

25/08/2020

(P.1)

HILAL AHMAD

ID: 13144

Q No 1(a)

$$I = \int (x^2 e^x) dx$$

using by parts.

$$I = x^2 \int e^x dx - \int \left( \int e^x dx \frac{d}{dx} x^2 \right) dx.$$

$$I = x^2 e^x - \int e^x (2x) dx$$

$$I = x^2 e^x - \int_{II} e^x (2x) dx_{I}$$

Now using again by parts

$$I = x^2 e^x - (2x \int e^x dx - \int (\int e^x dx \frac{d}{dx} 2x) dx)$$

$$I = x^2 e^x - (2x e^x - \int e^x (2) dx)$$

$$I = x^2 e^x - (2x e^x - (2 \int e^x dx - \int (\int e^x dx \frac{d}{dx} (2x)) dx))$$

$$I = x^2 e^x - (2x e^x - 2e^x - \int e^x \times (0) dx)$$

$$I = x^2 e^x - 2x e^x + 2e^x - 0 + C$$

$$I = x^2 e^x - 2x e^x + C$$

Q1 (b)

$$\int (1+3t) \cdot t^3 dt$$

using by parts formula

$$\begin{aligned} I &= t^3 \int (1+3t) dx - \int \left( \int (1+3t) dx \frac{d}{dx} t^3 \right) dx \\ &= \int (1+3t) dx \end{aligned}$$

$$\text{suppose } t = 1+3t$$

$$= \int (t) dx$$

$$= \frac{t^{1+1}}{1+1} + c$$

$$= \frac{t^2}{2} + c$$

Put the value of  $\boxed{\frac{t(1+3t)^2}{2} + c}$ 

$$I = t^3 \frac{(1+3t)^2}{2} - \int \frac{(1+3t)^2}{2} (3t^2) dx$$

$$I = t^3 \frac{(1+3t)^2}{2} - \frac{3}{2} \int (1+3t)^2 (t^2) dx$$

again by parts

$$I = t^3 \frac{(1+3t)^2}{2} - \frac{3}{2} \left( t^2 \int (1+3t) dx - \int \left( \int (1+3t) dx \frac{d}{dx} t^2 \right) dx \right)$$

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$$I = t^3 \left( \frac{1+3t}{2} \right)^2 - \frac{3}{2} \left( t^2 \left( \frac{1+3t}{2} \right)^2 - \int \frac{(1+3t^2)}{2} (t) dx \right)$$

again by parts

$$I = t^3 \left( \frac{1+3t}{2} \right)^2 - \frac{3}{2} \left( t^2 \left( \frac{1+3t}{2} \right)^2 - t \int (1+3t)^2 dx - \int (1+3t)^2 dx \cdot \frac{d}{dx} t \right) dx$$

$$I = t^3 \left( \frac{1+3t}{2} \right)^2 - \frac{3}{2} \left( \frac{t^2 (1+3t)^2}{2} - t \left( \frac{1+3t}{3} \right)^3 - \int \frac{(1+3t)^3}{3} (1) dx \right)$$

$$I = t^3 \left( \frac{1+3t}{2} \right)^2 - \frac{3}{2} \left( t^2 \left( \frac{1+3t}{2} \right)^2 - t \left( \frac{1+3t}{3} \right)^3 - \frac{(1+3t)^4}{4} \right)$$

$$I = t^3 \left( \frac{1+3t}{2} \right)^2 - t^2 \left( \frac{1+3t}{4} \right)^2 + t \left( \frac{1+3t}{6} \right)^3$$

$$+ \frac{(1+3t)^4}{8} + C$$

Ans

$$Q1 (c) \int (e^x - e^3) dx:$$

using sum rule.

$$\int (e^x - e^3) dx = \int e^x dx - \int e^3 dx.$$

$$= \frac{e^{x+1}}{x+1} - \frac{e^{3+1}}{3+1} + C$$

$$= \frac{e^{x+1}}{x+1} - \frac{e^4}{4} + C$$

$$\boxed{= \frac{e^{x+1}}{x+1} - \frac{e^4}{4} + C} \quad \text{Ans}$$

(Q: 2)

$$f(x) = e^{-6x}$$

$$\xi \quad n = -4$$

$$\text{Sol: } f(x) = e^{-6x}$$

$$\Rightarrow f(-4) = e^{-6(-4)}$$

$$f(-4) = (2.7183)^{24}$$

$$f(-4) = (2.7183)^{24} = 2.649337233 \times 10^{10}$$

Now

$$f'(x) = -6x \cdot e^{-6x-1} \frac{d}{dx} (-6x)$$

$$f'(x) = 6x \cdot e^{-6x-1} (-6)$$

$$f'(x) = 6(-4) \cdot e^{-6(-4)-1} (-6)$$

$$f'(x) = 24 \cdot e^{-25} (-6)$$

$$f'(x) = +144 \cdot e^{-25} \Rightarrow 144 (2.7183)^{-25}$$

$$f'(x) = 144 \times 7.201693401 \times 10^{-10}$$

$$f'(x) = 1.037043855 \times 10^{-7}$$

Q:2

(P.6)

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Now using Taylor's series as:

$$f(x) = f(a) + \frac{(x-a)^1 (f'(a))}{1!} + \frac{(x-a)^2 f''(a)}{2!} + \dots$$

for  $a = -4$

$$f(x) = f(-4) + (x+4) f'(-4) + \frac{(x+4)^2 f''(-4)}{2!} + \dots$$

$$e^{-6x} = 2.649337233 \times 10^{10} + \frac{(x+4) 1.037043855 \times 10^{15}}{2} + \dots$$

$$e^{-6x} = 2.649337233 \times 10^{10} + \frac{(x+4) (1.037043855 \times 10^{15})}{2} + \dots$$

Ans.

Q No: 03 (a)

find The derivative of the follo

$$(a) \quad f(x) = x \cdot \sin x$$

diff with respect to  $x$ 

$$\frac{dy}{dx} = \frac{d}{dx} (x \sin x)$$

Applying Product rule.

$$y' = x \frac{d}{dx} (\sin x) + \sin x \frac{d}{dx} (x)$$

$$y' = x \cos x + \sin x$$

$$\boxed{y' = x \cos x + \sin x} \quad \text{Answer}$$

Q: 3 (b)

(P. 8)

$$F(x) = x^2 \cos x$$

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$$F'(x) = \frac{d}{dx} (x^2 \cos x)$$

Applying product rules.

$$\begin{aligned} F'(x) &= x^2 \frac{d}{dx} (\cos x) + \cos x \frac{d}{dx} x^2 \\ &= (x^2 - \sin x) + (\cos x + 2x) \end{aligned}$$

$$F'(x) = -\sin x \cdot x^2 + 2x \cos x$$

Ans



Q 3 (C)

(P.9)

$$F(z) = z \cdot (2z - 2)^2$$

$$F(z) = z \cdot ((2z)^2 + (2)^2 - (2)(2)(2z))$$

$$= z \cdot (4z^2 + 4 - 8z)$$

$$\frac{d^2}{dz^2} = \frac{d}{dz} (4z^3 + 4z - 8z^2)$$

= Applying some rules.

$$\frac{d}{dz} (4z^3) + \frac{d}{dz} 4z - \frac{d}{dz} 8z^2$$

$$4 \frac{d}{dz} z^3 + 4 \frac{d}{dz} z - 8 \frac{d}{dz} z^2$$

$$= (4 \times 3) z^2 + 4(1) - 8(2) z^{2-1}$$

$$= \boxed{12z^2 + 4 - 16z}$$

Ans