{-7} + x^{-7} = -4x^{-7} \quad -(1)$$

$$W_1 = \begin{vmatrix} x^3 & x^{-5} \\ 3x^2 & -5x^{-6} \end{vmatrix} = -5x^{-3} - 3x^{-3} \Rightarrow -8x^{-3} \quad -(2)$$

$$W_2 = \begin{vmatrix} x^{-1} & x^2 \\ -x^{-2} & 3x^2 \end{vmatrix} = 3x + x = 4x \quad -(3)$$

the general solution is

$$y = y_c + y_p \quad -(1)$$

put (**) and (*) in (***)

$$y = c_1 x^{-1} + c_2 x^{-5} + \frac{13}{45} x^4 \rightarrow (\$)$$

$$\boxed{y_1 = c_1 x^{-1} + c_2 x^{-5} + \frac{13}{45} x^4}$$

Q#5 $(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$

$y = x^m$

$y' = m x^{m-1}$ $\quad y'' = m(m-1)x^{m-2}$

$(x^2+1+2x)\, m(m-1)x^{m-2} - 3(x+1)\, m x^{m-1} + 4x^m = 0$

$x^2 m(m-1)x^{m-2} + m(m-1)x^{m-2} + 2xm(m-1)x^{m-2} - 3(xmx^{m-1} + mx^{m-1}) + 4x^m = 0$

$m(m-1)x^m + m(m-1)x^{m-2} + 2m(m-1)x^{m-1} - 3mx^m - 3mx^{m-1} + 4x^m = 0$

$\Rightarrow m(m-1)x^m - 3mx^m + 4x^m + 2m(m-1)x^{m-1} + m(m-1)x^{m-2} = 0$

$\Rightarrow (m(m-1) - 3m + 4)x^m + (2m(m-1)x^{m-1}) + m(m-1)x^{m-2} = 0$

$\Rightarrow m(m-1) - 3m + 4 = 0$

$\Rightarrow m^2 - m - 3m + 4 = 0$

$\Rightarrow m^2 - 4m + 4 = 0$

$a = 1 \quad b = -4 \quad c = 4$

$\Rightarrow$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+4 \pm \sqrt{16 - 4(1)(4)}}{2}$$

$$= \frac{4 \pm \sqrt{0}}{2} = 4/2 = 2$$

$m_1 = m_2 = 2$

$$y = c_1 x^2 + c_2 x^2 \ln x$$

$c_2(x^2/x + \ln(2x))$

⑤ $y_1 = c_1x^2 \quad y_2 = c_2x^2\ln x$.

$$y'' - \frac{3}{(x+1)}y' + \frac{4}{(x+1)^2}y = \frac{x^2}{(x+1)^2} = \left(\frac{1}{x+1}\right)^2$$

$$f(x) = \frac{x^2}{(x+1)^2}$$

$$W = \begin{vmatrix} c_1x^2 & c_2x^2\ln x \\ 2c_1x & c_2x + c_2 2x\ln x \end{vmatrix} = \boxed{\quad}$$

$$W_1 = \begin{vmatrix} 0 & c_2x^2\ln x \\ \frac{x^2}{(x+1)^2} & c_2x + c_2 2x\ln x \end{vmatrix} = -\frac{x^2}{(x+1)^2}c_2x^2\ln x$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} c_1x^2 & 0 \\ 2c_1x & \frac{x^2}{(x+1)^2} \end{vmatrix}$$

$$W_2 = \frac{c_1x^4}{(x+1)^2}$$

$$u_1' = \frac{W_1}{W} \qquad u_2' = \frac{W_2}{W}$$

$$\boxed{Y_p = u_1y_1 + u_2y_2}$$

$$y = Y_h + Y_p$$