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| Subject | Theory of Structure II |
| Exam | Final term |
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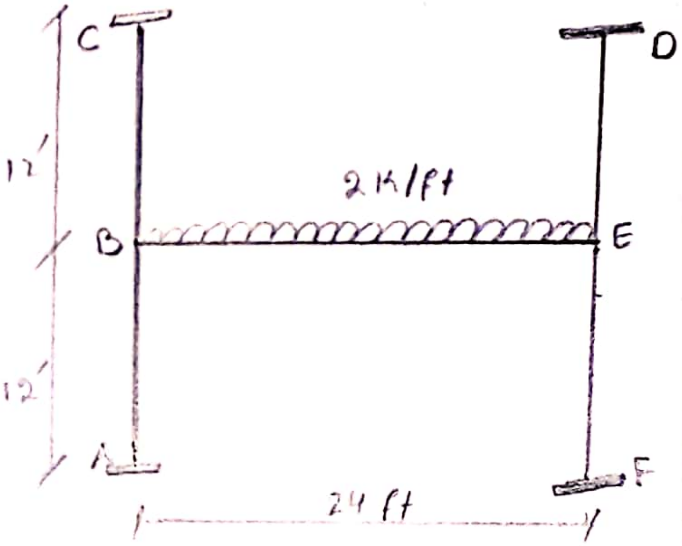
| Department | B.tech Civil |
| Date | 25/09/2020 |

# Q.NO 1

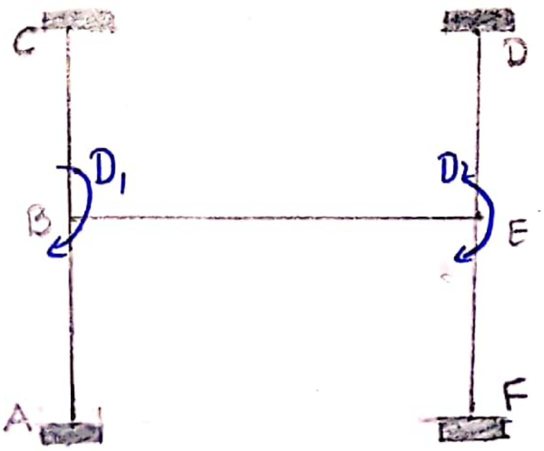
## Given data:

=> K.I = 2 degree

=> Take EI = Constant.



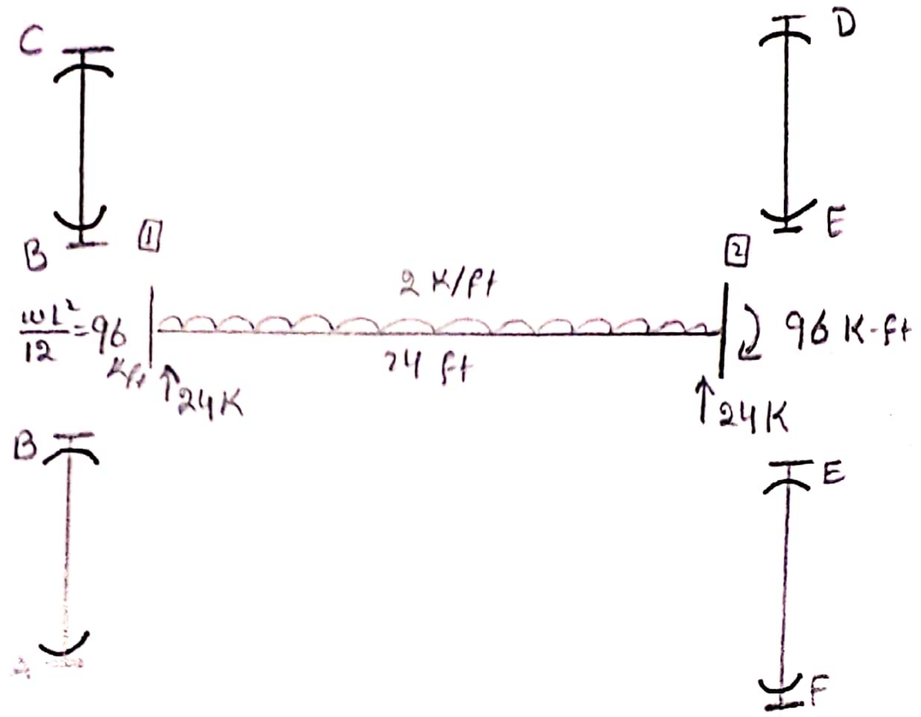

**Step 01** Selection of Redundant Joint displacement and assign Coordinates at those location. Also Compute AD values.




$$\Rightarrow [D]_{2\times1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\Rightarrow [AD]_{2\times1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

**Step No 2** Compute ADL matrix (Fixed end Action).




Now; $[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} -96 \\ 96 \end{bmatrix}$

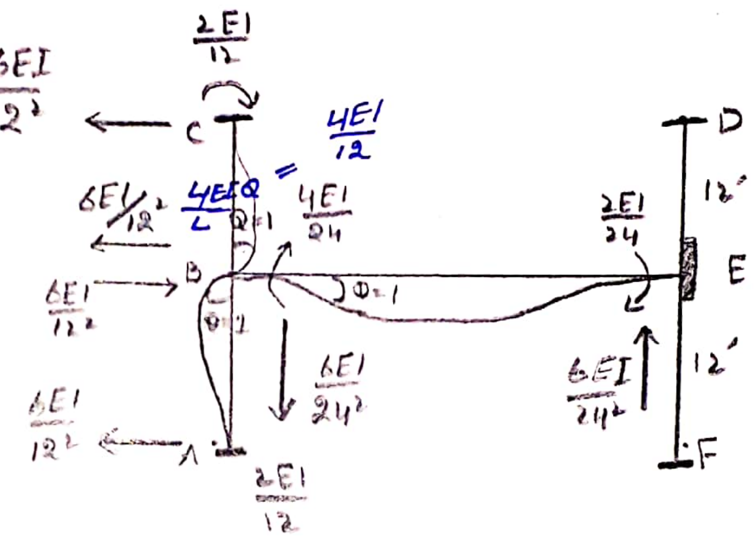
**Step No: 3** primary Structure acted upon by a unit value of D and Computation of Stifness Coefficients "S" values in the BKDS corresponding to the redundant joint displacement location.

* 1st a unit rotation is applied at location 1 & prevented at 2 as shown
  Compute the value of $S_{11}$ and $S_{21}$

* Then apply a unit rotation at the redundant displacement location 2 and prevented at 1 as shown. Compute the value of $S_{12}$ and $S_{22}$.

(i) 1st $D_1 = 1$ and $D_2 = 0$

Now;



Now;

$$S_{11} = \frac{4EI}{12} + \frac{4EI}{12} + \frac{4EI}{24}$$

$$\Rightarrow \boxed{S_{11} = 0.833\,EI}$$

$$S_{21} = \frac{2EI}{24}$$

$$\Rightarrow \boxed{S_{21} = 0.0833\,EI}$$

(ii) Now $D_2 = 1$ and $D_1 = 0$.

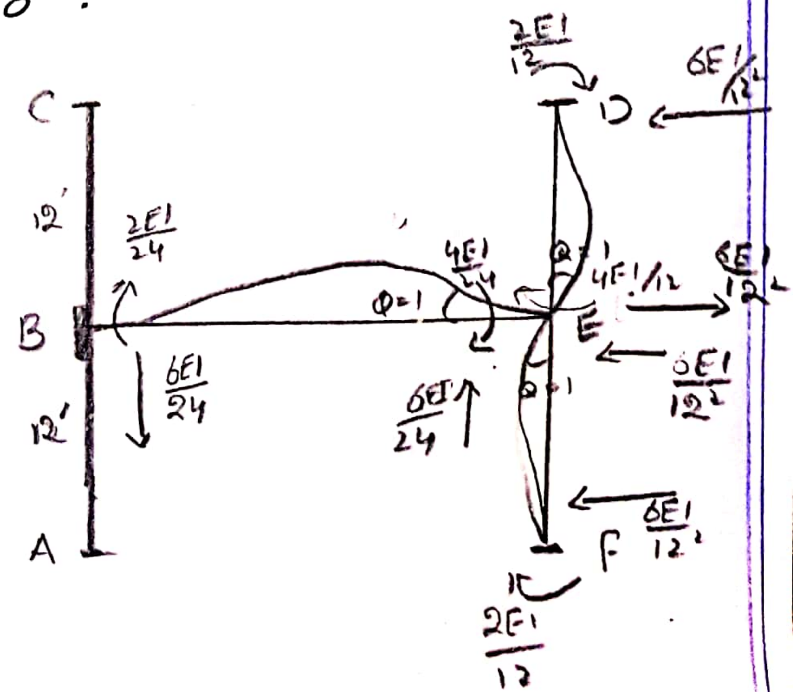
Now as;

$$\Rightarrow S_{12} = \frac{2EI}{24}$$

$$\Rightarrow \boxed{S_{12} = 0.083EI}$$

$$S_{22} = \frac{4EI}{24} + \frac{4EI}{12} + \frac{4EI}{12}$$

$$\Rightarrow \boxed{S_{22} = 0.833\,EI}$$



Now;

$\Rightarrow S_{11} = 0.833\, EI$

$\Rightarrow S_{12} = 0.0833 EI$

$\Rightarrow S_{21} = 0.0833 EI$

$\Rightarrow S_{22} = 0.833 EI$

$$\Rightarrow [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$\Rightarrow [S] = EI \begin{bmatrix} 0.833 & 0.0833 \\ 0.0833 & 0.833 \end{bmatrix} \rightarrow \text{stiffness co-efficient matrix.}$$

Step No 4 :

Apply Equilibrium condition at the location of the redundant joint displacement to write equilibrium equation and solve for unknow joint displacement.

$$\Rightarrow AD_1 = ADL_1 + S_{11} D_1 + S_{12} D_2$$

$$\Rightarrow AD_2 = ADL_2 + S_{21} D_1 + S_{22} D_2$$

$$\Rightarrow \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\Rightarrow [AD]_{2\times 1} = [ADL]_{2*1} + [S]_{2*2} \cdot [D]_{2\times 1}$$

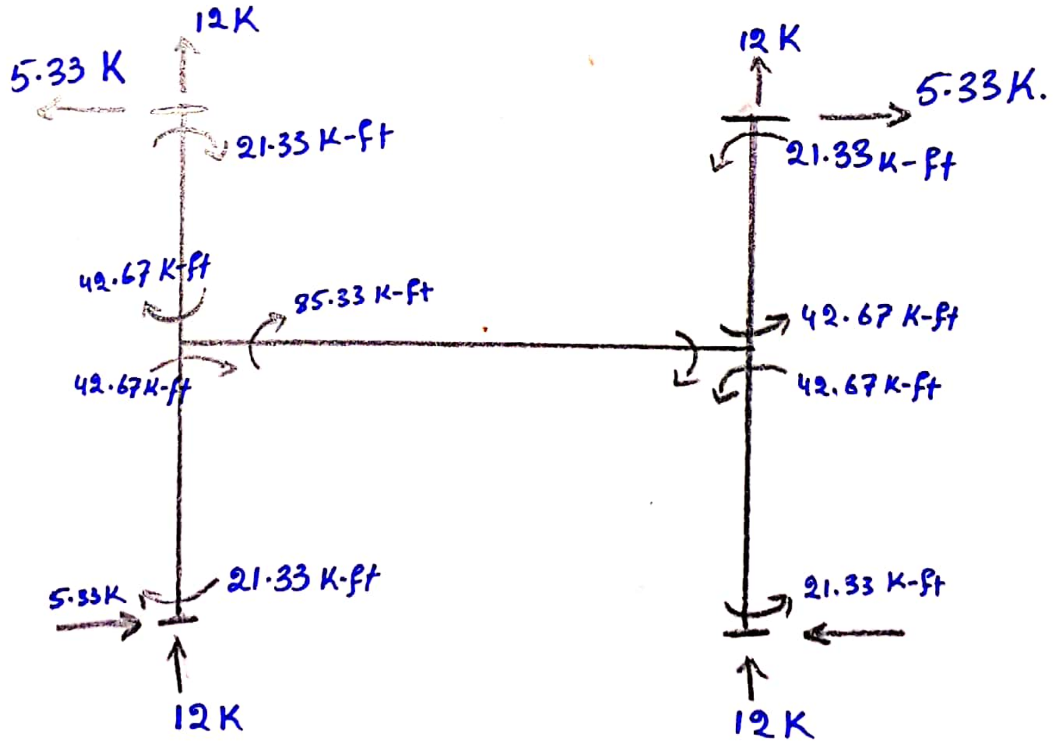
$$[D] = [S]^{-1} \cdot [AD - ADL]$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 0.833 & 0.0833 \\ 0.0833 & 0.833 \end{bmatrix} \begin{bmatrix} 0-(-96) \\ 0-96 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 128 \\ -128 \end{bmatrix} \frac{1}{EI}$$

Here -ive sign show that our assumed redundant joint displacement direction is wrong.



# QNO = 2

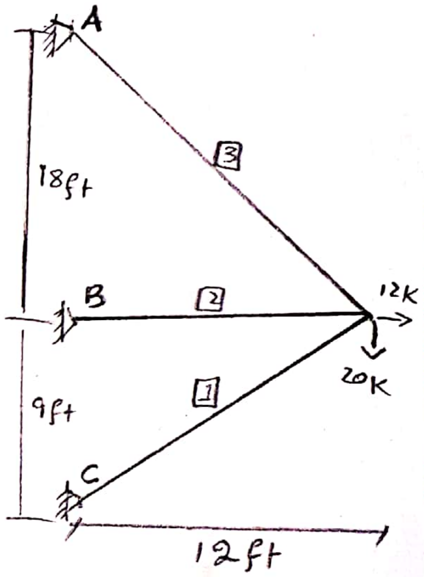
## Given data:

⇒ Take $EA$ = Constant

⇒ $L_1 = \sqrt{9^2+12^2} = 15\,ft$

⇒ $L_2 = 12\,ft$

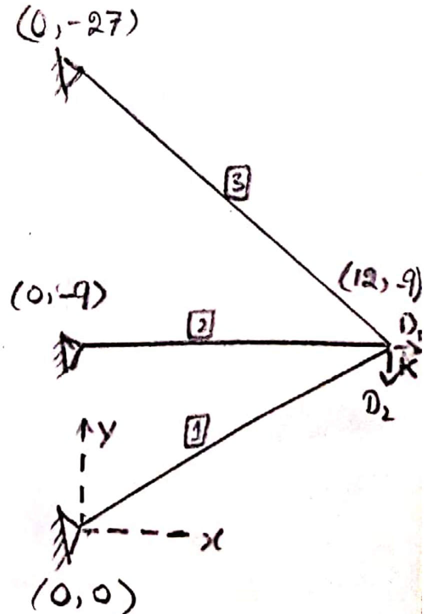
⇒ $L_3 = 21.63\,ft$




## Solution:

**Step#1** Identify the unknown joint displacements and compute the value of [AD] matrix.

⇒ Chose one point as an origin and assign Coordinates to each joint w.r.t the chosen origin. Here point is taken as origin.



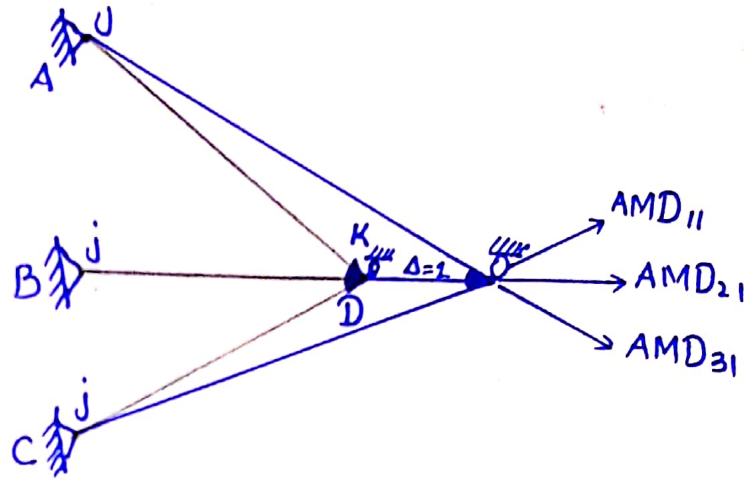

⇒ $[D]_{2\times1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$

⇒ $[AD]_{2\times1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \end{bmatrix}$

Step No2:-

Now; Computation of AMD and Stiffness matrices.

1. When $D_1 = 1$ and $D_2 = 0$




$$\Rightarrow AMD_{11} = \frac{EA}{L^2}(x_K - x_j) = \frac{EA}{15^2}(12-0)$$

$$= 0.0533\,EA$$

$$\Rightarrow AMD_{21} = \frac{EA}{L^2}(x_K - x_j) = \frac{EA}{12^2}(12-0)$$

$$= 0.0833\,EA$$

$$\Rightarrow AMD_{31} = \frac{EA}{L^2}(x_K - x_j) = \frac{EA}{21.63^2}(12-0)$$

$$= 0.0256\,EA$$

Now; $S_{11} = \frac{EA}{L^3}(x_K - x_j)^2 = \frac{EA}{15^3}(12-0)^2 +$

$$\frac{EA}{(12)^3}(12-0) + \frac{EA}{(21.63)^3}(12-0)^2$$

$$\Rightarrow S_{11} = 0.1402\,EA$$

$$\Rightarrow S_{21} = \frac{EA}{L^3}(x_K - x_j)(y_K - y_j)$$

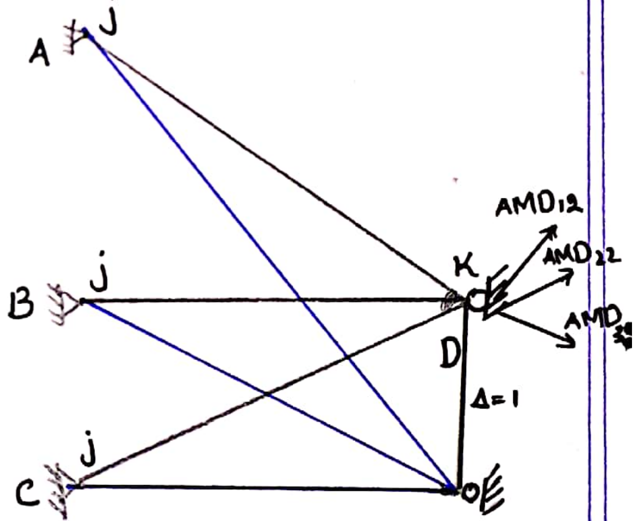
$$= \frac{EA}{15^3}(12-0)(-9-0) + \frac{EA}{12^3}(12-0)(-9-0) +$$

$$\frac{EA}{(21.63)^3}(12-0)(-9-0).$$

$$\Rightarrow S_{21} = 0.0107\ EA$$

Step 02

$\Rightarrow$ When $D_2 = 1$ and $D_1 = 0$




$$AMD_{12} = \frac{EA}{L^2}(y_k - y_j) = \frac{EA}{15^2}(-9+0) = 0.04EA$$

$$AMD_{22} = \frac{EA}{L^2}(y_k - y_j) = \frac{EA}{12^2}(-9-(-9)) = 0$$

$$AMD_{32} = \frac{EA}{L^2}(y_k - y_j) = \frac{EA}{21.63^2}(-9-(-12)) =$$

$$0.0385EA$$

$$\Rightarrow S_{12} = \frac{EA}{L^3}(x_k - x_j)(y_k - y_j) = \frac{EA}{15^3}(12-0) + \frac{EA}{12^3}(12-0)(-9-(-9)) + \frac{EA}{21.63^3}(12-0)(-9-(-27)) = 0.0107EA$$

$$\Rightarrow S_{22} = \frac{EA}{L^3}(y_k - y_j)^2 = \frac{EA}{15^3}(-9-0)^2 + \frac{EA}{12^3}(-9+9)^2 + \frac{EA}{21.63^3}(-9-27)^2 = 0.056EA$$

Now, AMD matrix will be

$\Rightarrow AMD_{11} = 0.0533EA$ ; $\Rightarrow AMD_{21} = 0.0833EA$

$\Rightarrow AMD_{31} = 0.0256EA$

$\Rightarrow AMD_{12} = -0.04EA$; $\Rightarrow AMD_{22} = 0$ $AMD_{32} = 0.0385EA$

$$\Rightarrow [AMD] = EA\begin{bmatrix} 0.0533 & -0.04 \\ 0.0833 & 0 \\ 0.0256 & 0.0385 \end{bmatrix}$$

Now;

$\Rightarrow$ Stiffness matrix will be;

$S_{11} = 0.1402EA$ $\quad S_{21} = -0.0107EA$

$S_{12} = -0.0107\,EA \qquad S_{22} = 0.056\,EA$

$$\Rightarrow [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \quad [S] = EA \begin{bmatrix} 0.1402 & -0.0107 \\ -0.0107 & 0.056 \end{bmatrix}$$

## Stiffness method for Truss Analysis

**Step# 03:** Apply equilibrum condition at The location of The redundant joint displacement to write equilibrium equation and Solve unknown joint displacment.

$$AD_1 = S_{11}\,D_1 + S_{12}\,D_2$$

$$AD_2 = S_{21}\,D_1 + S_{22}\,D_2$$

$$\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$[AD]_{2\times 1} = [S]_{2\times 2} \cdot [D]_{2\times 1}$$

$$[D] = [S]^{-1} \cdot [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix}$$

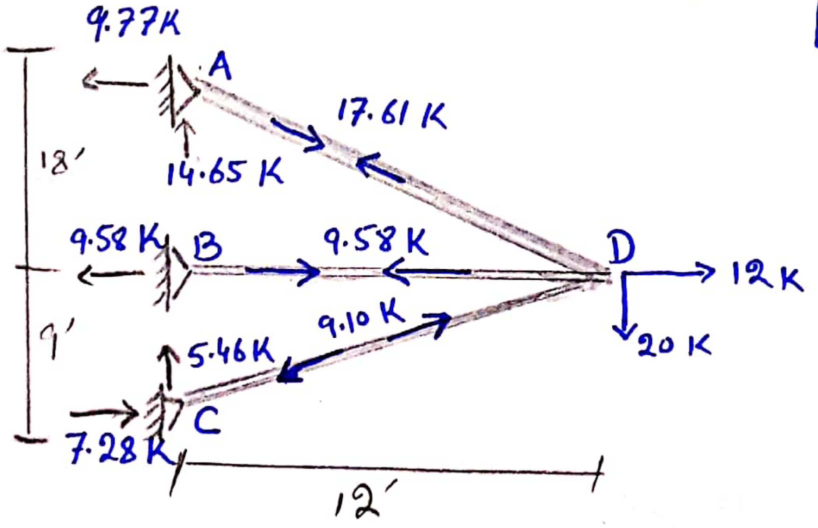
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} 0.1402 \\ -0.0107 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} 115.065 \\ 380.83 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \end{bmatrix} = \begin{bmatrix} AM_{11} & AMD_{12} \\ AM_{21} & AMD_{22} \\ AM_{31} & AMD_{32} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \end{bmatrix} = EA \begin{bmatrix} 0.0533 & -0.04 \\ 0.0833 & 0 \\ 0.0256 & 0.0385 \end{bmatrix} \begin{bmatrix} 115.065 \\ 380.83 \end{bmatrix}$$

$$\frac{1}{EA} = \begin{bmatrix} -9.10K \\ 9.58K \\ 17.61K \end{bmatrix}$$



## QN03

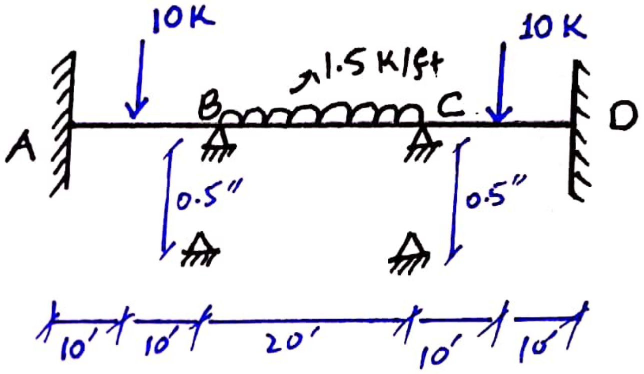
### Given data:

$\Rightarrow E = 30{,}000$ Ksi

$\Rightarrow I = 200$ $in^4$

$\Rightarrow EI = 41666.7$ $K\text{-}ft^2$

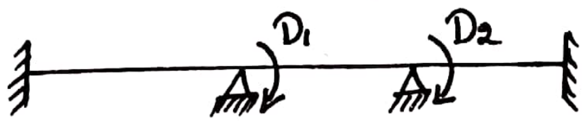
$\Rightarrow \Delta = \frac{1}{2}'' = \frac{1}{24}'$




### Solution:

#### Step # 01:

'Selection of Redundant Joint displacement

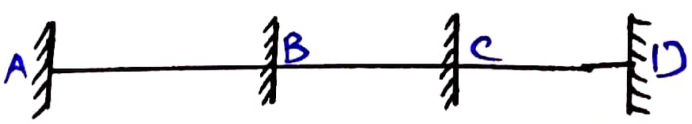



Now;

$$\Rightarrow [D] = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\Rightarrow AD = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

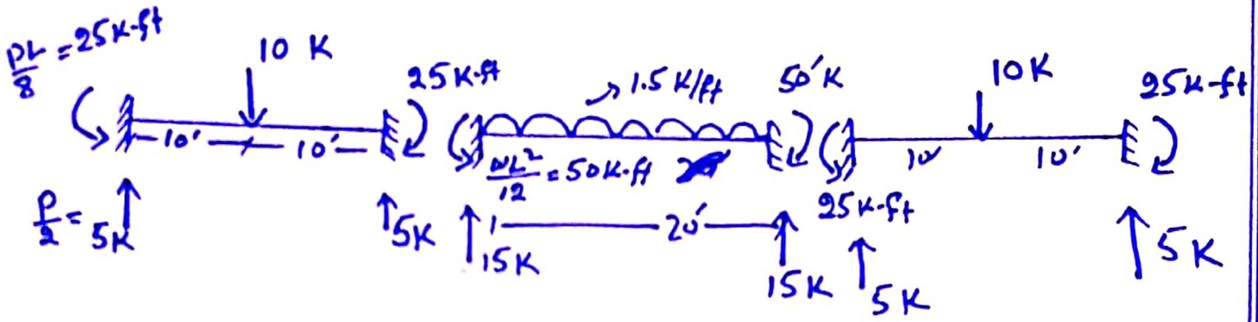
Now; Restrain all the degrees of freedom to get restrained structure.




$\Rightarrow$ RKDS or restrained structure.

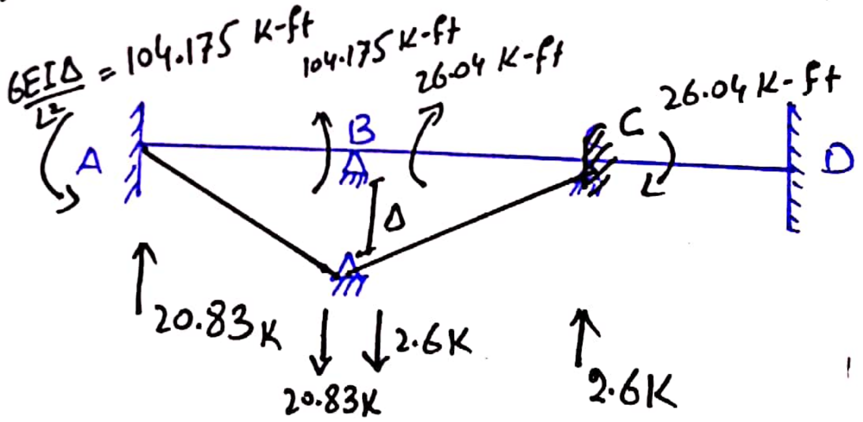
**Step No 2:** Compute ADL matrix.

(i) Due to direct Loading (ADL′)



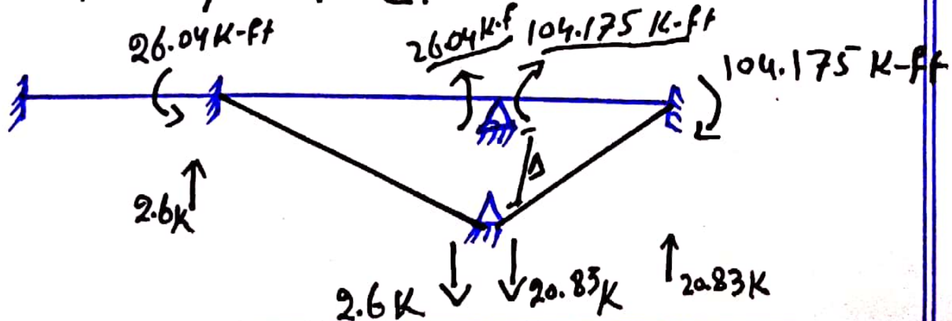

$$\Rightarrow [ADL'] = \begin{bmatrix} ADL_1' \\ ADL_2' \end{bmatrix} = \begin{bmatrix} -25 \\ 25 \end{bmatrix}$$

(ii) Due to Indirect loading i-e Due to Settlement at B.




$$\Rightarrow [ADL''] = \begin{bmatrix} ADL_1'' \\ ADL_2'' \end{bmatrix} = \begin{bmatrix} -78.135 \\ 26.04 \end{bmatrix}$$

(iii) Due to Settlement at C.



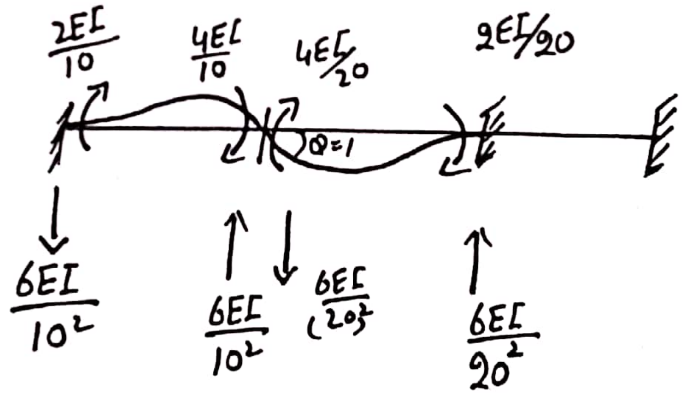
Now;
$$[ADL'''] = \begin{bmatrix} ADL_1''' \\ ADL_2''' \end{bmatrix} = \begin{bmatrix} -26.04 \\ 78.135 \end{bmatrix}$$

$$\Rightarrow [ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} ADL_1' + ADL_1'' + ADL''' \\ ADL_2' + ADL'' + ADL''' \end{bmatrix}$$

$$= \begin{bmatrix} -129.175 \\ +129.175 \end{bmatrix}$$

**Step No 3** Compute stiffness co-efficient matrix [S]

(i) when $D_1 = 1$ ; $D_2 = 0$




$$\Rightarrow S_{12} = 0.1\,EI$$

$$\Rightarrow S_{22} = 0.6\,EI$$

$$\Rightarrow [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = EI\begin{bmatrix} 0.6 & 0.1 \\ 0.1 & 0.6 \end{bmatrix}$$

**Step#4** Apply equilibrium condition at the location of the Redundant join displacement to write equilibrium equations and solve for unknown joint displacement.

$$\Rightarrow [AD] = [ADL] + [S][D]$$

oR

$$[D] = [S]^{-1} [AD - ADL]$$

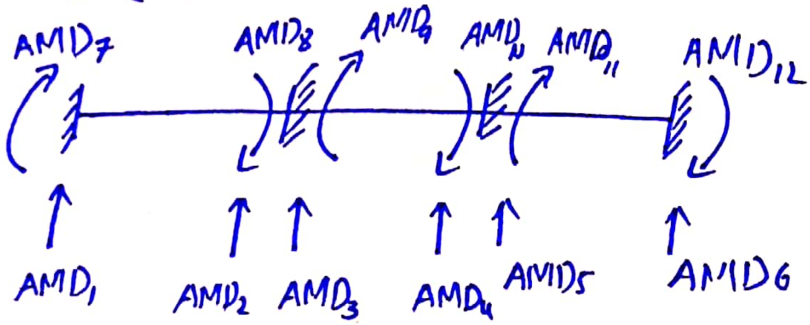
$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1.7143 & -0.2857 \\ -0.2857 & 1.7143 \end{bmatrix} \begin{bmatrix} 129.175 \\ -129.175 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 258.35 \\ -258.35 \end{bmatrix}$$

Step#5 Compute AMD values.

(i) first apply unit rotation at redundant location 1 and compute the member end action we will take these value from Step#3(i)

(ii) Then apply unit rotation at redundant location 2 and compute the member end action. we will take these values from Step #3 (ii).



$$AMD = EI\begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ \vdots & \vdots \\ AMD_{12.1} & AMD_{12.2} \end{bmatrix} = EI\begin{bmatrix} -0.06 & 0 \\ 0.06 & 0 \\ -0.015 & -0.015 \\ 0.015 & 0.015 \\ 0 & -0.06 \\ 0 & 0.06 \\ 0.2 & 0 \\ 0.4 & 0 \\ 0.2 & 0.1 \\ 0.1 & 0.2 \\ 0 & 0.4 \\ 0 & 0.2 \end{bmatrix}$$

(b) Compute AML values.

There are the member end action when the actual load is applied on the structure. we will take from step #2.

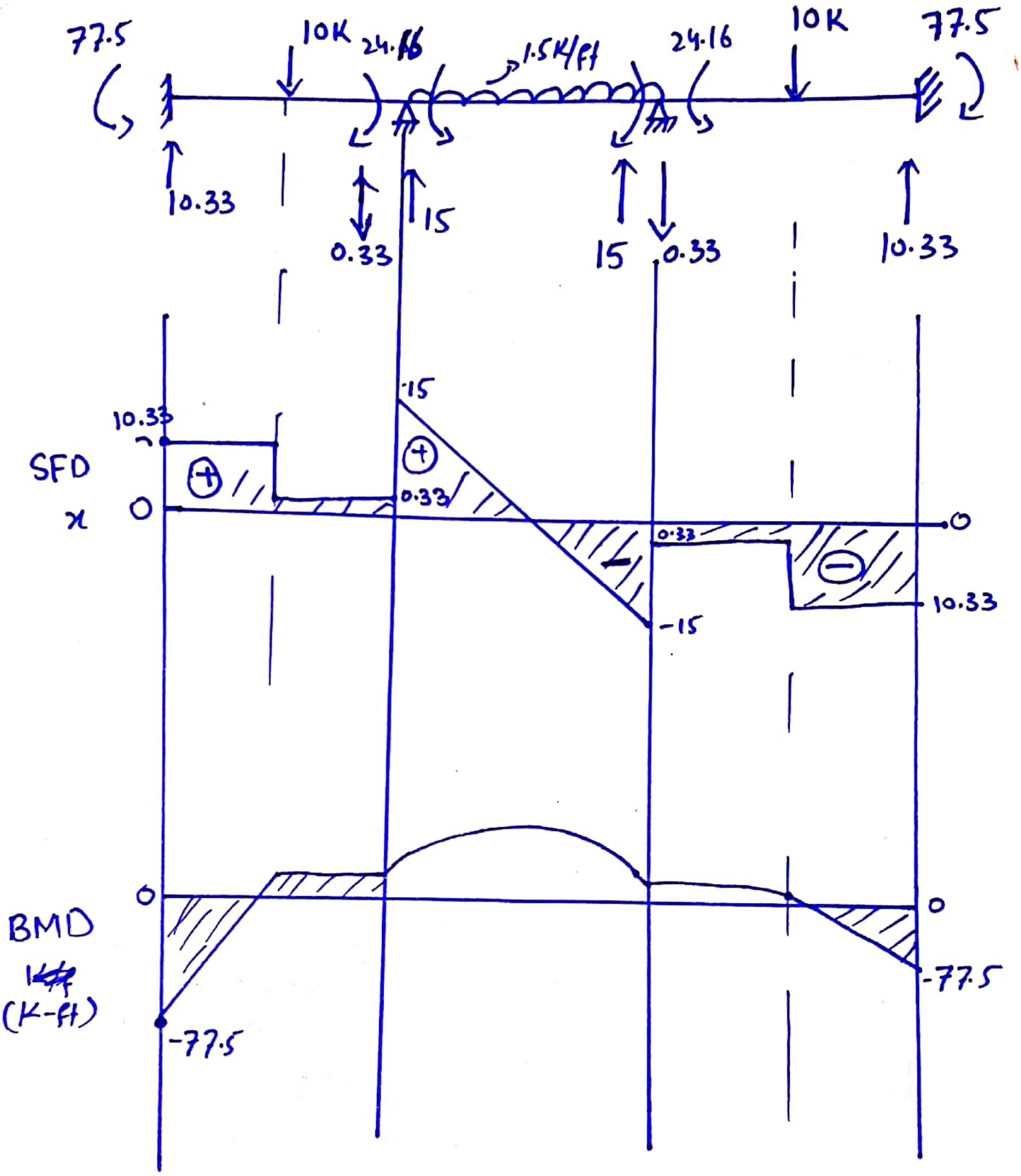
$$AML = \begin{bmatrix} AML_1 \\ \vdots \\ AML_{12} \end{bmatrix} = \begin{bmatrix} 25.83 \\ -15.83 \\ 15 \\ 15 \\ -15.83 \\ 25.83 \\ -129.175 \\ -79.175 \\ -50 \\ 50 \\ 79.175 \\ 129.175 \end{bmatrix}$$

Now member end action are Calculated as

$$[AM] = [AML] + [AMD][D]$$

$$\Rightarrow \begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \\ AM_8 \\ AM_9 \\ AM_{10} \\ AM_{11} \\ AM_{12} \end{bmatrix} = \begin{bmatrix} 25.83 \\ -15.83 \\ 15 \\ 15 \\ -15.83 \\ 25.83 \\ -129.175 \\ -79.175 \\ -50 \\ 50 \\ +79.175 \\ +129.175 \end{bmatrix} + \begin{bmatrix} -0.06 & 0 \\ 0.06 & 0 \\ -0.015 & -0.015 \\ 0.015 & 0.015 \\ 0 & -0.06 \\ 0 & 0.06 \\ 0.2 & 0 \\ 0.4 & 0 \\ 0.2 & 0.1 \\ 0.1 & 0.2 \\ 0 & 0.4 \\ 0 & 0.2 \end{bmatrix} \begin{bmatrix} 258.35 \\ -258.35 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} AM_1 \\ AM_2 \\ \vdots \\ \vdots \\ AM_{12} \end{bmatrix} = \begin{bmatrix} 10.33 \\ -0.33 \\ 15 \\ 15 \\ -0.33 \\ 10.33 \\ 77.5 \\ 24.16 \\ -24.16 \\ 24.16 \\ -24.16 \\ 77.5 \end{bmatrix}$$


77.5
10K
24.16
1.5K/ft
24.16
10K
77.5
10.33
0.33
15
15
0.33
10.33
SFD
10.33
0
x
15
0.33
0.33
-15
10.33
0
BMD
(K-ft)
0
-77.5
0
-77.5

End