Department of Electrical Engineering

Assignment

Date: 13/04/2020

Course Details

Course Title:	Digital Signal Processing	Module:	6th
Instructor:	Sir Peer Mehr Ali Shah	Total Marks:	30

Student Details

Name:	Hafiz Ayub hassan	Student ID:	6997

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n , & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \left\{2, \underset{\uparrow}{1}, -2, 3, -4\right\} \quad , h[n] = \left\{\underset{\uparrow}{3}, 1, 2, 1, 4\right\}$$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^n, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{4})^n, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \ge 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

(1)

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Question No: 1

Part (a) Consider the following analog Signal.

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t.$$

(i) Determine Sampling rate to Avoid Aliasing:

According to Sampling theorom:

$F_1 = 50Hz$ $f_2 = 100Hz$.

$F_s = 2F_{max} \Rightarrow 2\times100 = 200Hz$

$F_N = 200 Hz$

Nequist rate is $\boxed{200 Hz}$

(ii) $F_s = 100 Hz$.

$$\frac{100}{2} = 50Hz.$$

This is the max freq that can be represented by the sampled Signal as:

$$X_a(n) = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n.$$

$$= 3\cos 2\pi\left(\frac{1}{2}\right)n + 4\sin 2\pi(1)n.$$

$$= 3\cos \pi n + 4\sin 2\pi n.$$

⇒ As both the frequencies are not more than folding freq. Hence there will be no effect of sampling over nearly generated Discreate Time Signal.

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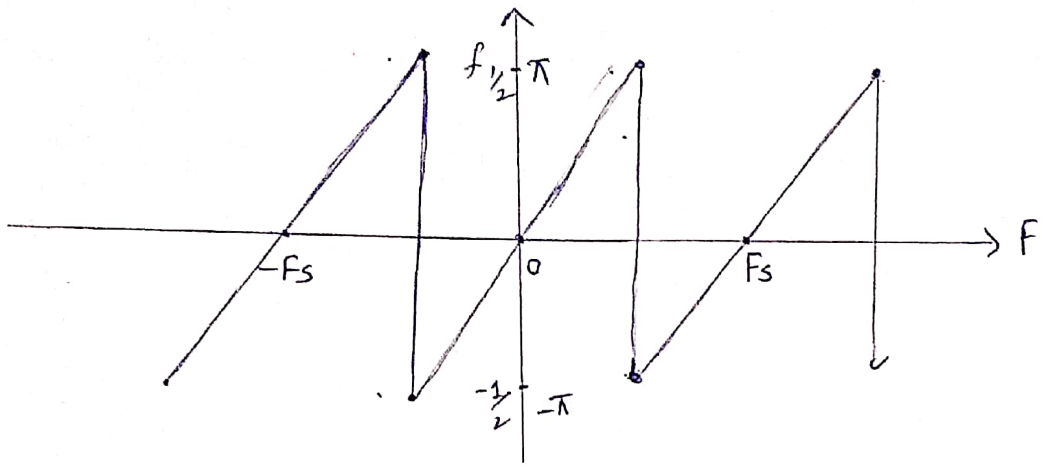

Discreate time Signal.

(iii) Since only the frequency components at 100 Hz are present on the Sampled Signal, the analog Signal we can recover or reconstruct is:

$$y_a(t) = 3\cos 100\pi t.$$ Ans.

X ——————————————— X

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Part (B) Q No: 1

Sol: Consider a discreate time Signal which is given by.

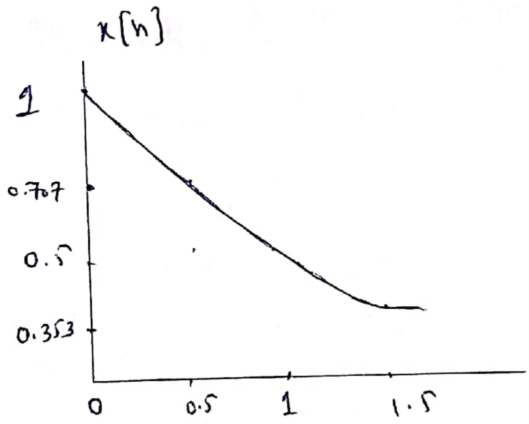
$$x(nT) = \begin{cases} 0.5^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$F_s = 2\,Hz.$$

$$F_s = 1/T \Rightarrow T = \frac{1}{F_s} = 1/2 = 0.5\,sec$$

(i) Draw the Sampled Signal:

$X(n)$	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



T = 0.5 sec

(ii) Quantization Level:

$L = 2^n$

$n = 3$ bits

$L = 2^3 = 8$ level.

$$\text{Resolutions} = \frac{X_{max} - X_{min}}{L} \Rightarrow \frac{1-0}{8} = 0.125$$

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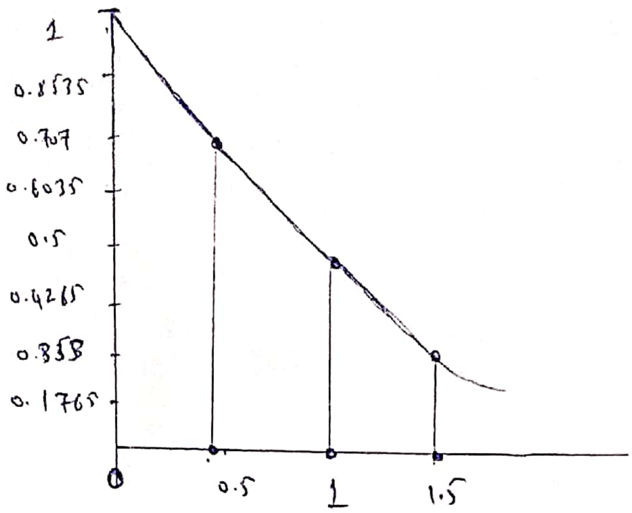

Quantization. Level.

(iii) Perform the Process of Truncation and ....

	Discreate Time Signal	Truncation	Rounding	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

X ———— X

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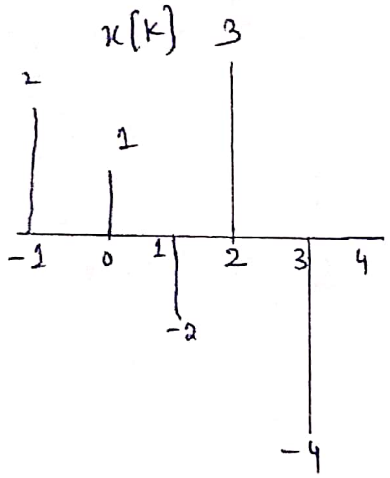
Question No: 2

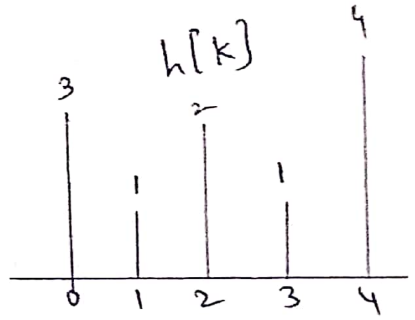
Part (a):

$$x[n] = \{ 2, \underset{\uparrow}{1}, -2, 3, -4 \}$$

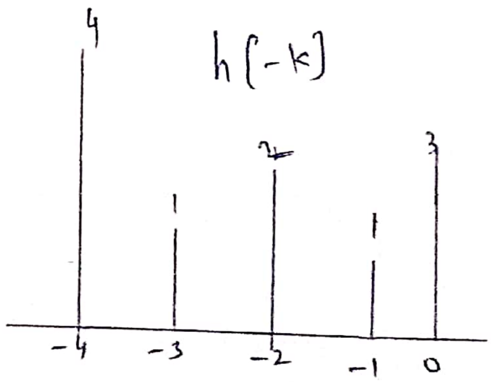
$$h[n] = \{ \underset{\uparrow}{3}, 1, 2, 1, 4 \}$$

Sol: $$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$





Folded signal ⇒



$$y[0] = \sum_{k=-1}^{0} x[-1]\, h[-1] + x(0)\, h(1)$$

$$= 2 \times 1 + 1 \times 3$$

$$= 2 + 3$$

$$= 5$$

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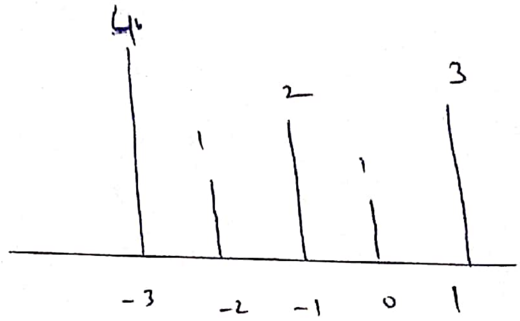
For $n = 1$



$$y[1] = \sum_{n=-1}^{1} x[k]\, h[1-k]$$

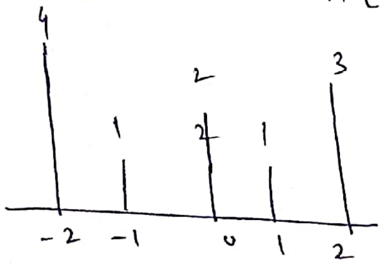
$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1)$$

$$= (2)(2) + (1)(1) + (3)(-2)$$

$$= 4 + 1 - 6$$

$$= -1$$

For $n = 2$: $h[2-k]$



$$y(2) = \sum_{k=-1}^{2} x(n)\, h(2-k)$$

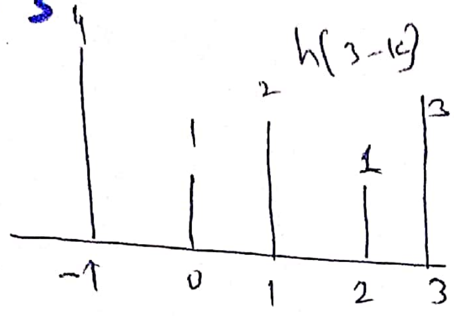
$$= x(-1)\, h(-1) + x(0)h(0) + x(1)(h(1) + x(2)h(2)$$

$$= (2)(1) + (1)(2) + (-2)(1) + (3)(3)$$

$$= 2 + 2 - 2 + 9$$

$$= 11$$

n = 3



$$y(3) = \sum_{k=-1}^{3} x(k)\, h(3-k)$$

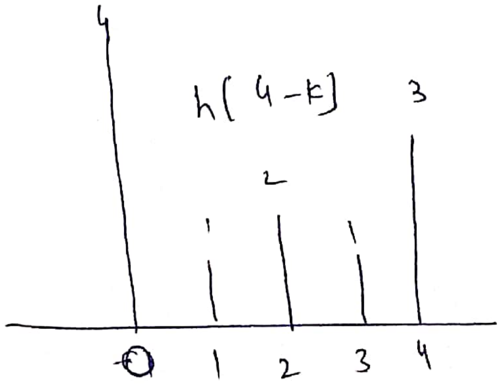
$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$

$= 2\times4 + 1\times1 + -2\times2 + 3\times1 + (-4\times3)$

$= 8 + 1 - 4 + 3 - 12$

$= -4$

n = 4



$$y[4] = \sum_{k=0}^{3} x(k)\, h(4-k)$$

$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$

$1\times4 + -2\times1 + 3\times2 + -4\times1$

$= 4 - 2 + 6 - 4$

$= 4$

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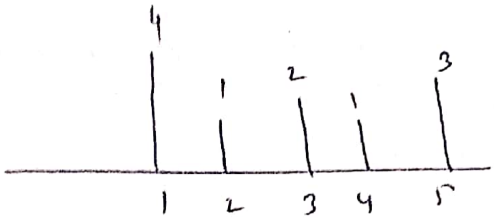
$n = 5$ $\quad h[5-k]$



$$y(5) = \sum_{k=1}^{5} x(k)\, h(5-k)$$

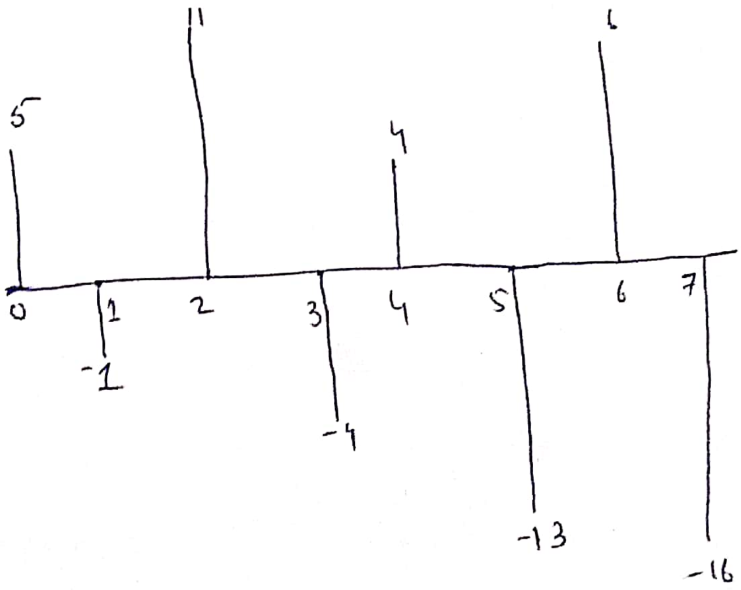
$= x(1)\,h(1) + x(2)\,h(2) + x(3)\,h(3)$

$= (-2)(4) + (3)(1) + (-4)(2)$

$= -8 + 3 - 8$

$= -13$

$y[n] =$



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Q No:2 (b)

$$x(n) = \begin{cases} a^{n+1} & -3 \le n \le 5 \\ 0, & \text{else where} \end{cases}$$

$$h(n) = \begin{cases} 2^{n} & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

Minimum limit = -3

Maximum limit = 5 for both.

$$y[n] = \sum_{k=-3}^{5} x(k)\, h(n-k) \quad \text{(Convolution formula)}$$

$$y(n) = \sum_{k=-3}^{5} a^{k+1} \cdot 2^{n-k} \quad \text{Put } k = n-k$$

$$= \sum_{k=-3}^{5} a^{k} \cdot a \cdot 2^{n} \cdot 2^{-k}$$

$$= a\, 2^{n} \sum_{k=-3}^{5} a^{k} \cdot 2^{-k}$$

$$y[n] = 2^{n} a \sum_{k=-3}^{5} (2^{-1} a)^{k}$$

$$y(n) = 2^{n} a \sum_{k=-3}^{5} (2^{-1} a)^{k}$$

$$\text{or} \quad y[n] = 2^{n} a \sum_{k=-3}^{5} \left(\frac{a}{2}\right)^{k}$$

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for $a = 1$ $$\sum_{k=M}^{N} a^k = N - M + 1$$

for $a \neq 1$ $$\sum_{k=M}^{N} a^k = a^M + a^{M+1} \text{----} a^N$$

$$(1-a) \sum_{k=M}^{N} a^k = a^M - a^{N+1}$$

Also:

$$\sum_{k=M}^{N} a^k = \begin{cases} \dfrac{a^M - a^{N+1}}{1-a} & a \neq 1 \\ N - M + 1 , & a = 1 \end{cases}$$

$$y(n) = a 2^n \sum_{k=-3}^{5} \left(\frac{a}{2}\right)^k$$

here $M = -3$

$N = 5$

$a = \frac{a}{2}$

Applying formula:

$$= a 2^n \left[a^{-3} + a^{-2} + a^{-1} + a^0 + a^1 + a^2 + a^3 + a^4 + a^5 \right]$$

$$y(n) = 2^n \left[a^{-2} + a^{-1} + a^0 + a^1 + a^2 + a^3 + a^4 + a^5 + a^6 \right]$$

Now:

$$y(-3) = \frac{1}{8} \left[a^{-2} + a^{-1} + \text{----} a^6 \right]$$

$$y(-4) = \frac{1}{16} \left[a^{-2} + a^{-1} \text{----} a^6 \right]$$

$$y(-2) = \frac{1}{4} \left[a^{-2} + a^{-1} \text{---} a^6 \right]$$

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$y(-1) = \frac{1}{2}\left[a^{-2} + a^{-1} + \cdots a^{6}\right]$

$y(0) = 1\left[a^{-2} + a^{-1} \cdots a^{6}\right]$

$y(1) = 2\left[a^{-2} + a^{-1} \cdots a^{6}\right]$

$y(2) = 4\left[a^{-2} + a^{-1} \cdots a^{6}\right]$

$y(3) = 8\left[a^{-2} + a^{-1} \cdots a^{6}\right]$

$+$

$\vdots$

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Question NO: 3

Determine the Z-Transform. Also sketch its Region of Convergence (ROC)

(i) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n , & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n} , & n < 0 \end{cases}$

Sol: As we know that, Z-Transform Pair is -

$x(n) = a^n u(n) \longleftrightarrow X(z) = \frac{1}{1-\alpha z^{-1}}$ ROC $|z|$ eq (A)

By using eq (A) we can put values

$$X(z) = \sum_{n>0}^{\alpha} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=-\alpha}^{\alpha} \left(\frac{1}{3}\right)^n z^{-n} - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\alpha} \left(\frac{1}{3}\right)^n z^n - 1$$

In simplest form.

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$$

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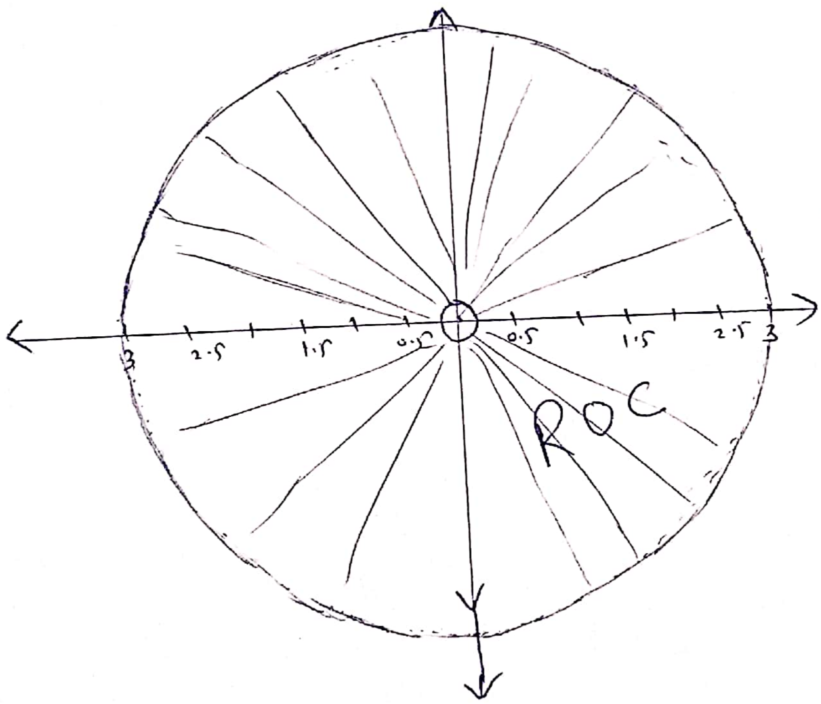
$$= \frac{7/12}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

= The ROC is $\frac{1}{4} < |z| < 3$

The sketch is as Under.



(ii) $x(n) = \begin{cases} \frac{1}{2}^n - 3^n & n \geq 0 \\ 0 & \text{Else where} \end{cases}$

Solution: Using z-Transform pair equation here.

i.e $x(n) = \alpha^n u(n) \longleftrightarrow X(z) = \dfrac{1}{1-\alpha z^{-1}}$

Roc $|z| > |\alpha|$

eq (B)

By putting values.

$$x(z) = \sum_{n=0}^{\alpha} \left(\tfrac{1}{2}\right) z^{n} - \sum_{n=0}^{\alpha} 3^n z^{-n}$$

$$= \frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3 z^{-1}}$$

$$= \frac{-\frac{5}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3 z^{-1}\right)}$$

A seen, The Roc uses $|z| > 2$

The sketch are.