Subject: Diffrential Equation

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Section A

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Paper Solved. Thanks

Q.No1: **Solve the following Objective type questions.**

i) the order of matrix AB is $\underline{m \times n}$.

ii) The number of non-zero rows in Echelon form is $\underline{\text{One}}$.

iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a Singular, matrix then $a = \underline{8}$.

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= 2i \times -i - i \times i$$

$$= -2i^2 - i^2$$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$= 3 \text{ Ans.}$$

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v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Ans:- The given matrix A is a Scaler matrix.

vi) Solution of $\frac{dy}{dx} + 2xy = y$ ?

Sol:-
$$\frac{dy}{dx} = y - 2xy$$
$$= y(1 - 2x)$$

$$\frac{dy}{y} = (1-2x)dx$$

Take $\int$ b. Side

$$\int \frac{dy}{y} = \int (1-2x)dx$$

$$\ln y = \int 1dx - \int 2x dx$$

$$\ln y = x - \cancel{2}\frac{x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

$$\cancel{e}^{\ln y} = e^{(x - x^2 + c)}$$

$$y = e^{x - x^2 + c}$$ Ans

vii) The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Ans Order = 1

Degree = 3

viii) The order and degree of differential Eqn $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is

Ans Order = 2

Degree = 1

ix) The differential Eqn $2\frac{dy}{dx} + x^2 y = 2x+3$ ; $y(0)=5$ ?

Ans :-

$$2\frac{dy}{dx} + x^2 y = 2x+3$$

$$2y' + \frac{x^2 y}{\cancel{2}} = x^2 + 3$$

$$y' + \left(\frac{x^2}{2}\right) y = \frac{x^2+3}{2} \quad \div \text{ by } 2 \text{ b. Side}$$

let $u = \frac{x^2}{2}$ To Much dificult to Solve.
Sorry Mam.

$\propto$

x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & a-b & a^2-b^2 \\ 0 & a-c & a^2-c^2 \end{vmatrix} \quad -R_2+R_1 \text{ \& } -R_3+R_1$$

$$= 1 \begin{vmatrix} a-b & a^2-b^2 \\ a-c & a^2-c^2 \end{vmatrix}$$

$$= (a-b)(a^2-c^2) - (a^2-b^2)(a-c)$$

$$= (a-b)(a+c)(a-c) - (a+b)(a-b)(a-c)$$

$$= (a-b)(a-c)\left[a+c-(a+b)\right]$$

$$= (a-b)(a-c)\left[\cancel{a}+c-\cancel{a}-b\right]$$

$$= \underline{(a-b)(a-c)(c-b)} \quad \text{Ans}$$

**QNO2:** i) Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$ as the product of factor which are linear in $a, b, c$.

Sol :- Expand by $R_1$

$$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3cb^2$$

$$= abc(bc^2 - b^2c - a^2c + a^2c + ab^2 - a^2b)$$

$$= abc\left[bc(c-b) - ac(c-a) + ab(b-a)\right]$$ Ans

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ii) $$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol :- Characteristic Eqn $|A - \lambda I| = 0$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by $R_1$

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$\longrightarrow$ (B)

Again $\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$ Expand by $R_1$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= -\lambda^3+8\lambda^2-18\lambda+8 \longrightarrow \text{(Aa)}$$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$ Expand by $C_1$

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+6\lambda-8 \longrightarrow \text{B (b)}$$

$$\Rightarrow \quad -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by $C_1$

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow (c)$$

put (a), (b) and (c) in (B)

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda$$
$$-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda+6\lambda$$
$$+16-16$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By synthatic division, we get

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$\lambda=0$$

$$\lambda-2=0 \Rightarrow \boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By factorization method

$$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$$

$$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda - 4) = 0$$

$$\lambda = 4, \quad \lambda = 4.$$

$$\boxed{\lambda_1 = 0, \ \lambda_2 = 2, \ \lambda_3 = 4, \ \lambda_4 = 4} \text{ Ans}$$

**QNO3** $(x^2 + 3y^2)dx - 2xy\,dy = 0$

$x = 2, \ y = 6$

**Sol:→** $(x^2 + 3y^2)dx - 2xy\,dy = 0$

$$\Rightarrow (x^2 + 3y^2)dx = 2xy\,dy$$

Divide b. side by $2xy\,dx$, we get

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \to (A)$$

Let $y = vx$

Diff;

$$dy = v\,dx + x\,dv$$

Dividing by $dx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \to (a)$$

Put ⓐ in Ⓐ

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{x}{xv} + \frac{3vx}{x}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying Both Side by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$= \frac{1+v^2}{v}$$

Multiplying b. Side by $\frac{dx}{dv}$, we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

xing b. Side by $\frac{v}{(1+v^2)}$, we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on b. Side

$$\int \frac{v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on b. Side

$$e^{\ln|1+v^2|} = e^{\ln(xc)}$$

$$1+v^2 = xc$$

put $v = \frac{y}{x}$

$$1+\left(\frac{y}{x}\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \longrightarrow (A)$$

put $x=2$, $y=6$ in eq (A)

$$4+36 = 8c$$

$$\frac{\cancel{40}^{5}}{\cancel{8}_{1}} = \frac{8c}{\cancel{8}} \Rightarrow \boxed{c=5} \text{ put in (A)}$$

So $x^2+y^2 = 5x^3$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking "$\sqrt{\ }$" on b. sides

$y = +x\sqrt{5x-1}$ , $y = -x\sqrt{5x-1}$ or

$$\boxed{y = \pm x\sqrt{5x-1}} \text{ Ans}$$