IQRA NATIONAL UNIVERSITY

Digital Signal Processing

Final Assignment Spring 2020

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Question No (1)

=> Part (A)

Determine the response $y(n)$, $n \geq 0$, of the system describe by the second order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

To the input $x(n) = (-1)^n u(n)$

And the initial condition are $y(-1) = y(-2) = 0$?

Solution :- $\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$ Hence

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is

$$y_p(n) = K(-1)^n u(n)$$

Subsituting this solution into the difference equation we obtain

$$\Rightarrow K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

$\Rightarrow$ For $n=2$, $K(1+4+4)=2$

$K = 2/9$ The total solution is

$$y(n) = \left[C_1 2^n + C_2 n 2^n + 2/9(-1)^n\right] u(n)$$

From the initial conditions

we obtain, $y(0) = 1$, $y(1) = 2$

Than

$$C_1 + \frac{2}{9} = 1$$

$$\Rightarrow C_1 = 7/9$$

$$\Rightarrow 2C_1 + 2C_2 - 2/9 = 2$$

$$\Rightarrow C_2 = 1/3$$

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Question (1)

⇒ Part (B)

Determine the impulse response and unit step response of systems described by the difference equation

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Solution :- The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = 1/2 , 1/5 \quad \text{Hence}$$

$$y_h(n) = C_1 \, 1/2^n + C_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y(1) - 0.7\, y(0) = 0$$

$$\Rightarrow y(1) = 1.4$$

Hence , $C_1 + C_2 = 2$

And ,

$$1/2\, C_1 + 1/5 = 1.4$$

$$1.4 = 7/5$$

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$\Rightarrow \quad C1 + \frac{2}{5} C2 = 14/5$

These equations yield

$C1 = \frac{10}{3}, \quad C2 = -4/3$

$$h(n) = \left[\frac{10}{3} (1/2)^n - \frac{4}{3} (1/5)^n \right] u(n)$$

The step response is

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$s(n) = \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$s(n) = \frac{10}{3} \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3} \left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$s(n) = \frac{10}{3} \left(\frac{1}{2}\right)^n \left(2^{n+1} - 1\right) u(n) - 1/3$$

$$(1/5)^n \left(5^{n+1} - 1\right) u(n).$$

$\longleftrightarrow$

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Question (2)

⇒ Part (A)

Determine the causal signal x(n) having the Z - transform

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Solution :- Taking inverse and Z - transform

$$\frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence

$$x(n) = [4(2)^n - 3 - n]\, u(n)$$

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Question No (2)

=> Part (B)

Evaluate the inverse z transform using the inverse complex invertion Integral

$$x(z) = \frac{1}{1-az^{-1}} \quad |z| > |a|$$

Solution:- using the complex inversion integral.

we have

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz = \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$$

where c is circle at radius greater than |a| we shall evaluate this Integral using (3,4,2) with $f(z) = z^n$

we distiguish two cases.

1 - If $n \geq 0$, $f(z)$ has radius only zeros and hence no poles inside c The only pole inside c is $z = a$

Hence $x(n) = f(z_0) = a^n \quad n \geq 0$

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⇒ 2 - if $n < 0$ $f(z) = z^n$ has an nth- order pole at $z = 0$, which is also inside c Thus there are Contribution from both poles For $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z^2(z-a)} dz = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

if $n = -2$, we have

$$x(-2) = \frac{1}{2\pi j} \oint \frac{1}{z^2(z-a)} dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By Continuing in same way we can show that $x(n) = 0$ for $n < 0$, Thus

$$x(n) = a^n u(n)$$

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Question No (3)

⇒ Part (A)

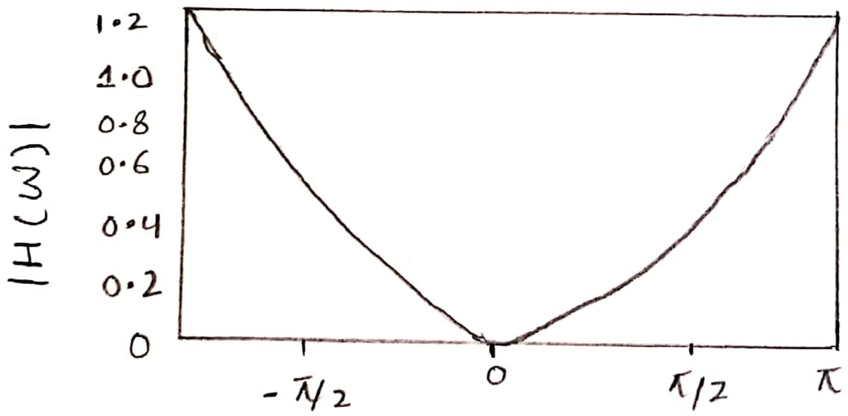
$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

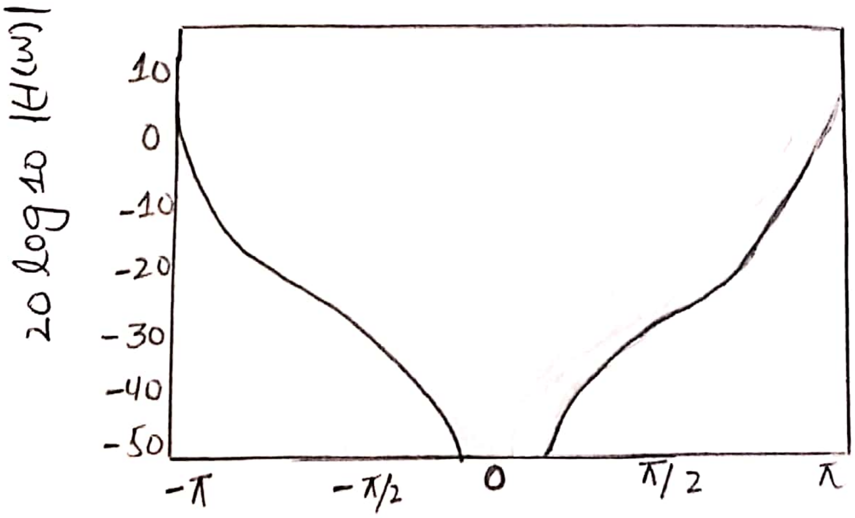
Determine b_0 and p such that $H(\omega)$ satisfies the condition $H(0) = 1 \quad |H(\pi/4)|^2 = 1/2$.

Solution:- At $\omega = 0$ we have

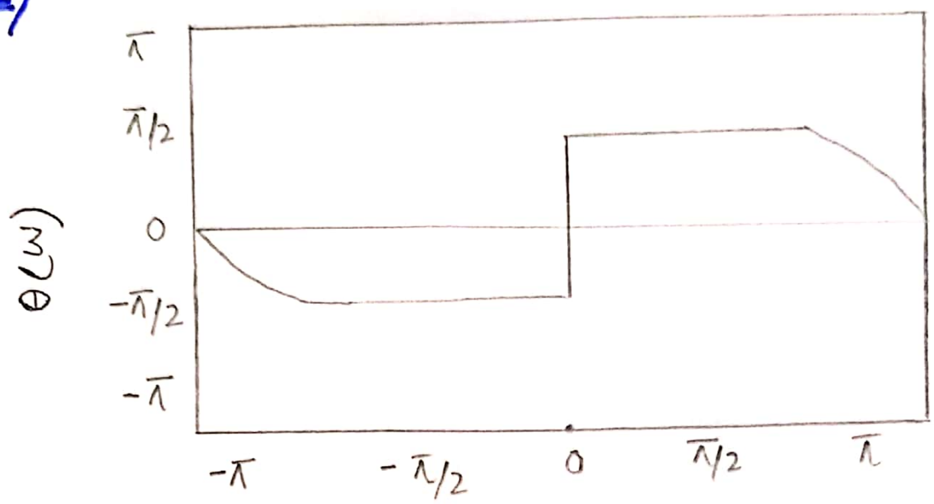
$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $b_0 = (1-p)^2$





⇒



At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{[1-P\cos(\pi/4) + jP\sin(\pi/4)^2]}$$

$$= \frac{(1-P)^2}{[1-P/\sqrt{2}+jP/\sqrt{2})^2}$$

Hence

$$= \frac{(1-P)^4}{[(1-P/\sqrt{2})^2+P^2/2]^2}$$

$$= 1/2$$

⇒ Equivalently

$$\sqrt{2}\,(1-p)^2 = 1 + p^2 - \sqrt{2}\,p$$

The system function for desired filter

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Question No (3)

⇒ part (B)

Design a two pole bandpass filter that has the center of the $\omega = \pi/2$ zero in the frequency response characteristic at $\omega = 0$ and $\omega = \pi$ and its magnitude response is $\frac{1}{\sqrt{2}}$ at $\omega = \frac{4\pi}{9}$.

Solution:- The filter must have poles at $p_{12} = re$

And zero at $z = 1$ And $z = -1$

Consequently. the same system function is.

$$\Rightarrow H(z) = G\,\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$\Rightarrow$ $$H(z) = G\frac{z^2 - 1}{z^2 + r^2}$$

The gain Factor is determined evaluating the frequency response $H(\omega)$ of the the filter at $\omega = \pi/2$

$$H = (\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$

we have

$$|H(4\pi/9)|^2 = \frac{(1-r^2)^2}{4}\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

$\Rightarrow$ Equivalently

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ Satisfies this equation therefor the System Function for the desired filter is.

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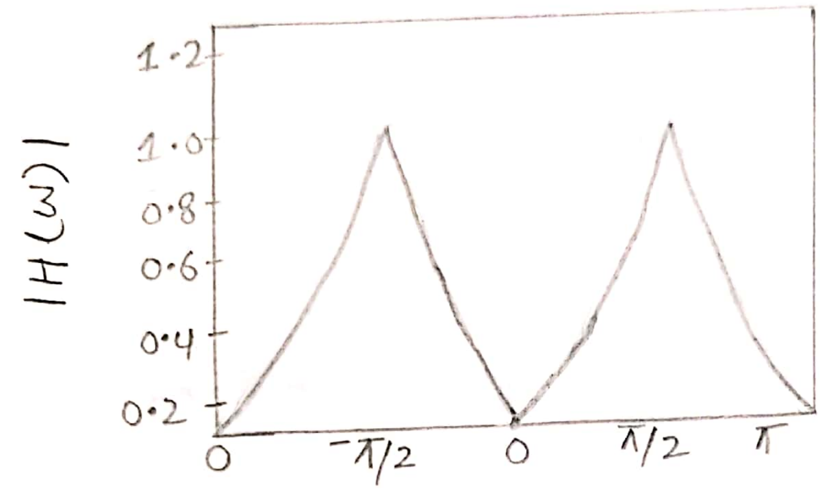
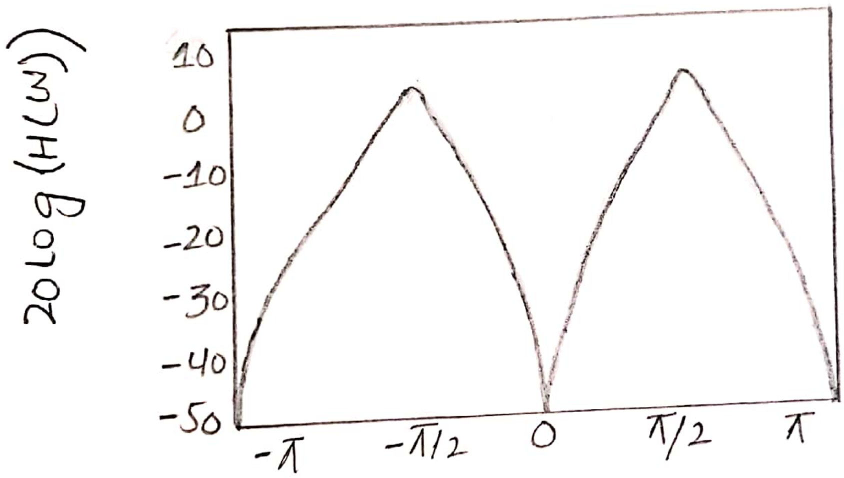
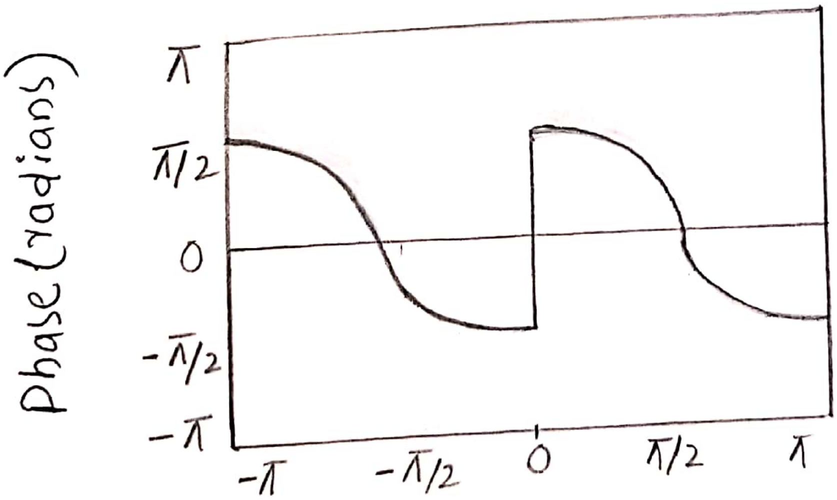
=> $H(z) = 0.15 \dfrac{1 - z^{-2}}{1 + 0.7 z^{-2}}$

its frequency response is illustrated







=> Magnitude and phase response of a simple bandpass filter is

$H(z) = 0.15 \left[(1 - z^{-2}) / (1 + 0.7 z^{-2}) \right]$

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Question No (4)

=> Part (A)

A finit duration sequence of length L given is

$$x[x] = \begin{cases} 1 & 0 \le n \ L-1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the N-point DFT $N \ge L$.

Solution :- The Fourier transform of this sequence is

$$X(w) = \sum_{n=0}^{L-1} x(n)\, e^{-jwn}$$

$$x(w) = \sum_{n=0}^{L=1} e^{-jwn} = \frac{1-e^{-jwL}}{1-e^{-jw}}$$

$$= \frac{\sin(wL/2)}{\sin(w/2)}\, e^{-jw(L-1)/2}$$

The magnitude end phase of $X(w)$ are illustrated for $L = 10$. The N point DFT of $x(n)$ is simply $X(w)$ evaluated at the set of N equally spaced frequency.

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$\Rightarrow$ $\omega_K = 2\pi K/N,$

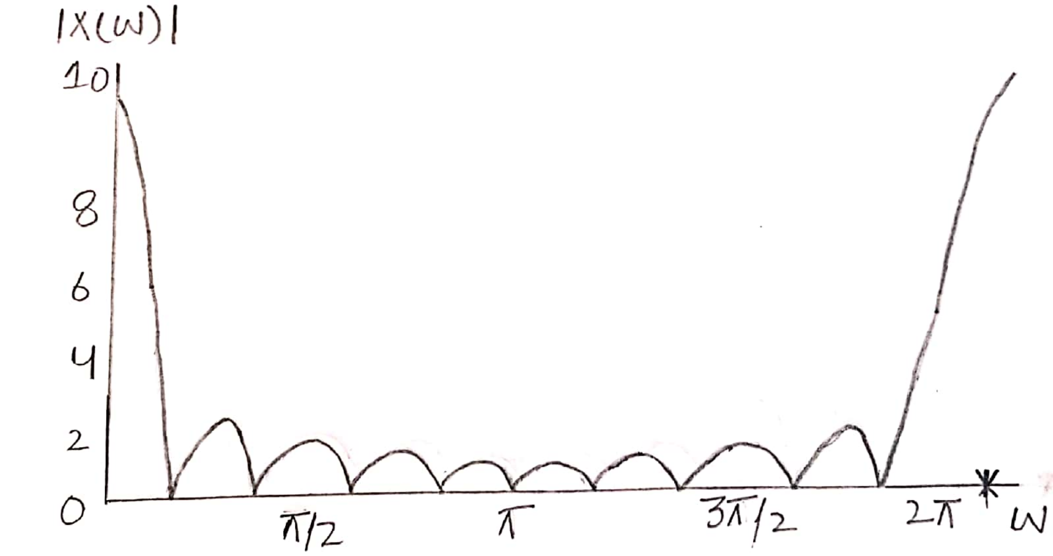
$K = 0, 1, 2, \ldots\ldots N-1$

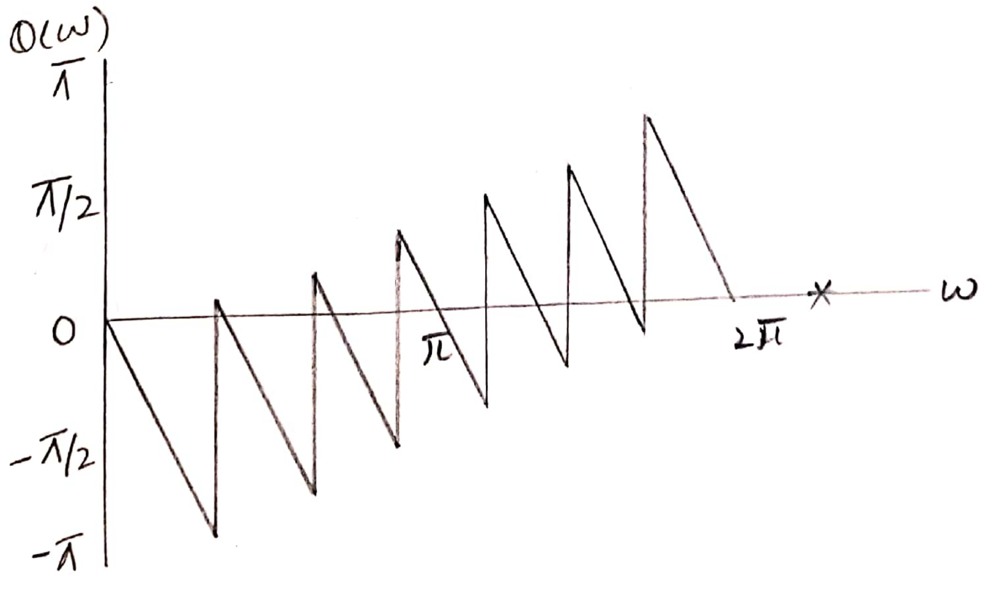
Hence,

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}}$$

$K = 0, 1, \ldots\ldots, N-1$

$$X(K) = \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)N}$$





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→ If N is selected such that $N = L$ then the Discrete Fourier Transform becomes.

$$X(K) = \begin{cases} L, & K = 0 \\ 0, & K = 1, 2, \ldots L-1 \end{cases}$$

Thus there is only one non zero value in the DFT This is apparent from observation of $X(w)$, since $X(w) = 0$ at the frequencies $wk = 2\pi K/L$

$K \neq 0$;

The reader should verify that $x(n)$ can be recoverd from $X(K)$ by performing an L- point IDFT provides a plot of the N- point DFT in magnitude Phase for $L = 10$ $N = 50$ and $N = 100$ Now the spectral characteristic of the sequence are more clearly evident. ⟷

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Question No (4)

=> Part (B)

Perform the circular convolution of the following two sequence.

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Solution:-

Each sequence consist of four nonzero points For the purpose of illustrating the operations involved in circular convolution it is desired to graph each sequence as points on a circle Thus the sequences $x_1(n)$ and $x_2(n)$ are graphed as illustrated we note that the sequences are graphed in a counterclockwise direction on a circle.

Now, $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with

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$\Rightarrow$ $x_1(n)$ with $x_2(n)$ as Specified

Begning with $m = 0$ we have

$$x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2[(-n)]_N$$

$x_2(-n)_4$ is Simply the Sequence $x_2(n)$ folded and graphed on a Circle. The product Sequence is obtained by multiplying $x_1(n)$ with $x_2(-n)_4$ point by point. Finaly we Sum the values in the product Sequence to obtain

$$x_3(0) = 14$$

For $m = 1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)_4$$

it is easily verified that $x_2(1-n)_4$ is Simply the Sequence $x_2(-n)_4$ rotated Counter Clockwise by one unit in the time.

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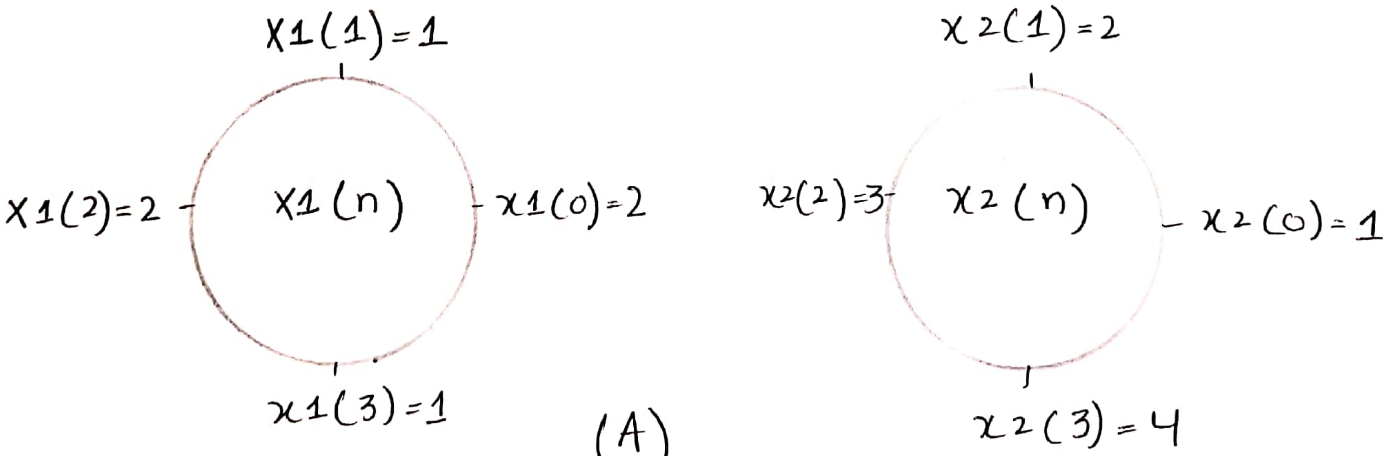
$\Rightarrow$ This rotated sequence multiplies $x_1(n)$ to yield the product sequence also finally we sum the values in the product sequence to obtain $x_3(1)$ Thus.

$$x_3(1) = 16$$

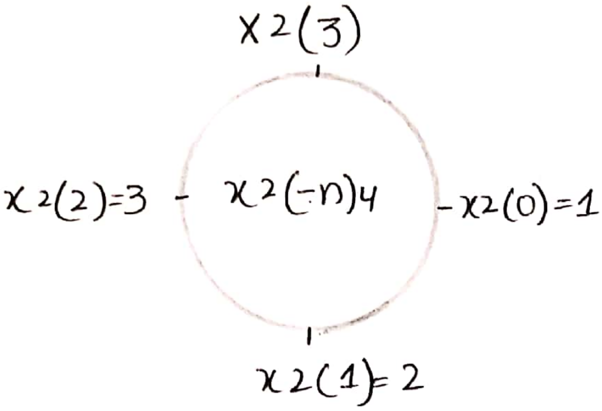
For $m = 2$ we have

$$x_3(2) \sum_{n=0}^{3} x_1(n)\, x_2(2-n)_4$$

Now $x_2(2-n)_4$ is the folded sequence.

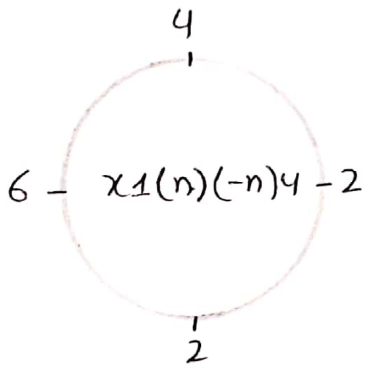


(A)

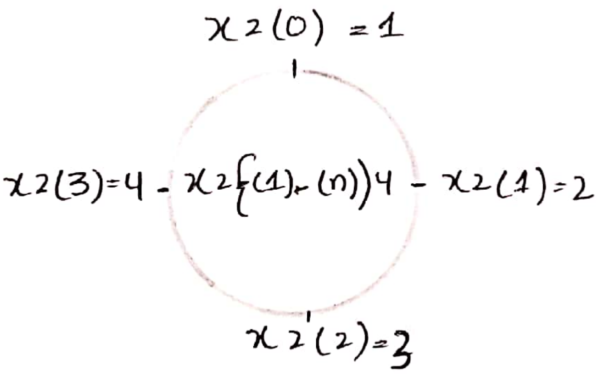


Folded Sequence

(B)

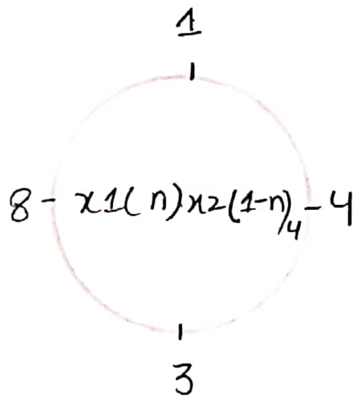


Product Sequence.

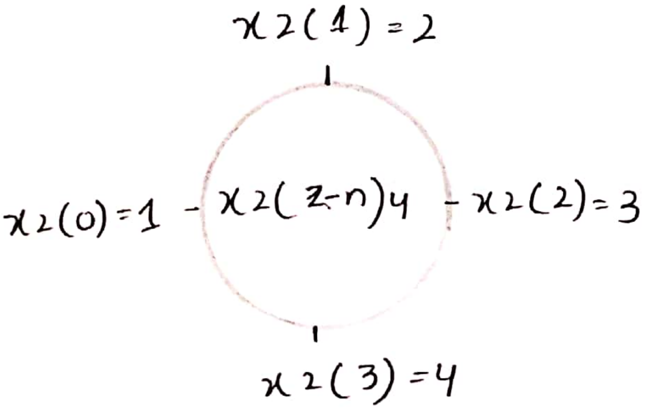


Folded Sequence rotated by one unit in time

(C)

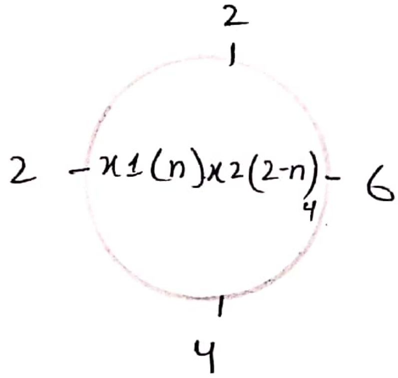


Product Sequence

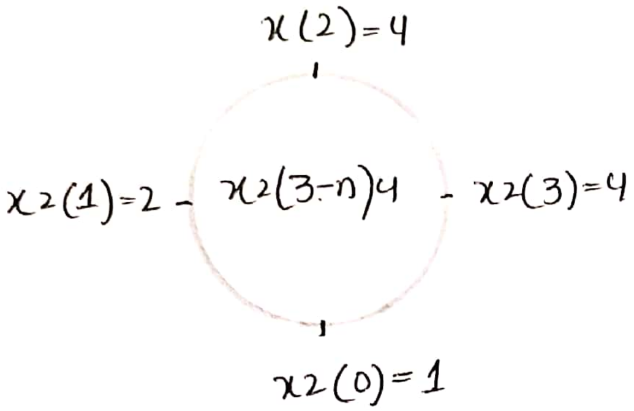


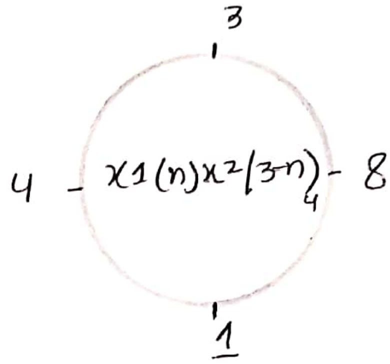
Folded Sequence rotated bd two unit in time

(D)



Product Sequence.





(E)

=> Folded Sequence rotated by three unit in time

=> Product Sequence.

Thank You