

Name : Muhammad Sulaiman

ID: 7925

Section : A

Subject : Numerical Analysis

Instructor : Shumaila Mazhar

Summer Mid term

Q.No 1

Find the root of the equation

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2 \quad \text{Use}$$

Newton Raphson method with

$$x_0 = 0.02$$

$$\text{Newton Raphson method} = x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Answer : Solution:

 \Rightarrow Rearranging the equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

Then

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(x_0) = \frac{0.00039}{0.00033}$$

$$f'(x_0) = 0.0054$$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$$x_1 = 0.081$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \rightarrow \textcircled{1}$$

$$x_1 = 0.081$$

$$f(x_1) = -0.00016$$

$$f'(x_1) = -0.0070$$

Put x_1 , $f(x_1)$ and $f'(x_1)$ value
of equation $\textcircled{1}$

$$x_2 = 0.081 - \frac{(-0.00016)}{-0.0070}$$

$$x_2 = 0.058$$

Now

1- (3)

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \rightarrow (2)$$

$$\boxed{x_2 = 0.058}, \quad \boxed{f(x_2) = 0.000030}$$

$$\boxed{f'(x_2) = -0.0090}$$

Put ^{$x_2, f(x_2)$ and $f'(x_2)$} This value in equation (2)

$$x_3 = 0.058 - \frac{(0.000030)}{(-0.0090)}$$

$$x_3 = 0.061$$

Required Answer

$$x_1 = 0.081, \quad x_2 = 0.058$$

$$x_3 = 0.061$$

Q. No. 2

P-04

Use the numbers $x_0 = 2$, $x_1 = 2.75$

$x_2 = 4$ Find the Lagrange
interpolation

polynomial for

$$f(x) = \frac{1}{x} \text{ at } n = 3$$

Answer

Solution:

Given Data

$$x_0 = 2, \quad y_0 = 0.5$$

$$x_1 = 2.75, \quad y_1 = 0.36$$

$$x_2 = 4, \quad y_2 = 0.25$$

As we know that

Lagrange interpolation formula

$$y = \frac{(x-x_1)(x-x_2) \dots (x-x_n)}{(x_0-x_1)(x_0-x_2) \dots (x_0-x_n)} y_0$$

$$x_0 = 2$$

$$x_1 = 2.75$$

$$x_2 = 4$$

$$y_0 = 0.5$$

$$y_1 = 0.364$$

$$y_2 = 0.25$$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

Put value

$$y = \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)} \times (0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} (0.364)$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)} \times (0.25)$$

p-06

$$y = (-0.083) + 0.388 + 0.025$$

$$y = 0.33$$

Answer

Exam

Q.No.3

Complete the divided difference table for the given data and construct the interpolation polynomial that uses all this data

$x =$	1.0	1.3	1.6	1.9	2.2
$y =$	0.7651977	0.6200820	0.4554079	0.2818186	0.1103623

↓

x	x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, x_i)$	$f(x_{i-4}, \dots)$
x_0	1	0.7651977	-0.4837056			
x_1	1.3	0.6200860	-0.548946	-0.108734	0.0658785	
x_2	1.6	0.4554022	-0.578612	-0.0494433		-0.00280
x_3	1.9	0.2818186	-0.571521	0.006818	0.06251255	
x_4	2.2	0.1103623				

j

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \rightarrow \textcircled{1}$$

$$f(x_0, x_1) = \frac{(0.6200860 - 0.7651977)}{(1.3 - 1)}$$

$$f(x_0, x_1) = -0.4837050$$

$f(x_1) = 0.6200860$
 $f(x_0) = 0.7651977$
 $x_1 = 1.3$
 $x_0 = 1$
 Put this value in eq ①

2)

$$f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \rightarrow \textcircled{2}$$

$$f(x_1, x_2) = \frac{(0.4554022 - 0.6200860)}{(1.6 - 1.3)}$$

$$f(x_1, x_2) = -0.548946$$

$f(x_1) = 0.6200860$
 $f(x_2) = 0.4554022$
 $x_1 = 1.3$
 $x_2 = 1.6$
 Put this value in eq ②

3)

$$f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2} \rightarrow \textcircled{3}$$

$$\Rightarrow f(x_2, x_3) = \frac{(0.2818186 - 0.4554022)}{(1.9 - 1.6)}$$

$$f(x_2, x_3) = -0.578612$$

$f(x_2) = 0.4554022$
 $f(x_3) = 0.2818186$
 $x_2 = 1.6$
 $x_3 = 1.9$
 Put this value in eq ③

p-10

4)

$$f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3} \rightarrow \textcircled{4}$$

$$f(x_3, x_4) = \frac{(0.1103623 - 0.2818186)}{(2.2 - 1.9)}$$

$$f(x_3, x_4) = -0.571521$$

Second Divided Difference

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$f(x_0, x_1, x_2) = \frac{(-0.548946 - (-0.4837056))}{(1.6 - 1)}$$

$$f(x_0, x_1, x_2) = -0.108734$$

$$f(x_3) = 0.2818186$$

$$f(x_4) = 0.1103623$$

$$x_3 = 1.9$$

$$x_4 = 2.2$$

put this
value
of eqs

④

P-11

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$f(x_1, x_2, x_3) = \frac{(-0.578612 - (-0.548946))}{(1.9 - 1.3)}$$

$$f(x_1, x_2, x_3) = -0.0494433$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$f(x_2, x_3, x_4) = \frac{(-0.571521 - (-0.578612))}{(2.2 - 1.6)}$$

$$f(x_2, x_3, x_4) = 0.006818$$

Third Divided Difference

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$f(x_0, x_1, x_2, x_3) = \frac{(-0.0494433 - (-0.108734))}{(1.9 - 1)}$$

$$f(x_0, x_1, x_2, x_3) = \boxed{0.0658785}$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{(0.006818 - (-0.049443))}{(2.2 - 1.3)}$$

$$f(x_1, x_2, x_3, x_4) = \boxed{0.06251255}$$

4th Divided ^{P-13} Difference

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{f(x_1, x_2, x_3, x_4) - f(x_0, x_1, x_2, x_3)}{x_4 - x_0}$$

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{(0.06251255 - 0.0658785)}{2.2 - 1}$$

$$f(x_0, x_1, x_2, x_3, x_4) = \boxed{0.0028049}$$

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4)$$

$$f(x) = (0.761977) + (x-1) - 0.4837056 + (x-1)(x-1.3) (-0.168734) + (x-1)(x-1.3)(x-1.6) (0.0658785) + (x-1)(x-1.3)(x-1.6)(x-1.9) (-0.0028049)$$