Iqra National University, Peshawar
Department of Electrical Engineering

FINAL – ASSIGNMENT SPRING2020
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Course Code: MTH 102 **Course Title:** Calculus and analytic geometry
Prerequisite: **Instructor:** HIMAYATULLAH
Module: 3 **Program:** BEE **Total Marks:** 50 **:** Mansoor Jadoon 16637

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	a	. **Estimate** $\int \theta \sqrt[4]{1-\theta^2}\ d\theta$	Marks 7 PLO2 C2
	b	**Estimate** $\int_0^1 x^3(1+x^4)^3 dx$ using substitution method.	Marks 7 PLO2 C2
Q2	(a)	**Illustrate** the centre and radius of the sphere $x^2+y^2+z^2+3x-4z+1$.	Marks 5 PLO1 C3
	(b)	The region between the curve $y=\sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. **Apply** the integration find the volume of solid.	Marks 4 PLO1 C3
Q3		If A= $2i-4j+\sqrt{5}k$, $and\ B=-2i+4j-\sqrt{5}k$ then **illustrate** the vector $proje_A\ B$	Marks 9 PLO1 C3
Q4		Find the area of the region between the graph and the x-axis Where $y=-x^2+5x-4$, $[0,\ 2]$.	Marks 9 PLO1 C3
Q5	(a)	**Estimate** the angle between $A=i-2j-2k\ and\ B=6i+3j+2k$	Marks 5 PLO1 C3
	(b)	**Change** into a spherical coordinate equation for the sphere x^2+y^2+	Marks 4 PLO1 C3

		$(z-1)^2 = 1$	PLO2 C2

Q NO 1 (Pa)

Ans:

Given

$$\int \varphi \sqrt[4]{1-\varphi^2}\, dn$$

Solution

let

$$1-\varphi^2 = u$$

$$\frac{d}{d\varphi}(1-\varphi^2) = \frac{d}{d\varphi}u$$

$$-2\varphi = \frac{dn}{d\varphi}$$

$$\varphi\, d\varphi = \frac{-1}{2}\, du$$

Now

$$= \int (u)^{1/4} \cdot \left(\frac{-1}{2}\right) du$$

$$= \frac{-1}{2}\int u^{1/4}\, du \quad \because \frac{1}{4}+1$$

$$= \frac{-1}{2} \cdot \frac{4}{5}\, u^{5/4} + C$$

$$= \frac{-2}{5}\, u^{5/4} + C$$

By back substitution

$$\boxed{= \frac{-2}{5}(1-\varphi^2)^{5/4} + C}$$

1

Q# 1 P(b):

Ans:-

$\int_0^1 x^3(1+x^4)^3\,dx$

Taking $(1+x^4) = u$

$u = (1+x^4)$

applying d/dx b.s

$\frac{du}{dx} = \frac{d}{dx}(1+x^4)$

$\frac{du}{dx} = \frac{d}{dx}1 + \frac{d}{dx}x^4$

$\frac{du}{dx} = 4x^3$

$du = 4x^3\,dx$

$\int_0^1 x^3(1+x^4)^3\,dx$

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(2)

$\therefore 4x^3\,dx = du$

So

$$\frac{1}{4}\int_0^1 u^3\,du$$

So take

$u = (1+x^4)$

put $x = 0$

Now $u(1+0)$

$x = 1$

$u = (1+1^4)$

$u = 2$

So limits are

$$\frac{1}{4}\int_1^2 u^3\,du$$

(3)

(4)

apply intregation

$$\frac{1}{4}\int_1^2 x^3 dx$$

$$= \frac{1}{4}\left(3x^2 \Big|_1^2\right)$$

putting limits

$$= \frac{1}{4}\left(3(2)^2 - 3(1)^2\right)$$

$$= \frac{1}{4}\left(3(4) - 3\right)$$

$$= \frac{1}{4}\left(12 - 3\right)$$

$$= \frac{1}{4}(9)$$

$$\boxed{= \frac{9}{4}}$$

(4)

Q#2 (a). (5)

Find the center of the radius of the sphere.

$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$

Solution.

$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$

$(x^2 + 3x) + y^2 + z^2 - 4z + 1 = 0$

$(x^2 + 3x + (3/2)^2) + (y)^2 + (z^2 - 4z + (4/2)^2)$
$= -1 + (3/2)^2 + (-4/2)^2$

$(x + 3/2)^2 + (y + 0)^2 + (z - 2)^2$
$= \frac{21}{4}$

So $(x_0, y_0, z_0) \Rightarrow$ Center
$= (-3/2, 0, 2)$

find $\boxed{\text{Radius } a = \sqrt{\frac{21}{4}}}$

(5)

(c)

Question No 2 (Part b)

Answer

Given :-

$y = \sqrt{x}$

$0 \leq x \leq 4 \Rightarrow a \leq x \leq b$

~~We~~

Solution :

We know that

$V = \int_a^b \pi y^2 dx$

$= \int_a^4 \pi (\sqrt{x})^2 dx$

$= \pi \int_a^4 x dx$

$= \pi \left. \frac{x^2}{2} \right|_a^4$

$= \frac{\pi}{2}(4^2 - 0) \Rightarrow \frac{\pi}{2} 16$

Result $\boxed{V = 8\pi}$ Ans (6)

Q NO 3

$A = 2i - 4j + \sqrt{5}k$ and

$B = 2i + 4j - \sqrt{5}k$

then illustrate the vector proje AB

Solution.

By pot project

$B \cdot A = (-2i + 4j - \sqrt{5}k)$

$(2i - 4j + \sqrt{5}k)$

$B \cdot A = (-2j + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$B \cdot A = (-\cancel{2}j + \cancel{2}j + \cancel{4}j - \cancel{4}j - \sqrt{5}\cancel{k} + \sqrt{5}k)$

$B \cdot A = -4 - 16 - 5$

$\boxed{B \cdot A = -25}$

Now

$A \cdot A = (2j - 4j + \sqrt{5}k) \cdot (2j - 4j + \sqrt{5}k)$

$A \cdot A = (2ij - 2i) + (-4j \cdot 4j + (\sqrt{5}k + \sqrt{5}k)$

$A \cdot A = 4 + 16 + 5$

$= 25$

So proj $A^B = \left(\frac{B \cdot A}{A \cdot A}\right) A$

As proj $A^B = \left(\frac{B \cdot A}{A \cdot A}\right) A$

7

8

By putting value

$$= \frac{-25}{25}\left(2\hat{i} - 4\hat{j} + \sqrt{5}\hat{k}\right)$$

$$= (-1)\left(2\hat{i} + 4\hat{j} + \sqrt{5}\hat{k}\right)$$

$$\boxed{= -2\hat{i} + 4\hat{j} - \sqrt{5}\hat{k}}$$ An

9

Question No 4

Find area of region between the graph and the x-axis where $y = -x^2 + 5x - 4$, $(0, 2)$

Solution.

$y = fx = -x^2 + 5x - 4$

and

$[a, b] = [0, 2]$

as

$a = 0$

$b = 2$

So area under graph will

$A = \int_a^b f(x)\,dx$ (by putting value)

$A = \int_0^2 (-x^2 + 5x - 4)\,dx$

$-\frac{x^3}{3} + \frac{5x^2}{2} - 4 \Big|_0^2$

$= \frac{(-2)^2}{3} + \frac{5(2)^2}{2} - 4(2) - \frac{(0)^2}{3} + \frac{5(0)^2}{2} - 4(0)^2$

$= \left(\frac{-4}{3} + \frac{20}{2} - 8\right) - 0 + 0 - 0$

$= \frac{-4}{3} + 10 - 8$

$= \frac{-4}{3}\ 2 \Rightarrow \frac{-4+6}{3}$

$\boxed{A = 2/3}$ (9)

Q# 5 (a) (10)

Estimate the angle between $A = \hat{i} - 2\hat{j} - 2\hat{k}$, $B = 6\hat{i} + 3\hat{j} + 2\hat{k}$

Solution.

$A = \hat{i} - 2\hat{j} - 2\hat{k}$

$|A| = \sqrt{1 + 4 + 4} = \sqrt{9}$ $\boxed{\Rightarrow 3}$

$|B| = 6\hat{i} + 3\hat{j} + 2\hat{k}$

$= \sqrt{36 + 9 + 4}$

$= \sqrt{49}$

$\boxed{|B| \Rightarrow 7}$

$\theta = \cos^{-1} \dfrac{A - B}{|A| - |B|}$

$\theta = \cos^{-1} \dfrac{((\hat{i} - 2\hat{j} - 2\hat{k}) - (6\hat{i} + 3\hat{j} + 2\hat{k}))}{3 \times 7}$

$\theta = \cos^{-1} \dfrac{((1)(6) + (-2)(3) + (-2)(2))}{21}$

$\boxed{\theta = \cos^{-1}(-4/21)}$

(10)

Q # 5 (3) (11)

find a sperad coordinate equation for the sphere.

$$x^2+y^2+(z-1)^2=1$$

Solution.

$$x^2+y^2+(z-1)^2=1$$

$$(\rho \sin\phi \cos\phi)^2 + (\rho \sin\phi \sin\phi)^2 + (\rho \cos\phi - 1)^2 = 1$$

$$\rho^2 \sin^2\phi \cos\phi^2 + \rho^2 \sin^2\phi \sin^2\phi$$

$$\rho^2 \cos\phi^2 + 1 - 2\rho \cos\phi = 1$$

$$\rho^2 \sin^2\phi (\cos\phi^2 + \sin^2\phi) + \rho^2 \cos\phi^2 + 1 - 2\rho\cos\phi = 1$$

$$\rho^2 (\sin^2\phi) + \rho^2 \cos\phi^2 - 2\cdot\rho(\cos\phi = 1-1$$

$$\rho^2 (\sin\phi^2 + \cos^2\phi) - 2\rho \cos\phi = 0$$

$$\rho^2 = 2\rho \cos\phi$$

$$\boxed{\rho = 2\cos\phi}$$

11